

Solutions Manual  
to accompany

# **APPLIED STRENGTH OF MATERIALS**

**Fourth Edition**

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# CHAPTER 1 Basic Concepts in Strength of Materials

1-1 TO 1-15 ANSWERS IN TEXT.

1-16  $W = m \cdot g = 1800 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 17658 \text{ kgm/s}^2 = 17.7 \times 10^3 \text{ N}$   
 $W = 17.7 \text{ kN}$

1-17 TOTAL WT.  $= m \cdot g = 4000 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 39.24 \text{ kN}$   
 EACH FRONT WHEEL:  $F_F = \left(\frac{1}{2}\right)(0.40)(39.24 \text{ kN}) = 7.85 \text{ kN}$   
 EACH REAR WHEEL:  $F_R = \left(\frac{1}{2}\right)(0.60)(39.24 \text{ kN}) = 11.77 \text{ kN}$

1-18 LOADING = TOTAL FORCE / AREA  
 TOTAL FORCE  $= 6800 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 66.7 \text{ kN}$   
 AREA  $= (5.0 \text{ m})(3.5 \text{ m}) = 17.5 \text{ m}^2$   
 LOADING  $= 66.7 \text{ kN} / 17.5 \text{ m}^2 = 3.81 \text{ kN/m}^2 = 3.81 \text{ kPa}$

1-19 FORCE  $= WT = m \cdot g = 25 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 245 \text{ N}$   
 $K = \text{SPRING SCALE} = 4500 \text{ N/m} = F / \Delta L$   
 $\Delta L = \frac{F}{K} = \frac{245 \text{ N}}{4500 \text{ N/m}} = 0.0545 \text{ m} = 54.5 \times 10^{-3} \text{ m} = 54.5 \text{ mm}$

1-22  $W = 17.7 \text{ kN} = 17700 \text{ N} \times 0.2248 \text{ lb/N} = 3980 \text{ lb}$

1-23  $F_F = 7.85 \text{ kN} = 7850 \text{ N} \times 0.2248 \text{ lb/N} = 1765 \text{ lb}$   
 $F_R = 11.77 \text{ kN} = 11770 \text{ N} \times 0.2248 \text{ lb/N} = 2646 \text{ lb}$

1-24 LOADING  $= 3.81 \text{ kPa} = 3.81 \times 10^3 \frac{\text{N}}{\text{m}^2} \times \frac{0.2248 \text{ lb}}{\text{N}} \times \frac{1 \text{ m}^2}{(3.28 \text{ ft})^2} = 77.6 \frac{\text{lb}}{\text{ft}^2}$

1-25  $F = 245 \text{ N} \cdot 0.2248 \text{ lb/N} = 55.1 \text{ lb}$   
 $K = \frac{4500 \text{ N}}{\text{m}} \times \frac{0.2248 \text{ lb}}{\text{N}} \times \frac{1 \text{ m}}{39.37 \text{ in}} = 25.7 \text{ lb/in}$   
 $\Delta L = \frac{F}{K} = \frac{55.1 \text{ lb}}{25.7 \text{ lb/in}} = 2.14 \text{ in}$

$$\underline{1-26} \quad m = \frac{W}{g} = \frac{2750 \text{ LB}}{32.2 \text{ FT/S}^2} = 85.4 \frac{\text{LB} \cdot \text{S}^2}{\text{FT}} = \underline{85.4 \text{ SLUGS}}$$

$$\underline{1-27} \quad m = \frac{W}{g} = \frac{12800 \text{ LB}}{32.2 \text{ FT/S}^2} = 398 \frac{\text{LB} \cdot \text{S}^2}{\text{FT}} = \underline{398 \text{ SLUGS}}$$

$$\underline{1-29} \quad p = 1200 \text{ psi} \times 6.895 \text{ kPa/psi} = \underline{8274 \text{ kPa}}$$

$$\underline{1-30} \quad \sigma = 21600 \text{ psi} \times 6.895 \text{ kPa/psi} = 149,000 \text{ kPa} = \underline{149 \text{ MPa}}$$

$$\underline{1-31} \quad S_M = 14000 \text{ psi} \times 6.895 \text{ kPa/psi} = 96,500 \text{ kPa} = \underline{96.5 \text{ MPa}}$$

$$S_M = 76000 \text{ psi} \times 6.895 \text{ kPa/psi} = 524,000 \text{ kPa} = \underline{524 \text{ MPa}}$$

$$\underline{1-32} \quad n = 1750 \frac{\text{REV}}{\text{MIN}} \times \frac{2\pi \text{ RAD}}{\text{REV}} \times \frac{1 \text{ MIN}}{60 \text{ S}} = \underline{183 \text{ RAD/S}}$$

$$\underline{1-33} \quad A = 14.1 \text{ in}^2 \times \frac{(25.4 \text{ mm})^2}{\text{in}^2} = \underline{9097 \text{ mm}^2}$$

$$\underline{1-34} \quad h = 0.08 \text{ IN} \times 25.4 \text{ mm/IN} = \underline{2.03 \text{ mm}}$$

$$\underline{1-35} \quad \text{DIMENSIONS: } 18 \text{ IN} \times 25.4 \text{ mm/IN} = 457 \text{ mm}$$

$$12 \text{ IN} \times 25.4 \text{ mm/IN} = 305 \text{ mm}$$

$$\text{AREA} = (18 \text{ IN})^2 = \underline{324 \text{ IN}^2}$$

$$\text{AREA} = (457 \text{ mm})^2 = \underline{209 \times 10^3 \text{ mm}^2}$$

$$\text{VOLUME} = V = \text{AREA} \times \text{HEIGHT}$$

$$V = 324 \text{ IN}^2 \times 12 \text{ IN} = \underline{3888 \text{ IN}^3}$$

$$V = (1.5 \text{ FT})^2 \times 1.0 \text{ FT} = \underline{2.25 \text{ FT}^3}$$

$$V = (209 \times 10^3 \text{ mm}^2) \times 305 \text{ mm} = \underline{6.37 \times 10^7 \text{ mm}^3}$$

$$V = (0.457 \text{ m})^2 \times 0.305 \text{ m} = 0.0637 \text{ m}^3 = \underline{6.37 \times 10^{-2} \text{ m}^3}$$

$$\underline{1-36} \quad A = \pi D^2/4 = \pi (0.505 \text{ IN})^2/4 = \underline{0.200 \text{ IN}^2}$$

$$A = 0.200 \text{ IN}^2 \times \frac{(25.4 \text{ mm})^2}{\text{IN}^2} = \underline{129 \text{ mm}^2}$$

$$\underline{1-37} \quad \sigma = \frac{P}{A} = \frac{3200 \text{ N}}{\pi D^2/4} = \frac{3200 \text{ N}}{\pi (10 \text{ mm})^2/4} = 40.7 \text{ N/mm}^2 = \underline{40.7 \text{ MPa}}$$

$$\underline{1-38} \quad \sigma = \frac{P}{A} = \frac{20 \times 10^3 \text{ N}}{(40)(30) \text{ mm}^2} = 66.7 \text{ N/mm}^2 = \underline{66.7 \text{ MPa}}$$

$$\underline{1-39} \quad \sigma = \frac{P}{A} = \frac{860 \text{ LB}}{(0.40 \text{ IN})^2} = \underline{5375 \text{ psi}}$$

$$\underline{1-40} \quad \sigma = \frac{P}{A} = \frac{1850 \text{ LB}}{\pi (0.375 \text{ IN})^2/4} = \underline{16750 \text{ psi}}$$

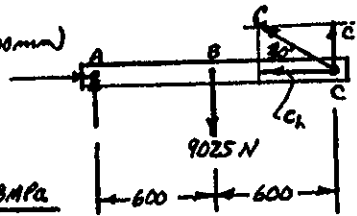
1-41  $LOAD\ ON\ SHELF = W = mg = 1810\ kg \cdot 9.81\ m/s^2 = 18050\ N$   
 $W/2 = 9025\ N\ ON\ EACH\ SIDE$

$\sum M_A = 0 = (9025\ N)(600\ mm) - C_v(1200\ mm)$

$C_v = 4512\ N$

$C = C_v / \sin 30^\circ = 9025\ N$

$\sigma = \frac{P}{A} = \frac{C}{A} = \frac{9025\ N}{\pi(12\ mm)^2/4} = 79.8\ MPa$



1-42  $\sigma = \frac{P}{A} = \frac{70000\ LB}{\pi(8\ in)^2/4} = 1393\ psi$

1-43  $\sigma = \frac{P}{A} = \frac{29500\ LB/3}{(3.5\ in)^2} = 803\ psi$

1-44  $\sigma = \frac{P}{A} = \frac{3500\ N}{(8.0\ mm)^2} = 54.7\ MPa$

1-45  $W = mg = 4200\ kg \cdot 9.81\ m/s^2 = 41.2\ kN$

$AB_x = AB \sin 35^\circ$

$AB_y = AB \cos 35^\circ$

$BC_x = BC \sin 55^\circ$

$BC_y = BC \cos 55^\circ$

$\sum F_x = 0 = AB_x - BC_x$

$0 = AB \sin 35^\circ - BC \sin 55^\circ$

$AB = BC \cdot \frac{\sin 55^\circ}{\sin 35^\circ} = 1.428\ BC$

$\sum F_y = 0 = AB_y + BC_y - 41.2\ kN = AB \cos 35^\circ + BC \cos 55^\circ - 41.2\ kN$

$0 = (1.428\ BC) \cos 35^\circ + BC \cos 55^\circ - 41.2\ kN$

$41.2\ kN = BC [1.170 + 0.574] = 1.743\ BC$

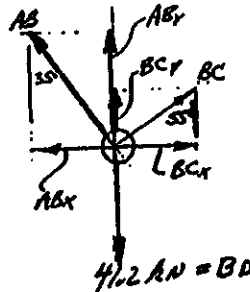
$BC = 41.2\ kN / 1.743 = 23.63\ kN$

$AB = 1.428\ BC = 33.75\ kN$

STRESS IN ROD AB:  $\sigma_{AB} = \frac{AB}{A} = \frac{33.75 \times 10^3\ N}{\pi(20\ mm)^2/4} = 107.4\ MPa$

STRESS IN ROD BC:  $\sigma_{BC} = \frac{BC}{A} = \frac{23.63 \times 10^3\ N}{\pi(20\ mm)^2/4} = 75.2\ MPa$

STRESS IN ROD BD:  $\sigma_{BD} = \frac{BD}{A} = \frac{41.2 \times 10^3\ N}{\pi(20\ mm)^2/4} = 131.1\ MPa$



1-46  $F = 0.01097 \text{ m}^2 \text{ Rm}^2 = (0.01097)(0.40)(0.60)(3000)^2 \text{ N}$   
 $F = 23695 \text{ N}$   
 $A = \pi(16 \text{ mm})^2/4 = 201 \text{ mm}^2$   
 $\sigma = \frac{F}{A} = \frac{23695 \text{ N}}{201 \text{ mm}^2} = \underline{118 \text{ MPa}}$

1-47  $A = (30 \text{ mm})^2 = 900 \text{ mm}^2$   
 FOR AB:  $F_{AB} = (110 - 40 + 80) \text{ kN} = 150 \text{ kN}$   
 $\sigma_{AB} = \frac{F_{AB}}{A} = \frac{150 \times 10^3 \text{ N}}{900 \text{ mm}^2} = \underline{167 \text{ MPa}} \text{ TENSION}$   
 FOR BC:  $F_{BC} = 110 - 40 = 70 \text{ kN}$   
 $\sigma_{BC} = \frac{F_{BC}}{A} = \frac{70 \times 10^3 \text{ N}}{900 \text{ mm}^2} = \underline{77.8 \text{ MPa}} \text{ TENSION}$   
 FOR CD:  $F_{CD} = 110 \text{ kN}$   
 $\sigma_{CD} = \frac{F_{CD}}{A} = \frac{110 \times 10^3 \text{ N}}{900 \text{ mm}^2} = \underline{122 \text{ MPa}} \text{ TENSION}$

1-48 AREAS: A-C;  $A_1 = \pi(25)^2/4 = 491 \text{ mm}^2$   
 C-D;  $A_2 = \pi(16)^2/4 = 201 \text{ mm}^2$   
 FOR AB:  $F_{AB} = -9.65 - 12.32 + 4.45 = -17.52 \text{ kN}$   
 $\sigma_{AB} = \frac{F_{AB}}{A_1} = \frac{-17.52 \times 10^3 \text{ N}}{491 \text{ mm}^2} = \underline{-35.7 \text{ MPa}} \text{ COMP.}$   
 FOR BC:  $F_{BC} = -9.65 - 12.32 = -21.97 \text{ kN}$   
 $\sigma_{BC} = \frac{F_{BC}}{A_1} = \frac{-21.97 \times 10^3 \text{ N}}{491 \text{ mm}^2} = \underline{-44.7 \text{ MPa}} \text{ COMP.}$   
 FOR CD:  $F_{CD} = -9.65 \text{ kN}$   
 $\sigma_{CD} = \frac{F_{CD}}{A_2} = \frac{-9.65 \times 10^3 \text{ N}}{201 \text{ mm}^2} = \underline{-48.0 \text{ MPa}} \text{ COMP.}$

1-49  $A = \pi[(1.90)^2 - (0.61)^2]/4 = 0.799 \text{ in}^2 \text{ (1½ in Pipe - App. A-12)}$   
 FOR BC:  $\sigma_{BC} = \frac{F_{BC}}{A} = \frac{2500 \text{ lb}}{0.799 \text{ in}^2} = \underline{3129 \text{ psi}} \text{ TENSION}$   
 FOR AB:  $F_{AB} = 2500 + 2(8000 \cos 30^\circ) = 16356 \text{ lb}$   
 $\sigma_{AB} = \frac{F_{AB}}{A} = \frac{16356 \text{ lb}}{0.799 \text{ in}^2} = \underline{20471 \text{ psi}} \text{ TENSION}$

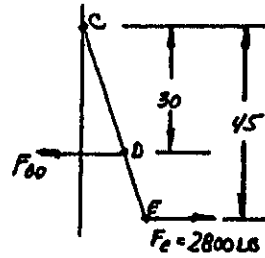
1-50

$$\sum M_c = 0 = 2800(45) - F_{BD}(30)$$

$$F_{BD} = 4200 \text{ LB}$$

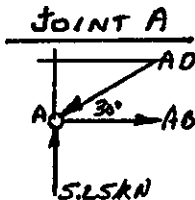
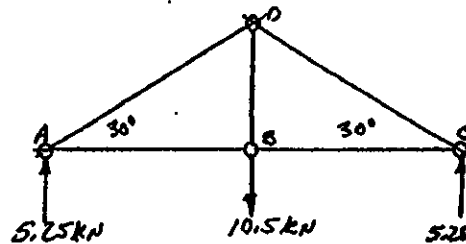
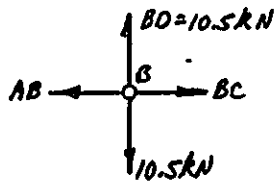
$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{4200 \text{ LB}}{(2.0)(0.65) \text{ in}^2} = 3231 \text{ psi}$$

TENSION



1-51

JOINT B



$$AD \sin 30^\circ = 5.25 \text{ kN}$$

$$AD = 10.5 \text{ kN} = CD$$

$$AB = AD \cos 30^\circ = 9.09 \text{ kN} = BC$$

STRESSES:

$$AB, BC: \sigma_{AB} = \sigma_{BC} = \frac{9.09 \times 10^3 \text{ N}}{(12)(30) \text{ mm}^2} = 25.3 \text{ MPa TENSION}$$

$$BD: \sigma_{BD} = \frac{10.5 \times 10^3 \text{ N}}{(2)(10)(30) \text{ mm}^2} = 17.5 \text{ MPa TENSION}$$

$$AD, CD: A = (30)^2 - (20)^2 = 500 \text{ mm}^2$$

$$\sigma_{AD} = \sigma_{CD} = \frac{-10.5 \times 10^3 \text{ N}}{500 \text{ mm}^2} = -21.0 \text{ MPa COMPRESSION}$$

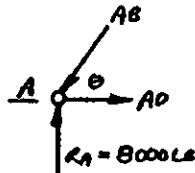
# 1-52

$$\sum M_A = 0 = 6000(6) + 12000(12) - R_F(18)$$

$$R_F = 10000 \text{ LB}$$

$$\sum M_F = 0 = 12000(6) + 6000(12) - R_A(18)$$

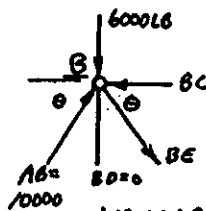
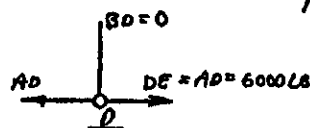
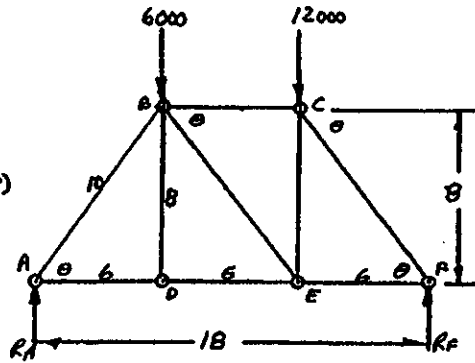
$$R_A = 8000 \text{ LB}$$



$$R_A = AB \sin \theta = AB(0.8)$$

$$AB = R_A / 0.8 = 8000 / 0.8 = 10000 \text{ LB COMP.}$$

$$AD = AB \cos \theta = 10000(0.6) = 6000 \text{ LB TENS.}$$

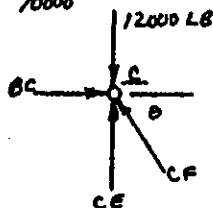


$$DE \sin \theta + 6000 - AB \sin \theta = 0$$

$$BE = \frac{AB \sin \theta - 6000}{\sin \theta} = \frac{10000(0.8) - 6000}{0.8} = 2500 \text{ LB TENS.}$$

$$BC = AB \cos \theta + BE \cos \theta = 10000(0.6) + 2500(0.6) =$$

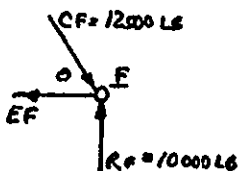
$$BC = 7500 \text{ LB COMP.}$$



$$BC = CF \cos \theta$$

$$CF = BC / \cos \theta = 7500 / 0.6 = 12500 \text{ LB COMP.}$$

$$CE = 12000 - CF \sin \theta = 12000 - 12500(0.8) = 2000 \text{ LB T}$$



$$EF = CF \cos \theta = 12500(0.6) = 7500 \text{ LB TENS.}$$

AREAS OF MEMBERS: (APP. A5, A6)

$$AD, DE, EF - 2(0.484) = 0.968 \text{ IN}^2$$

$$BD, BE, CE - 0.484 \text{ IN}^2$$

$$AB, BC, CF - 2(1.21) = 2.42 \text{ IN}^2$$

NOTE: COMPRESSION MEMBERS MUST BE CHECKED FOR COLUMN BUCKLING

## STRESSES:

$$\sigma_{AD} = \sigma_{DE} = 6000 / 0.968 = 6199 \text{ psi}$$

$$\sigma_{EF} = 7500 / 0.968 = 7748 \text{ psi}$$

$$\sigma_{BD} = 0$$

$$\sigma_{BE} = 2500 / 0.484 = 5165 \text{ psi}$$

$$\sigma_{CE} = 2000 / 0.484 = 4132 \text{ psi}$$

$$\sigma_{AB} = -10000 / 2.42 = -4132 \text{ psi}$$

$$\sigma_{BC} = -7500 / 2.42 = -3099 \text{ psi}$$

$$\sigma_{CF} = -12500 / 2.42 = -5165 \text{ psi}$$

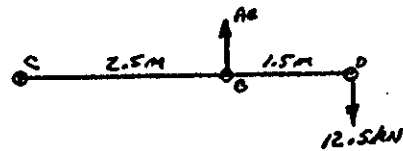


1-53

$$\sum M_C = 0 = (2.5)(4.0) - AB(2.5)$$

$$AB = 20 \text{ kN}$$

$$\sigma = \frac{20 \times 10^3 \text{ N}}{(20)^2 \text{ mm}^2} = 50 \text{ MPa}$$



1-54

$$A = \pi(0.505)^2/4 = 0.200 \text{ in}^2$$

$$\sigma = F/A = 12600 \text{ lb} / 0.200 \text{ in}^2 = 63000 \text{ psi}$$

1-55

$$A = (2.65)(1.40) + 2[(0.40)(0.5)(t)] = 4.41 \text{ in}^2$$

$$\sigma = F/A = (52000 \text{ lb}) / 4.41 \text{ in}^2 = 11791 \text{ psi}$$

1-56

$$A = (80)(40) - (60)(15) + \pi(40)^2/4 = 3557 \text{ mm}^2$$

$$\sigma = F/A = 640 \times 10^3 \text{ N} / 3557 \text{ mm}^2 = 180 \text{ MPa}$$

1-57 DIRECT SHEAR - SINGLE SHEAR

$$A_s = [\pi(12.0)^2/4] \text{ mm}^2 = 113 \text{ mm}^2$$

$$\tau = F/A_s = 16.5 \times 10^3 \text{ N} / 113 \text{ mm}^2 = 146 \text{ MPa}$$

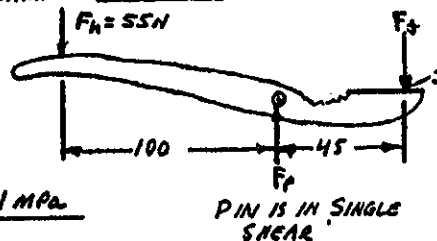
1-58

$$\sum F_y = 0 = 55(145) - F_p(45)$$

$$F_p = 177 \text{ N}$$

$$A_s = \pi(3.0)^2/4 = 7.07 \text{ mm}^2$$

$$\tau = F_p/A_s = 177 \text{ N} / 7.07 \text{ mm}^2 = 25.1 \text{ MPa}$$



1-59

$$\text{FROM PROB 1-46: } F = 23695 \text{ N}$$

$$A_s = 2[\pi(10)^2/4] = 157 \text{ mm}^2 \text{ DOUBLE SHEAR}$$

$$\tau = F/A_s = 23695 \text{ N} / 157 \text{ mm}^2 = 151 \text{ MPa}$$

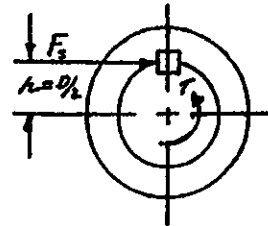
1-60  $A_s = (3.0)(3.5) = 10.5 \text{ in}^2$   
 $T = F/A_s = (800 \text{ Lb}) / (10.5 \text{ in}^2) = \underline{171 \text{ psi}}$

1-61  $A_s = [2(35) + \pi(8)](5.0) = 475.7 \text{ mm}^2$   
 $T = F/A_s = 38.6 \times 10^3 \text{ N} / 475.7 \text{ mm}^2 = \underline{81.1 \text{ MPa}}$

1-62  $L = \sqrt{.4^2 + .6^2} = 0.721 \text{ in.}$   
 $A_s = [2(1.60) + \pi(0.8)/2 + 2(0.721)] 0.194$   
 $A_s = 1.144 \text{ in}^2$   
 $T = F/A_s = 45000 \text{ Lb} / 1.144 \text{ in}^2 = \underline{39324 \text{ psi}}$



1-63  $T = F_s \cdot R$   
 $F_s = T/R = \frac{95 \text{ N} \cdot \text{m}}{35 \text{ mm}/2} = 5429 \text{ N}$   
 $A_s = b \cdot L = (10)(22) = 220 \text{ mm}^2$   
 $T = F_s/A_s = 5429 \text{ N} / 220 \text{ mm}^2 = \underline{24.7 \text{ MPa}}$



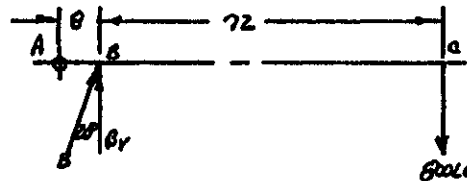
1-64  $F_s = T/R = 8000 \text{ Lb} \cdot \text{in} / 10 \text{ in} = 8000 \text{ Lb}$   
 $A_s = b \cdot L = (10.50)(2.25) = 1.125 \text{ in}^2$   
 $T = F_s/A_s = 8000 \text{ Lb} / 1.125 \text{ in}^2 = \underline{7111 \text{ psi}}$

1-65 PIN DOUBLE SHEAR;  $A_s = 2[\pi(0.5)^2/4] = 0.393 \text{ in}^2$   
 $T = F/A_s = 20000 \text{ Lb} / 0.393 \text{ in}^2 = \underline{50930 \text{ psi}}$

COLLAR SHEAR COLLAR FROM CONNECTOR BODY

$A_s = \pi d \cdot t = \pi(0.875)(0.0875) = 0.5154 \text{ in}^2$   
 $T = F/A_s = 20000 \text{ Lb} / 0.5154 \text{ in}^2 = \underline{38800 \text{ psi}}$

1-66  $\sum M_A = 0 = 800(80) - B_v(8)$   
 $B_v = 8000 \text{ Lb}$   
 $B = B_v/\cos 20^\circ = 8513 \text{ Lb}$   
 $A_s = 2[\pi(0.375)^2/4] = 0.221 \text{ in}^2$   
 $T = B/A_s = 8513 \text{ Lb} / 0.221 \text{ in}^2 = \underline{38570 \text{ psi}}$



1-67  $A_s = (40)(12) = 480 \text{ mm}^2$   
 $T = F/A_s = 88 \times 10^3 \text{ N} / 480 \text{ mm}^2 = \underline{183 \text{ MPa}}$

1-68  $A_s = (40)(120) = 4800 \text{ mm}^2$   
 $T = F/A_s = 88.2 \times 10^3 \text{ N} / 4800 \text{ mm}^2 = \underline{18.4 \text{ MPa}}$

1-69  $A_s = \pi D t = \pi (12)(8) = 301.6 \text{ mm}^2$   
 $T = F/A_s = 22.3 \times 10^3 \text{ N} / 301.6 \text{ mm}^2 = \underline{73.9 \text{ MPa}}$

1-70  $A_s = 2[\pi (12)^2 / 4] = 226.2 \text{ mm}^2$  TWO RIVETS-SINGLE SHEAR  
 $T = F/A_s = 10.2 \times 10^3 \text{ N} / 226.2 \text{ mm}^2 = \underline{45.1 \text{ MPa}}$

1-71  $A_s = 4[\pi (12)^2 / 4] = 452.4 \text{ mm}^2$  TWO RIVETS-DOUBLE SHEAR  
 $T = F/A_s = 10.2 \times 10^3 \text{ N} / 452.4 \text{ mm}^2 = \underline{22.55 \text{ MPa}}$

### BEARING STRESS

1-72 A) W6X15 ON STEEL PLATE:  $A_b = 4.43 \text{ in}^2$  (APP. A-7)

$\sigma_b = F/A_b = 26000 \text{ LB} / 4.43 \text{ in}^2 = \underline{5869 \text{ PSI}}$

B) STEEL PLATE ON CONCRETE:  $A_b = (12)^2 = 144 \text{ in}^2$

$\sigma_b = F/A_b = 26000 \text{ LB} / 144 \text{ in}^2 = \underline{181 \text{ PSI}}$

C) CONCRETE PIER ON CONCRETE FOOTING:  $A_b = (18)^2 = 324 \text{ in}^2$

$\sigma_b = F/A_b = 26000 \text{ LB} / 324 \text{ in}^2 = \underline{80.2 \text{ PSI}}$

D) CONCRETE FOOTING ON SOIL:  $A_b = (36)^2 = 1296 \text{ in}^2$

$\sigma_b = F/A_b = 26000 \text{ LB} / 1296 \text{ in}^2 = \underline{20.1 \text{ PSI}}$

1-73 a) PIPE ON FLOOR:  $A_b = \frac{\pi}{4}(2.375^2 - 2.067^2) = 1.075 \text{ in}^2$   
 $\sigma_b = F/A_b = 2350 \text{ LB} / 1.075 \text{ in}^2 = \underline{2187 \text{ PSI}}$  (APP. A-12)

b) 2.375 IN DIA. ROUND PLATE:  $A_b = 2.375^2 \left(\frac{\pi}{4}\right) = 4.43 \text{ in}^2$   
 $\sigma_b = F/A_b = 2350 \text{ LB} / 4.43 \text{ in}^2 = \underline{530 \text{ PSI}}$

1-74 a) BOLT HEAD ON WASHER:  $A_b = A_{\text{hex}} - A_{\text{sd}}$  (SEE APP. A-1)

$A_b = 0.866(0.75)^2 - \pi(0.562)^2/4 = 0.239 \text{ in}^2$

$\sigma_b = F/A_b = 385 \text{ LB} / 0.239 \text{ in}^2 = \underline{1610 \text{ PSI}}$

b) WASHER ON WOOD:  $A_b = \frac{\pi}{4}(1.375^2 - 0.562^2) = 1.237 \text{ in}^2$

$\sigma_b = F/A_b = 385 \text{ LB} / 1.237 \text{ in}^2 = \underline{311 \text{ PSI}}$

1-75 DATA FROM PROB. 1-64:  $F = 8000 \text{ LB}$ ,  $L = 2.25 \text{ IN}$ ,  $h = 0.375 \text{ IN}$ .  
 $\sigma_b = F/A_b = 8000 \text{ LB} / (2.25)(0.375/2) \text{ IN}^2 = \underline{18963 \text{ PSI}}$

1-76 DATA FROM PROB. 1-65:  $F = 20000 \text{ LB}$ , FIG. 1-38  
 a) PIN/TUBE:  $A_b = D_p(D - d) = (0.50)(1.25 - 0.875) = 0.1875 \text{ IN}^2$   
 $\sigma_b = F/A_b = 20000 \text{ LB} / 0.1875 \text{ IN}^2 = \underline{106700 \text{ PSI (VERY HIGH)}}$   
 b) COLLAR/TUBE:  $A_b = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(1.25^2 - 0.875^2) = 0.626 \text{ IN}^2$   
 $\sigma_b = F/A_b = 20000 \text{ LB} / 0.626 \text{ IN}^2 = \underline{31950 \text{ PSI}}$

1-77 FROM FIG. 1-43:  $A_b = 2(d)(t) = 2(12)(15) = 360 \text{ mm}^2$   
 $\sigma_b = F/A_b = 10.2 \times 10^3 \text{ N} / 360 \text{ mm}^2 = \underline{28.3 \text{ MPa}}$

1-78 FROM FIG. 1-44:  
 a) ON MIDDLE PART:  $A_b = 2dt = (2)(12)(15) = 360 \text{ mm}^2$   
 $\sigma_b = F/A_b = 10.2 \times 10^3 \text{ N} / 360 \text{ mm}^2 = \underline{28.3 \text{ MPa}}$   
 b) ON OUTER PARTS:  $A_b = 4dt_o = (4)(12)(10) = 480 \text{ mm}^2$   
 $\sigma_b = F/A_b = 10.2 \times 10^3 \text{ N} / 480 \text{ mm}^2 = \underline{21.25 \text{ MPa}}$

1-79 FIG. 1-47:  $A_b = (10)(6) + \frac{1}{2}(\frac{\pi}{4})(10)^2 = 99.3 \text{ mm}^2$   
 $\sigma_b = F/A_b = 535 \text{ N} / 99.3 \text{ mm}^2 = \underline{5.39 \text{ MPa}}$

## CHAPTER 2 Design Properties of Materials

ONLY THOSE PROBLEMS REQUIRING NUMERICAL DATA ARE SHOWN.

- 2-14  $S_u = 90 \text{ ksi}$  (621 MPa);  $S_y = 60 \text{ ksi}$  (414 MPa); 25% ELONG.  
BECAUSE % ELONGATION  $> 5\%$ , IT IS DUCTILE. (APP. A-13)
- 2-15 1020 HR: 36% ELONGATION - GREATER DUCTILITY  
1040 HR: 25% ELONGATION (APP. A-13)
- 2-16 AISI 1141 OQT 700: HIGH SULFUR ALLOY STEEL WITH 0.41% CARBON, QUENCHED IN OIL, TEMPERED AT 700°F. (APP. A-13)
- 2-17 YES.  $S_y = 172 \text{ ksi}$  @ OQT 700,  $S_y = 129 \text{ ksi}$  @ OQT 900  
BY INTERPOLATION  $S_y = 2150 \text{ ksi}$  @ OQT 800. (APP. A-13)
- 2-18  $E = 30 \times 10^6 \text{ psi}$  (207 GPa) FOR ALL CARBON AND ALLOY STEELS.  
(APP. A-13)
- 2-19 WT = DENSITY  $\times$  VOLUME =  $(0.283 \text{ lb/in}^3)(1.0)(4.0)(14.5) \text{ in}^3 = 16.4 \text{ lb}$   
(APP. A-13) VALUE OF  $\text{lb}_m = \text{VALUE OF LB FORCE (WT.)}$
- 2-20 VOLUME = AREA  $\times$  LENGTH =  $\frac{\pi}{4}(50)^2 \times 250 = 4.909 \times 10^5 \text{ mm}^3$   
STEEL BAK  
MASS =  $\frac{7680 \text{ kg}}{\text{m}^3} \times \frac{4.909 \times 10^5 \text{ mm}^3}{1} \times \frac{1 \text{ m}^3}{(10^3 \text{ mm})^3} = 3.77 \text{ kg}$   
(APP. A-13)  
WT =  $m \cdot g = 3.77 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 36.98 \text{ kg} \cdot \text{m/s}^2 = 36.98 \text{ N}$
- 2-21 MAGNESIUM WOULD BECAUSE IT HAS A LOWER  $E$ .  
 $E_{\text{Mg}} = 45 \text{ GPa}$ ;  $E_{\text{Ti}} = 114 \text{ GPa}$ ; Ti IS STIFFER. (APP. A-14)
- 2-22 ALLOY OF ALUMINUM WITH SILICON AND MAGNESIUM.  
HEAT TREATED TO T6 TEMPER.
- 2-23
- |         | $S_u$  | $S_y$  | $E$                          | DENSITY                 | (APP. A-17) |
|---------|--------|--------|------------------------------|-------------------------|-------------|
| 6061-O  | 18 ksi | 8 ksi  | $10 \times 10^6 \text{ psi}$ | 0.10 lb/in <sup>3</sup> |             |
| 6061-T4 | 35 ksi | 21 ksi | "                            | "                       |             |
| 6061-T6 | 45 ksi | 40 ksi | "                            | "                       |             |
- 2-29  $S_{uc} = 40 \text{ ksi}$ ;  $S_{uc} = 140 \text{ ksi}$  (APP. A-16)
- 2-31 BENDING  $\sigma_b = 1450 \text{ psi}$ ; TENSION  $\sigma_t = 850 \text{ psi}$ ; COMP. 1000 psi PARALLEL TO GRAIN, 385 psi PERPENDICULAR TO GRAIN; SHEAR  $\tau_s = 95 \text{ psi}$   
(APP. A-18)
- 2-32 2000 TO 7000 psi (SECTION 2-10)

2-44 Graphite fibers.

2-45 S-glass, quartz fibers, tungsten fibers coated with silicon carbide.

2-51	<i>Material</i>	<i>Specific strength (in)</i>	<i>Ratio to AISI 1020</i>
	Graphite/Epoxy (High Strength)	$4.86 \times 10^6$	25.0
	Aramid/Epoxy Composite	$4.00 \times 10^6$	20.6
	Boron/Epoxy Composite	$3.60 \times 10^6$	18.5
	Graphite/Epoxy (Ultra-hi mod)	$2.76 \times 10^6$	14.2
	Glass/Epoxy Composite	$1.87 \times 10^6$	9.63
	Titanium Ti-6Al-4V	$1.00 \times 10^6$	5.15
	AISI 5160 OQT 700 Steel	$0.929 \times 10^6$	4.78
	Aluminum 7075-T6	$0.822 \times 10^6$	4.23
	Aluminum 6061-T6	$0.459 \times 10^6$	2.36
	AISI 1020 HR Steel	$0.194 \times 10^6$	1.00

2-52	<i>Material</i>	<i>Specific modulus (in)</i>	<i>Ratio to AISI 1020</i>
	Graphite/Epoxy (Ultra-hi mod)	$8.28 \times 10^8$	7.81
	Boron/Epoxy Composite	$4.00 \times 10^8$	3.77
	Graphite/Epoxy (High Strength)	$3.45 \times 10^8$	3.25
	Aramid/Epoxy Composite	$2.20 \times 10^8$	2.07
	AISI 1020 HR Steel	$1.06 \times 10^8$	1.00
	AISI 5160 OQT 700 Steel	$1.06 \times 10^8$	1.00
	Titanium Ti-6Al-4V	$1.03 \times 10^8$	0.97
	Aluminum 6061-T6	$1.02 \times 10^8$	0.96
	Aluminum 7075-T6	$0.99 \times 10^8$	0.93
	Glass/Epoxy Composite	$0.66 \times 10^8$	0.62

2-60  $V_m = 1 - V_f = 1.0 - 0.60 = 0.40$

2-61 See Equation (2-5).

2-62 See Equations (2-6), (2-7), (2-8), (2-9).

2-63 Given:  $V_f = 0.50$ ; Fibers are high strength carbon-PAN; Matrix is Epoxy

See Table 2-9 for data.  $V_m = 1 - V_f = 1 - 0.50 = 0.50$

Use Equation (2-5):  $s_{uc} = s_{uf} V_f + \sigma_m' V_m$

Strain at which fibers would fail:  $\epsilon_f = s_{uf} / E_f = (820 \times 10^3 \text{ psi}) / (40 \times 10^6 \text{ psi})$

$$\epsilon_f = 0.0205$$

Stress in matrix at this strain:  $\sigma_m' = E_m \epsilon = (0.56 \times 10^6 \text{ psi})(0.0205) = 11\,480 \text{ psi}$

Then:  $s_{uc} = (820 \times 10^3 \text{ psi})(0.50) + (11\,480 \text{ psi})(0.50) = \underline{415 \times 10^3 \text{ psi}}$

Modulus of elasticity:  $E_c = E_f V_f + E_m V_m = (40 \times 10^6)(0.5) + (0.56 \times 10^6)(0.50)$

$$\underline{E_c = 20.3 \times 10^6 \text{ psi}}$$

Specific weight:  $\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.065)(0.50) + (0.047)(0.50)$

$$\underline{\gamma_c = 0.056 \text{ lb/in}^3}$$

2-64 Given:  $V_f = 0.50$ ; Fibers are high modulus carbon; Matrix is Epoxy

See Table 2-9 for data.  $V_m = 1 - V_f = 1 - 0.50 = 0.50$

Use Equation (2-5):  $s_{uc} = s_{uf} V_f + \sigma_m' V_m$

Strain at which fibers would fail:  $\epsilon_f = s_{uf} / E_f = (325 \times 10^3 \text{ psi}) / (100 \times 10^6 \text{ psi})$

$$\epsilon_f = 0.00325$$

Stress in matrix at this strain:  $\sigma_m' = E_m \epsilon = (0.56 \times 10^6 \text{ psi})(0.00325) = 1820 \text{ psi}$

Then:  $s_{uc} = (325 \times 10^3 \text{ psi})(0.50) + (1820 \text{ psi})(0.50) = \underline{163 \times 10^3 \text{ psi}}$

Modulus of elasticity:  $E_c = E_f V_f + E_m V_m = (100 \times 10^6)(0.5) + (0.56 \times 10^6)(0.50)$

$$\underline{E_c = 50.3 \times 10^6 \text{ psi}}$$

Specific weight:  $\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.078)(0.50) + (0.047)(0.50)$

$$\underline{\gamma_c = 0.0625 \text{ lb/in}^3}$$

2-65 Given:  $V_f = 0.50$ ; Fibers are aramid; Matrix is Epoxy

See Table 2-9 for data.  $V_m = 1 - V_f = 1 - 0.50 = 0.50$

Use Equation (2-5):  $s_{uc} = s_{uf} V_f + \sigma_m' V_m$

Strain at which fibers would fail:  $\epsilon_f = s_{uf} / E_f = (500 \times 10^3 \text{ psi}) / (19 \times 10^6 \text{ psi})$

$$\epsilon_f = 0.0263$$

Stress in matrix at this strain:  $\sigma_m' = E_m \epsilon = (0.56 \times 10^6 \text{ psi})(0.0263) = 14\,740 \text{ psi}$

Then:  $s_{uc} = (500 \times 10^3 \text{ psi})(0.50) + (14\,740 \text{ psi})(0.50) = \underline{257 \times 10^3 \text{ psi}}$

Modulus of elasticity:  $E_c = E_f V_f + E_m V_m = (19 \times 10^6)(0.5) + (0.56 \times 10^6)(0.50)$

$$\underline{E_c = 9.78 \times 10^6 \text{ psi}}$$

Specific weight:  $\gamma_c = \gamma_f V_f + \gamma_m V_m = (0.052)(0.50) + (0.047)(0.50)$

$$\underline{\gamma_c = 0.0495 \text{ lb/in}^3}$$

**Solutions to Problems 2-66 to 2-67:** Some data approximated from Figure P2-66.  
Most accurate values are for Ultimate strength (b.) and % elongation (f).  
Elastic limit (d.) estimated between proportional limit (c.) and yield strength (a.)  
Modulus of elasticity (e.) computed from ( $\Delta$  stress /  $\Delta$  strain). Data are approximated  
Materials found from Appendixes A-13 through A-17 matching  $s_u$ ,  $s_y$ , % Elongation, and E

- 2-66**
- a.  $s_y = 73$  ksi - Offset
  - b.  $s_u = 83$  ksi
  - c.  $s_p = 60$  ksi
  - d.  $s_{el} = 67$  ksi
  - e.  $E = 10.0 \times 10^6$  psi
  - f. 11% Elongation
  - g. Ductile
  - h. Aluminum
  - i. 7075-T6

- 2-67**
- a.  $s_y = 173$  ksi Yield point
  - b.  $s_u = 187$  ksi
  - c.  $s_p = 162$  ksi
  - d.  $s_{el} = 168$  ksi
  - e.  $E = 29.0 \times 10^6$  psi
  - f. 15% Elongation
  - g. Ductile
  - h. Steel
  - i. AISI 4140 OQT 900

- 2-68**
- a.  $s_y = 62$  ksi Offset
  - b.  $s_u = 75$  ksi
  - c.  $s_p = 50$  ksi
  - d.  $s_{el} = 56$  ksi
  - e.  $E = 16.7 \times 10^6$  psi
  - f. 15% Elongation
  - g. Ductile
  - h. Copper Alloy
  - i. C54400 Bronze-hard

- 2-69**
- a.  $s_y = 49$  ksi - Yield point
  - b.  $s_u = 65$  ksi
  - c.  $s_p = 46$  ksi
  - d.  $s_{el} = 48$  ksi
  - e.  $E = 26.5 \times 10^6$  psi
  - f. 36% Elongation
  - g. Ductile
  - h. Steel
  - i. AISI 1020 CD

- 2-70**
- a. No  $s_y$  - Brittle
  - b.  $s_u = 55$  ksi
  - c.  $s_p = 50$  ksi
  - d.  $s_{el} = 53$  ksi
  - e.  $E = 20.0 \times 10^6$  psi
  - f. 0.5% Elongation
  - g. Brittle
  - h. Cast Iron
  - i. ASTM A48 Grade 60

- 2-71**
- a.  $s_y = 53$  ksi - Offset
  - b.  $s_u = 59$  ksi
  - c.  $s_p = 31$  ksi
  - d.  $s_{el} = 42$  ksi
  - e.  $E = 12.0 \times 10^6$  psi
  - f. 5.0% Elongation
  - g. Borderline Brittle/Ductile
  - h. Zinc
  - i. Cast ZA-12

- 2-72**
- a.  $s_y = 35$  ksi - Yield point
  - b.  $s_u = 57$  ksi
  - c.  $s_p = 30$  ksi
  - d.  $s_{el} = 27$  ksi
  - e.  $E = 26 \times 10^6$  psi
  - f. 21% Elongation
  - g. Ductile
  - h. Structural Steel
  - i. ASTM A36

- 2-73**
- a.  $s_y = 19$  ksi - Offset
  - b.  $s_u = 40$  ksi
  - c.  $s_p = 14$  ksi
  - d.  $s_{el} = 17$  ksi
  - e.  $E = 6 \times 10^6$  psi
  - f. 5% Elongation
  - g. Borderline Brittle/Ductile
  - h. Magnesium
  - i. ASTM AZ 63A-T6



- 2-74**
- a.  $s_y = 155$  ksi - Offset
  - b.  $s_u = 170$  ksi
  - c.  $s_p = 142$  ksi
  - d.  $s_{el} = 149$  ksi
  - e.  $E = 16.5 \times 10^6$  psi
  - f. 8% Elongation
  - g. Ductile
  - h. Titanium
  - i. 6Al-4V

- 2-76**
- a.  $s_y = 80$  ksi - Offset
  - b.  $s_u = 90$  ksi
  - c.  $s_p = 62$  ksi
  - d.  $s_{el} = 71$  ksi
  - e.  $E = 26 \times 10^6$  psi
  - f. 15% Elongation
  - g. Ductile
  - h. Stainless Steel
  - i. AISI 430 full hard

- 2-75**
- a.  $s_y = 40$  ksi - Offset
  - b.  $s_u = 45$  ksi
  - c.  $s_p = 30$  ksi
  - d.  $s_{el} = 35$  ksi
  - e.  $E = 10.0 \times 10^6$  psi
  - f. 17% Elongation
  - g. Ductile
  - h. Aluminum
  - i. 6061-T6

- 2-77**
- a.  $s_y = 80$  ksi - Offset
  - b.  $s_u = 95$  ksi
  - c.  $s_p = 55$  ksi
  - d.  $s_{el} = 68$  ksi
  - e.  $E = 26 \times 10^6$  psi
  - f. 2.0% Elongation
  - g. Brittle, but does yield
  - h. Malleable Iron
  - i. ASTM A220 Grade 80002

## CHAPTER 3 Design of Members Under Direct Stresses

3-1  $\sigma = P/A = \frac{8.50 \times 10^3 \text{ N}}{\pi (10 \text{ mm})^2 / 4} = 108 \text{ MPa} = \sigma_0 = S_y / 2$

REQ'D  $S_y = 2 \sigma_0 = 2 (108 \text{ MPa}) = \underline{216 \text{ MPa}}$

ALUMINUM 2014-T4 HAS  $S_y = 290 \text{ MPa}$  (APPENDIX A-17)

3-2  $\sigma = P/A = \frac{20,000 \text{ N}}{(10)(30) \text{ mm}^2} = 66.7 \text{ MPa} = \sigma_0 = S_u / 8$

REQ'D  $S_u = 8 \sigma_0 = 8 (66.7 \text{ MPa}) = \underline{533 \text{ MPa}}$  PLUS GOOD DUCTILITY

AISI 1141 ANNEALED HAS  $S_u = 600 \text{ MPa}$ ; 26% ELONGATION, (A-13)

3-3  $\sigma = P/A = \frac{1720 \text{ LB}}{(0.40 \text{ in})^2} = 10750 \text{ PSI} = \sigma_0 = S_u / 8$

REQ'D  $S_u = 8 \sigma_0 = 8 (10750) = \underline{86000 \text{ PSI}}$  PLUS GOOD DUCTILITY

AISI 1040 WQT B00 HAS  $S_u = 87 \text{ ksi}$ ; 32% ELONGATION (A-13)

3-4  $\sigma = P/A = \frac{1850 \text{ LB}}{\pi (0.375 \text{ in})^2 / 4} = 16750 \text{ psi} = \sigma_0 = 0.60 S_y$  (AISC)

REQ'D  $S_y = \sigma_0 / 0.60 = 16750 \text{ psi} / 0.60 = \underline{27900 \text{ psi}}$

ASTM A36 STRUCTURAL STEEL HAS  $S_y = 36000 \text{ psi}$  (A-15)

3-5  $\sigma = P/A = \frac{5200 \text{ LB}}{5.25 \text{ in}^2} = 990 \text{ psi}$  TOO HIGH;  $\sigma_{\text{ALLOW}} = 575 \text{ psi}$  (A-18)

3-6 a) NO. 1 GRADE DOUGLAS FIR HAS  $\sigma_{\text{ALLOW}} = 1050 \text{ psi}$  (A-18)

b) TO USE NO. 2 GRADE SOUTHERN PINE:  $\sigma_{\text{ALLOW}} = 575 \text{ psi}$

REQ'D AREA =  $\frac{P}{\sigma} = \frac{5200 \text{ LB}}{575 \text{ LB/in}^2} = 9.04 \text{ in}^2$ ; USE 2x8 OR 4x4  
OR TWO 2x4.

3-7  $\sigma = P/A$ ;  $A = \frac{P}{\sigma} = \frac{6400 \text{ LB}}{12000 \text{ LB/in}^2} = 0.533 \text{ in}^2 = \pi D^2 / 4$

REQ'D  $D = \sqrt{4A/\pi} = \sqrt{4(0.533 \text{ in}^2)/\pi} = \underline{0.824 \text{ in}}$

SPECIFY  $7/8 \text{ in}$  (0.875 in) OR 1.00 in. DIA.

3-8 TOTAL MASS =  $1150 + 6350 = 7500 \text{ kg}$ ; 1875 kg ON EACH STRAP.

$F = m \cdot g = 1875 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 18394 \text{ N}$

REQ'D  $A = \frac{P}{\sigma_0} = \frac{18394 \text{ N}}{70 \text{ N/mm}^2} = 263 \text{ mm}^2 = (w)(8 \text{ mm})$

REQ'D  $w = 263 \text{ mm}^2 / 8 \text{ mm} = \underline{32.8 \text{ mm}}$

SPECIFY  $w = 35.0 \text{ mm}$

3-9

FORCE ON SHELF =  $1840 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 18050 \text{ N} = 9025 \text{ N/SIDE}$

$$C_V = A_V = 9025 \text{ N} / 2 = 4513 \text{ N}$$

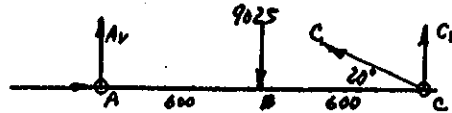
$$C = C_V / \sin 20^\circ = 13194 \text{ N}$$

C = FORCE IN ROD

$$REQD A = \frac{C}{\sigma_s} = \frac{13194 \text{ N}}{110 \text{ N/mm}^2} = 120 \text{ mm}^2 = \pi D^2 / 4$$

$$REQD D = \sqrt{4A/\pi} = \sqrt{4(120 \text{ mm}^2)/\pi} = 12.4 \text{ mm}$$

SPECIFY  $D = 14.0 \text{ mm}$



3-10

$$\sigma = \frac{P}{A} = \frac{70000 \text{ LB}}{\pi (8.0 \text{ IN})^2 / 4} = 1393 \text{ PSI} = \frac{1}{4} \times \text{RATED STRENGTH (SEC. 2-10)}$$

$$REQD \text{ RATED STRENGTH} = 4(1393) = 5570 \text{ PSI}$$

SPECIFY 6000 PSI RATED STRENGTH

3-11

LOAD ON EACH BLOCK =  $29500 \text{ LB} / 3 = 9833 \text{ LB}$

$$\sigma = \frac{P}{A} = \frac{9833 \text{ LB}}{12.25 \text{ IN}^2} = 803 \text{ PSI}$$

IF COMPRESSION IS PERPENDICULAR TO GRAIN, NO SUITABLE WOOD LISTED.

IF PARALLEL TO GRAIN: NO. 1 SOUTH. PINE -  $\sigma_{ALL} = 850 \text{ PSI}$  (A-18)

NO. 2 HEMLOCK -  $\sigma_{ALL} = 800 \text{ PSI}$

NO. 2 DOUGLAS FIR -  $\sigma_{ALL} = 1000 \text{ PSI}$

3-12

$$\sigma_{ALLN.} = \frac{\text{RATED STRENGTH}}{4} = \frac{20.7 \text{ MPa}}{4} = 5.18 \text{ MPa (SEC. 2-10)}$$

$$REQD A = \frac{P}{\sigma} = \frac{1.50 \times 10^6 \text{ N}}{5.18 \text{ N/mm}^2} = 2.90 \times 10^5 \text{ mm}^2 = \pi D^2 / 4$$

$$REQD D = \sqrt{4A/\pi} = 607 \text{ mm}; \text{ SPECIFY } \underline{700 \text{ mm DIA.}}$$

3-13

$$S_u = 483 \text{ MPa} = \sigma = P/A; P = \sigma \cdot A \quad (\text{A-17})$$

$$A = \pi (12^2 - 10^2) / 4 = 34.56 \text{ mm}^2$$

$$P = \sigma \cdot A = 483 \frac{\text{N}}{\text{mm}^2} \cdot 34.56 \text{ mm}^2 = 16.7 \times 10^3 \text{ N} = \underline{16.7 \text{ kN}}$$

3-14

$$A = (40 \text{ mm})^2 = 1600 \text{ mm}^2; \sigma_{ALLN.} = 1.69 \text{ MPa } \perp \text{ GRAIN (A-18)}$$

$$\sigma_{ALLN.} = 5.52 \text{ MPa } \parallel \text{ GRAIN (\#2 HEMLOCK)}$$

$$P = \sigma \cdot A = (1.69 \text{ N/mm}^2)(1600 \text{ mm}^2) = 2.70 \text{ kN} - \perp \text{ GRAIN}$$

$$P = \sigma \cdot A = (5.52 \text{ N/mm}^2)(1600 \text{ mm}^2) = 8.83 \text{ kN} - \parallel \text{ GRAIN}$$

3-15

$$\sigma_s = 0.60 S_y = 0.60 (50 \text{ ksi}) = 30.0 \text{ ksi} = P/A \quad (\text{AISC}) \quad (\text{A-15}) \quad S_y = 50 \text{ ksi}$$

$$REQD A = \frac{P}{\sigma_s} = \frac{4000 \text{ LB}}{30.000 \text{ LB/IN}^2} = 0.133 \text{ IN}^2 = \pi D^2 / 4 \quad \text{IF } D < 0.75 \text{ IN}$$

$$REQD D = \sqrt{4A/\pi} = \sqrt{4(0.133 \text{ IN}^2)/\pi} = 0.412 \text{ IN}$$

SPECIFY  $D = \underline{7/16 \text{ IN. (0.4375 IN.)}}$  OR  $\underline{0.500 \text{ IN.}}$  (OK -  $D < 0.75 \text{ IN.}$ )

3-16  $A = (2.65)(1.40) + 2 \left[ \frac{1}{2} (1.40)(0.5) \right] = 4.41 \text{ in}^2$   
 $\sigma = P/A = \frac{52000 \text{ LB}}{4.41 \text{ in}^2} = 11791 \text{ PSI} = \sigma_B = S_{uc}/N$   
 $N = \frac{S_{uc}}{\sigma} = \frac{80000 \text{ PSI}}{11791 \text{ PSI}} = 6.78$

3-17 FOR SHOCK LOADING - DUCTILE METAL:  $\sigma_B = S_{uc}/12$   
 $\sigma_B = 1650 \text{ MPa}/12 = 137.5 \text{ MPa} \quad (\text{A-16})$   
 $\text{REQ'D } A = \frac{P}{\sigma_B} = \frac{135 \times 10^3 \text{ N}}{137.5 \text{ N/mm}^2} = 982 \text{ mm}^2 = BH = B(28) = 2B^2$   
 $\text{REQ'D } B = \sqrt{A/2} = \sqrt{982/2} = 22.2 \text{ mm}; H = 44.4 \text{ mm}$   
SPECIFY  $B = 25.0 \text{ mm}; H = 50.0 \text{ mm}$

3-18  $\sigma_B = T.S./8 = 9000 \text{ PSI}/8 = 1125 \text{ PSI} = P/A \quad (\text{A-19})$   
 $\text{REQ'D } A = \frac{P}{\sigma_B} = \frac{110 \text{ LB}}{1125 \text{ LB/in}^2} = 0.0978 \text{ in}^2 = (0.20)(w)$   
 $\text{REQ'D } w = A/0.20 = 0.0978 \text{ in}^2/0.20 \text{ in} = 0.489 \text{ in}$   
SPECIFY  $w = 0.500 \text{ in}$

3-19  $A = (80)(40) - (60)(15) + \pi(40)^2/4 = 3557 \text{ mm}^2$   
 $\sigma = \frac{P}{A} = \frac{640 \times 10^3 \text{ N}}{3557 \text{ mm}^2} = 180 \text{ MPa} = \sigma_B$   
 FOR DUCTILE METALS:  $\sigma_B = S_y/2$   
 $\text{REQ'D } S_y = 2(180) = 360 \text{ MPa}$   
 POSSIBLE METALS: AISI 1040 HR,  $S_y = 414 \text{ MPa} \quad (\text{A-13})$   
 AISI 4140 ANNEALED,  $S_y = 414 \text{ MPa} \quad (\text{A-13})$   
 SOFT- C54400 BRASS,  $S_y = 393 \text{ MPa} \quad (\text{A-14})$   
 ALUMINUM 2014-T6,  $S_y = 444 \text{ MPa} \quad (\text{A-17})$   
 AISI 1020 CD,  $S_y = 441 \text{ MPa} \quad (\text{A-13})$

3-20 SEE PROB. 1-48:  $\sigma_{\text{MAX}} = -48.0 \text{ MPa}$  COMPRESSION  
 $\sigma_B = 0.6 S_y = 0.6(248 \text{ MPa}) = 149 \text{ MPa} \quad (\text{AISC}) \quad (\text{A-15})$   
OK FOR COMPRESSIVE STRESS.  
COLUMN BUCKLING SHOULD BE CHECKED.

3-21  $\sigma = 50 \text{ MPa}$  IN MEMBER AB - SEE PROB 1-53.

$$\sigma_a = S_u / A; \text{REQ'D } S_u = 8\sigma = 8(50) = 400 \text{ MPa}$$

ALUMINUM 2014-TY,  $S_u = 427 \text{ MPa}$ ; 20% ELONG. (A-17)

3-22  $A = 0.938 \text{ in}^2$ ;  $\sigma_a = 0.60 S_y = 0.60(36000 \text{ psi}) = 21600 \text{ psi}$

$$P_{\text{allow.}} = \sigma_a \cdot A = (21600 \text{ lb/in}^2)(0.938 \text{ in}^2) = \underline{20260 \text{ lb}}$$

### BEARING STRESS

3-23 FORCE =  $20 \text{ kN} = 425 \text{ kN} (4.0 \text{ m} / 2.5 \text{ m})$

a) TENSION IN MEMBER 1 AT PIN HOLES:

$$A_b = 2[(20-12)(14)] = 224 \text{ mm}^2$$

$$\sigma = \frac{P}{A} = \frac{20 \times 10^3 \text{ N}}{224 \text{ mm}^2} = 89.3 \text{ MPa} \text{ TOO HIGH}$$

$$\text{FOR 6061-TY: } \sigma_a = S_y / 2 = 145 \text{ MPa} / 2 = 72.5 \text{ MPa} \quad (\text{A-17})$$

b) BEARING AT THE PIN - MEMBER 1

$$\sigma_b = \frac{P}{A_b} = \frac{20 \times 10^3 \text{ N}}{(12)(14)(2) \text{ mm}^2} = 59.5 \text{ MPa}$$

$$\text{FOR 6061-TY; } \sigma_{bd} = 0.65 S_y = 0.65(145 \text{ MPa}) = 94.3 \text{ MPa} \text{ OK}$$

c) BEARING AT THE PIN - MEMBER 2

$$\sigma_b = \frac{P}{A_b} = \frac{20 \times 10^3 \text{ N}}{(12)(20) \text{ mm}^2} = 83.3 \text{ MPa}$$

$$\text{FOR 2014-TY; } \sigma_{bd} = 0.65 S_y = 0.65(290 \text{ MPa}) = 189 \text{ MPa} \text{ OK}$$

d) PIN IN BEARING: - MEMBER 2 (EQ. 3-12)

$$\sigma_{bmn} = 83.3 \text{ MPa} \text{ AT MEMBER 2}$$

$$\text{FOR 2014-T6; } \sigma_{bd} = 0.65 S_y = 0.65(414 \text{ MPa}) = 269 \text{ MPa} \text{ OK}$$

e) PIN IN SHEAR - DOUBLE SHEAR

$$A_s = \frac{\pi}{4} (12)^2 (2) = 226 \text{ mm}^2$$

$$\tau = P / A_s = 20 \times 10^3 \text{ N} / 226 \text{ mm}^2 = 88.4 \text{ MPa}$$

$$\text{FOR 2014-T6; } \tau_d = \frac{0.55 S_y}{2} = \frac{0.55(414 \text{ MPa})}{2} = 103 \text{ MPa} \text{ OK}$$

3-24  $W = 90 \text{ kN}$  TOTAL;  $45 \text{ kN}$  ON TWO LEGS;  $22.5 \text{ kN}$  ON EACH LEG

a) STEEL PLATE:  $\sigma_b = \frac{22.5 \times 10^3 \text{ N}}{(0.10 \text{ m})^2} = 2.25 \text{ MPa} \quad (\text{EQ. 3-8})$

$$\text{FOR A36 STEEL: } \sigma_{bd} = 0.9 S_y = 0.9(248 \text{ MPa}) = 223 \text{ MPa} \text{ OK}$$

b) TOP OF CONCRETE:  $A_b = 2(0.20 \text{ m})^2 = 0.08 \text{ m}^2$  (TWO LEGS)

$$\sigma_b = \frac{45 \times 10^3 \text{ N}}{0.08 \text{ m}^2} = 0.563 \text{ MPa}$$

$$\text{CONCRETE: } 2000 \text{ psi} = 2.0 \text{ ksi} \times 6.895 \text{ MPa/ksi} = 13.79 \text{ MPa} = \sigma_c$$

$$(\text{TABLE 3-6}) - \sigma_b = 0.35 \sigma_c \left[ \frac{A_c}{A_s} \right] = 0.35(13.79 \text{ MPa}) \left[ \frac{0.30 \text{ m}^2}{0.08 \text{ m}^2} \right] = 9.35 \text{ MPa} \text{ OK}$$

c) SOIL:  $\sigma_b = \frac{45 \times 10^3 \text{ N}}{(0.8)(2.5) \text{ m}^2} = 22.5 \text{ kPa}$

$$\text{ON COMPACT GRAVEL } \sigma_{bd} = 380 \text{ kPa} \text{ OK} \quad (\text{TABLE 3-6})$$

3-25 ON SOFT ROCK:  $\sigma_{bd} = 480 \text{ kPa} = 480 \times 10^3 \text{ N/m}^2$  (TABLE 3-6)  
 REQ'D.  $A = \frac{P}{\sigma_{bd}} = \frac{160 \times 10^3 \text{ N}}{480 \times 10^3 \text{ N/m}^2} = 0.333 \text{ m}^2 = S^2$   
REQ'D. SIDE DIMENSION:  $S = 0.577 \text{ m}$

3-26  $W_b = \frac{S_y - 13}{20} (0.66dL) = \frac{36 - 13}{20} (0.66)(3.00)(16.0) = 36.4 \text{ KIPS}$

3-27  $W_b = \frac{46 - 13}{20} (0.66)(3.00)(16.0) = 52.3 \text{ KIPS}$  (EQ. 3-9)

3-28 a)  $W_b = \frac{36 - 13}{20} (0.66)(5.00)(8.00) = 30.26 \text{ KIPS}$

b)  $W_b = \frac{46 - 13}{20} (0.66)(5.00)(8.00) = 43.56 \text{ KIPS}$

3-29 LOAD ON EACH FOOT =  $F = 10,000/4 = 2500 \text{ LB}$   
 $A_b = 1.59 \text{ IN}^2$  (TABLE A-9 2x2 x 1/4)  
 $\sigma_b = F/A_b = 2500 \text{ LB}/1.59 \text{ IN}^2 = 1572 \text{ PSI}$   
 REQ'D  $A_b = F/\sigma_{bd} = 2500/400 = 6.25 \text{ IN}^2$  USE SQUARE PLATE  
 SIDE =  $\sqrt{A_b} = \sqrt{6.25 \text{ IN}^2} = 2.50 \text{ IN}$

3-30 ALLOWABLE REACTION =  $W_b$  FROM EQ. 3-9.

$W_b = \frac{S_y - 13}{20} (0.66)(d)(L)$  U.S. CUSTOMARY UNITS

$S_y = 36 \text{ KSI}$

$d = 2(200 \text{ mm})(1.0 \text{ IN}/25.4 \text{ mm}) = 15.75 \text{ IN.}$

$L = (150 \text{ mm})(1.0 \text{ IN}/25.4 \text{ mm}) = 5.91 \text{ IN.}$

$W_b = \frac{36 - 13}{20} (0.66)(15.75)(5.91) = 70.6 \text{ KIPS}$

$W_b = (70.6 \text{ KIPS})(4.448 \text{ kN/KIP}) = 314 \text{ kN}$

OR - USING EQ 3-10 FOR SI UNITS

$d = 2d = 2(200 \text{ mm}) = 400 \text{ mm}, L = 150 \text{ mm}$

$S_y = 248 \text{ MPa}$

$W_b = (248 - 89.6)(3.3 \times 10^{-5})(400)(150) = 314 \text{ kN}$

## SHEARING STRESS

3-31  $T = 5000 \text{ LB-IN}$   
 FORCE ON SIDE OF KEY  $= F = T/R = 5000/1.125 = 4444 \text{ LB}$   
SHEAR:  $T = F/A_s = F/bL$   
 LET  $T = T_s = \frac{0.5S_y}{2} = \frac{(0.5)(64000 \text{ PSI})}{2} = 16000 \text{ PSI}$   
 $\text{REQ'D. } L = \frac{F}{b T_s} = \frac{4444 \text{ LB}}{(0.50 \text{ IN})(16000 \text{ LB/IN}^2)} = 0.556 \text{ IN}$   
BEARING:  $\sigma = \frac{F}{A_b} = \frac{F}{(b/2)(L)}$   
 LET  $\sigma = \sigma_b = 0.9S_y = 0.9(64000 \text{ PSI}) = 57600 \text{ PSI}$   
 $\text{REQ'D. } L = \frac{F}{(b/2) \sigma_b} = \frac{4444}{(0.50/2)(57600 \text{ LB/IN}^2)} = 0.309 \text{ IN.}$

3-32  $T = F/A_s = F/(8 \text{ IN})(a)$ ; LET  $T = T_s = 6000 \text{ PSI}$   
 $\text{REQ'D. } a = \frac{F}{(8 \text{ IN})(T_s)} = \frac{21000 \text{ LB}}{(8 \text{ IN})(6000 \text{ LB/IN}^2)} = 0.438 \text{ IN.}$

3-33 SHEAR: TWO PINS, EACH IN DOUBLE SHEAR;  $A_s = 4(\pi D^2/4) = \pi D^2$   
 $T = F/A_s$ ; FOR STATIC LOAD,  $T = T_s = 0.5S_y/2 = \frac{0.5(82000)}{2} = 20500 \text{ PSI}$   
 $\text{REQ'D. } A_s = F/T_s = \frac{42000 \text{ LB}}{20500 \text{ PSI}} = 2.049 \text{ IN}^2 = \pi D^2$   
 $\text{REQ'D. } D = \sqrt{A_s/\pi} = \sqrt{2.049 \text{ IN}^2/\pi} = 0.808 \text{ IN.}$   
 SPECIFY  $D = 1.00 \text{ IN.}$   
CHECK BEARING:  $\sigma_b = 0.9(S_y) = 0.9(82000 \text{ PSI}) = 73800 \text{ PSI}$   
 $\sigma_b = \frac{F}{A_b} = \frac{F}{(D)(2L)}$   
 $\text{REQ'D. } L = \frac{F}{(D)(2)(\sigma_b)} = \frac{42000 \text{ LB}}{(1.0 \text{ IN})(2)(73800 \text{ LB/IN}^2)} = 0.285 \text{ IN.}$   
 VERY SMALL

3-34  $T_s = 0.5S_y/b = (0.5)(365 \text{ MPa})/6 = 47.1 \text{ MPa}$   
 $T = F/A_s = P/[2(\pi D^2/4)]$   
 $F_{\text{ALLOW}} = T_s A_s = 47.1 \text{ N/mm}^2 \cdot \frac{2\pi(16 \text{ mm})^2}{4} = 18.9 \text{ kN}$

3-35 FROM PROB. 1-59:  $F = 23695 \text{ N}$ ;  $T = 151 \text{ MPa}$  ON PIN.  
 LET  $T = T_s = \frac{S_{us}}{2} \approx \frac{0.82 S_u}{2}$ ;  $\text{REQ'D. } S_u = \frac{8T}{0.82} = \frac{8(151)}{0.82} = 1473 \text{ MPa}$   
 POSSIBLE STEEL: AISI 4140 OQT 100,  $S_u = 1593 \text{ MPa}$ ; 12% ELONG.

3-36  $A_s = (\pi D)(t) = \pi(20)(8) \text{ mm}^2 = 503 \text{ mm}^2$   
 $T = S_{us} \approx 0.82 S_u = 0.82(448 \text{ MPa}) = 367 \text{ MPa} = 367 \text{ N/mm}^2$   
 $\text{REQ'D. } F = T \cdot A_s = \frac{367 \text{ N}}{\text{mm}^2} \cdot 503 \text{ mm}^2 = 185 \text{ kN}$

3-37  $T = S_{us} = 165 \text{ MPa}$  - GIVEN IN APP. A17  
 $\text{REQ'D. } F = T \cdot A_s = 165 \text{ N/mm}^2 \cdot 503 \text{ mm}^2 = 83 \text{ kN}$

3-38  $T = S_{MS} \approx 0.90 S_u = 0.90 (331 \text{ MPa}) = 298 \text{ MPa}$  COPPER  
 REQ'D:  $F = T \cdot A_s = \frac{298 \text{ N}}{\text{mm}^2} \cdot 503 \text{ mm}^2 = 150 \text{ kN}$

3-39  $T = S_{MS} \approx 0.82 S_u = 0.82 (621 \text{ MPa}) = 509 \text{ MPa}$   
 REQ'D:  $F = T \cdot A_s = \frac{509 \text{ N}}{\text{mm}^2} \cdot 503 \text{ mm}^2 = 256 \text{ kN}$

3-40  $F = T \cdot A_s$ ; LET  $T = S_{MS} \approx 0.82 S_u = 0.82 (448 \text{ MPa}) = 367 \text{ MPa}$   
 $A_s = [2(35 \text{ mm}) + \pi(8 \text{ mm})](5.0 \text{ mm}) = 476 \text{ mm}^2$   
 $F = (367 \text{ N/mm}^2)(476 \text{ mm}^2) = 175 \text{ kN}$

3-41  $F = T \cdot A_s$ ; LET  $T = S_{MS} = 16 \text{ ksi}$  (FROM APP. A-17)  
 $A_s = 1.144 \text{ in}^2$  (SEE PROB. 1-62)  
 $F = (16,000 \text{ lb/in}^2)(1.144 \text{ in}^2) = 18,300 \text{ LB}$

3-42  $A_s = (3.5 \text{ in})(3.0 \text{ in}) = 10.5 \text{ in}^2$   
 $T = \frac{F}{A_s} = \frac{1800 \text{ LB}}{10.5 \text{ in}^2} = 171 \text{ PSI}$  (UNSAFE)  
 MAXIMUM ALLOWABLE SHEAR STRESS LISTED IN TABLE A-18 IS 95 PSI.

3-43  $F = T \cdot A_s$ ; LET  $T = S_{MS} \approx 0.82 S_u = 0.82 (97,000 \text{ PSI}) = 79,540 \text{ PSI}$   
 $F = T \cdot A_s = (79,540 \text{ lb/in}^2)(7.5 \text{ in} \cdot 0.105 \text{ in}) = 62,650 \text{ LB}$

3-44 LET  $T = S_{MS} \approx 0.82 S_u = 0.82 (263,000 \text{ PSI}) = 215,660 \text{ PSI}$   
 $F = T \cdot A_s = (215,660 \text{ lb/in}^2)(7.5 \cdot 0.105 \text{ in}^2) = 169,800 \text{ LB}$

3-45 LET  $T = S_{MS} \approx 0.82 S_u = 0.82 (185,000 \text{ PSI}) = 151,700 \text{ PSI}$   
 $F = T \cdot A_s = (151,700 \text{ lb/in}^2)(7.5 \cdot 0.105 \text{ in}^2) = 119,500 \text{ LB}$

3-46 C36000 BRASS: LET  $T = S_{MS} \approx 0.9 (68,000 \text{ PSI}) = 61,200 \text{ PSI}$   
 $F = T \cdot A_s = (61,200 \text{ lb/in}^2)(7.5 \cdot 0.105 \text{ in}^2) = 48,200 \text{ LB}$

3-47 ALUM. 5154-H32: LET  $T = S_{MS} = 152,000 \text{ PSI}$   
 $F = T \cdot A_s = (152,000 \text{ lb/in}^2)(7.5 \cdot 0.105 \text{ in}^2) = 119,700 \text{ LB}$

3-48 1. FIND  $F_2$ :

$$\sum M_O = 0 = (F_1 \cos 30^\circ)(30) + (F_1 \sin 30^\circ)(40) + (F_2 \sin 10^\circ)(10) - (F_2 \cos 10^\circ)(25)$$

$$0 = 165.5 - 22.88 F_2$$

$$F_2 = 7.23 \text{ kN}$$

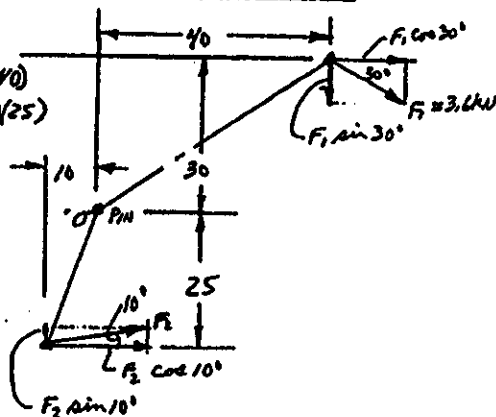
2. FIND RESULTANT  $F$  ON PIN

$$R_p = \sqrt{(F_{1x} + F_{2x})^2 + (F_{1y} + F_{2y})^2}^{1/2}$$

$$R_p = \sqrt{(3.12 + 7.12)^2 + (1.8 - 1.26)^2}^{1/2}$$

$$R_p = 10.7 \text{ kN}$$

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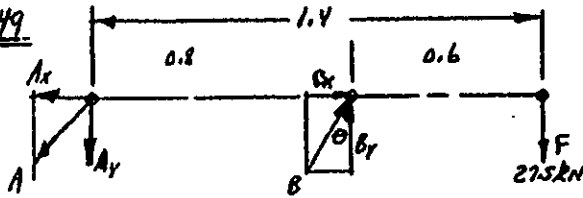
### 3-48 (CONTINUED)

3. DESIGN SHEAR STRESS:  $T_s = \frac{S_{AS}}{B} = \frac{0.954}{8} = \frac{(0.90)(14.32 \text{ MPa})}{8} = 1.67 \text{ MPa}$

4. REQ'D  $A_s = \frac{R}{T_s} = \frac{10.7 \times 10^3 \text{ N}}{127 \text{ N/mm}^2} = 64.2 \text{ mm}^2 = \frac{\pi D^2}{4}$

5. REQ'D  $D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(64.2)}{\pi}} = 9.04 \text{ mm}$

### 3-49



$\tan \theta = 0.8/1.50$   
 $\theta = 28.1^\circ$

$\sum M_A = 0 = 27.5(1.4) - B_y(0.8)$

$B_y = 48.125 \text{ kN}$

$A_y = B_y - F = 20.625 \text{ kN}$

$B = B_y / \cos \theta = 54.5 \text{ kN}$

$B_x = B \sin \theta = 25.7 \text{ kN}$

$A_x = B_x = 25.7 \text{ kN}$

$A = \sqrt{A_x^2 + A_y^2} = 32.95 \text{ kN}$

$C = B = 54.5 \text{ kN}$

$T_s = \frac{0.55V}{2} = \frac{0.5(441 \text{ MPa})}{2} = 110 \text{ MPa}$

PIN A: REQ'D  $A_s = \frac{A}{T_s} = \frac{32.95 \times 10^3 \text{ N}}{110 \text{ N/mm}^2} = 299 \text{ mm}^2 = \left[ \frac{\pi D^2}{4} \right] 2 = \frac{\pi D^2}{2}$

REQ'D  $D_A = \sqrt{2A/\pi} = \sqrt{2(299)/\pi} = 13.8 \text{ mm}$

PINS B AND C: REQ'D  $A_s = \frac{54.5 \times 10^3 \text{ N}}{110 \text{ N/mm}^2} = 494 \text{ mm}^2 = \frac{\pi D^2}{2}$

REQ'D  $D_B = D_C = \sqrt{2A/\pi} = \sqrt{2(494)/\pi} = 17.7 \text{ mm}$

### 3-50 FORCES ON JOINTS FOUND FROM PROBLEM 1-51.

JOINT A:  $F_2 = AD = 10.5 \text{ kN} = \text{JOINT C}$

JOINT B:  $F_3 = \sqrt{10.5^2 + 9.09^2} = 13.9 \text{ kN} = \text{JOINT D}$

$T_s = \frac{0.55V}{2} = \frac{(0.5)(345)}{2} = 86.3 \text{ MPa}$

REQ'D  $A_s = \frac{F_s}{T_s} = \frac{10.5 \times 10^3 \text{ N}}{86.3 \text{ N/mm}^2} = 121.7 \text{ mm}^2 = \frac{\pi D^2}{4}$

$D = \sqrt{4A_s/\pi} = 12.4 \text{ mm}$  FOR JOINTS A AND C

REQ'D  $A_s = \frac{F_3}{T_s} = \frac{13.9 \times 10^3 \text{ N}}{86.3 \text{ N/mm}^2} = 161.1 \text{ mm}^2$

$D = \sqrt{4A_s/\pi} = 14.3 \text{ mm}$  FOR JOINTS B AND D

3-51

$$x = 53 \cos 20^\circ = 46.8 \text{ in.}$$

$$\sum M_W = 0 = 280(46.8) - F_L(6)$$

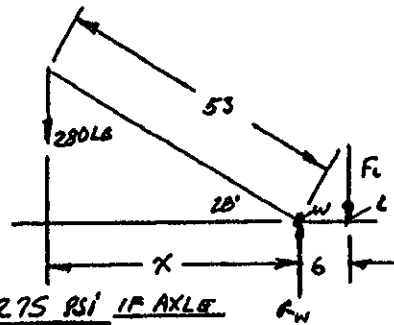
$$F_L = 2184 \text{ LB}$$

$$\sum M_L = 0 = 280(52.8) - R_W(6)$$

$$R_W = 2464 \text{ LB}$$

SHEAR STRESS ON AXLE:

$$\tau = \frac{R_W}{A_s} = \frac{2464 \text{ LB}}{2(\pi(0.5)^2/4) \text{ IN}^2} = \frac{6275 \text{ PSI IF AXLE IS IN DOUBLE SHEAR}}$$



3-52 a) SHEAR OF PIN:  $\tau = F/A_s$ ; LET  $\tau = \tau_0 = \frac{S_u}{B} = \frac{0.82 S_u}{B} = \frac{0.82(97)}{B} = 9.94 \text{ ksi}$

$$F_{\text{ALLOW}} = \tau_0 A_s = 9.94 \frac{\text{LB}}{\text{IN}^2} \cdot \frac{2\pi(0.63)^2 \text{IN}^2}{4} = 62.60 \text{ LB}$$

b) BEARING:  $\sigma_b = F/A_b$ ; LET  $\sigma_b = \sigma_{bs} = 0.9 S_y = 0.9(82) = 73.8 \text{ ksi}$

$$F_{\text{ALLOW}} = \sigma_{bs} A_b = 73.8 \frac{\text{LB}}{\text{IN}^2} \cdot (0.63)(2 \times 0.38) \text{IN}^2 = 35.34 \text{ LB}$$

c) TENSION:  $\sigma = \frac{F}{A_t} K_t$ ; LET  $\sigma = \sigma_0 = \frac{S_u}{B} = \frac{97}{B} = 12.125 \text{ ksi}$

$$A_t = (1.50 - 0.63)(2)(0.38) = 0.661 \text{ IN}^2$$

$$K_t \text{ FROM APP. A-21-4 CURVE B; } d/w = 0.63/1.50 = 0.42; K_t = 2.83$$

$$F_{\text{ALLOW}} = \frac{\sigma_0 A_t}{K_t} = \frac{(12.125 \text{ LB/IN}^2)(0.661 \text{ IN}^2)}{2.83} = 2832 \text{ LB}$$

3-53 IMPACT:  $T_0 = \frac{S_y}{12} = \frac{90 \text{ ksi}}{12} = 7.50 \text{ ksi} = 7500 \text{ psi}$

$$A_s = 2\pi D^2/4 = \pi D^2/2$$

$$\text{REQ'D. } A_s = \frac{F/T_0}{7500 \text{ LB/IN}^2} = \frac{500 \text{ LB}}{7500 \text{ LB/IN}^2} = 0.0667 \text{ IN}^2 = \pi D^2/2$$

$$\text{REQ'D. } D = \sqrt{\frac{2A_s}{\pi}} = \sqrt{\frac{2(0.0667)}{\pi}} = 0.206 \text{ IN}; \text{ SPECIFY } D = 0.250 \text{ IN.}$$

3-54 FALLOW. ON EACH BOLT:  $F = A_s T_0 = \frac{\pi(1.25)^2 \text{IN}^2}{4} \cdot \frac{6000 \text{ LB}}{\text{IN}^2} = 7363 \text{ LB}$

$$\text{TORQUE} = F \cdot \text{RADIUS} = \frac{7363 \text{ LB}}{\text{BOLT}} \cdot 8 \text{ BOLTS} \cdot 4.5 \text{ IN} = 2.65 \times 10^5 \text{ LB-IN}$$

3-55  $F = 25 \text{ kN}$  REPEATED. AISI 4140 OQT MOD  $S_u = 1014 \text{ MPa}$

$$\sigma_0 = S_u/N; \text{ LET } \sigma_{\text{MAX}} = \sigma_0; \text{ THEN } N = S_u/\sigma_{\text{MAX}}$$

HOLE:  $d/D = 10/25 = 0.40$ ;  $K_{t2} = 5.10$  (A-21-5)

$$\sigma_0 = \frac{F}{A} = \frac{F}{\pi D^2/4} = \frac{25000 \text{ N}}{\pi(25)^2/4 \text{ mm}^2} = 50.9 \text{ MPa}$$

$$\sigma_{\text{MAX}} = K_{t2} \cdot \sigma_0 = (5.10)(50.9) = 260 \text{ MPa}$$

$$N = S_u/\sigma_{\text{MAX}} = 1014/260 = 3.90 - \text{LOW - SHOULD BE } > 8.$$

FILLET:  $D/d = 25/20 = 1.25$ ;  $r/d = 2/20 = 0.10$ ;  $K_t = 1.77$  (A-21-2)

$$\sigma_{\text{NOM}} = \frac{F}{A} = \frac{F}{\pi d^2/4} = \frac{25000 \text{ N}}{\pi(20)^2/4 \text{ mm}^2} = 79.6 \text{ MPa}$$

$$\sigma_{\text{MAX}} = K_t \sigma_{\text{NOM}} = (1.77)(79.6) = 141 \text{ MPa}$$

$$N = S_u/\sigma_{\text{MAX}} = 1014/141 = 7.20 - \text{LOW}$$

3-56  $r/d = \frac{0.50}{6.0} = 0.083$ ;  $D/d = \frac{9}{6} = 1.50$ ; THEN  $K_t = 1.95$  (A-21-2)  
 $\sigma_{max} = K_t \sigma_{nom} = \frac{(1.95)(900 \text{ N})}{\pi(6 \text{ mm})^2/4} = 62.1 \text{ MPa}$

3-57 ASSUME CIRCULAR GROOVES: APPENDIX A-21-1  
 $\frac{r}{d_g} = \frac{6 \text{ mm}}{75 \text{ mm}} = 0.08$ ;  $\frac{d}{d_g} = \frac{85}{75} = 1.13$ ; THEN  $K_t = 2.35$   
 $\sigma_{max} = \frac{K_t F}{A} = \frac{2.35(36 \times 10^3 \text{ N})}{\pi(75)^2/4 \text{ mm}^2} = 19.15 \text{ MPa}$

3-58 STRESS AT HOLE:  $r/d = \frac{0.50}{1.00} = 0.50$ ; APP. A-21-5  $K_t = 6.3$

$$\sigma_{max} = \frac{K_t F}{A_g} = \frac{6.3(F)}{\pi(1.0)^2/4} = 8.02 F$$

STRESS AT FILLET: LET  $K_t = 1.7$  AND  $\sigma_{max} = 8.02 F$

$$\sigma_{max} = \frac{K_t F}{A} = \frac{1.7 F}{A} = 8.02 F$$

$$\text{REQ'D } A = \frac{1.7 F}{8.02 F} = 0.212 \text{ IN}^2 = \pi d^2/4$$

$$\text{REQ'D } d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.212)}{\pi}} = 0.519 \text{ IN}$$

$$\text{THEN } D/d = 1.00/0.519 = 1.93$$

$$\text{FOR } K_t = 1.7 \text{ AND } D/d = 1.93; r/d = 0.17$$

$$\text{THEN } r = 0.17 d = 0.17(0.519) = 0.088 \text{ IN.}$$

# ADDITIONAL PROBLEMS - DIRECT TENSION OR COMPRESSION

3-59  $F = 125 \text{ kN}$ .  $A = (32)(40) \text{ mm}^2 = 1280 \text{ mm}^2$ . SPECIFY MATERIAL.

$$a) \sigma_{\text{MAX}} = F/A = \frac{125 \times 10^3 \text{ N}}{1280 \text{ mm}^2} = 97.7 \text{ MPa}$$

b) CASE B: LET  $N=B$ :  $\sigma_b = \sigma_u/8$ ; REQ'D  $S_u = 8 \sigma_{\text{MAX}}$

$$S_u = 8(97.7 \text{ MPa}) = 781 \text{ MPa}$$

SPECIFY AISI 4140 OQT 1300,  $\sigma_u = 814 \text{ MPa}$ , 23% ELONGATION  
GOOD DUCTILITY. OTHER MATERIALS AVAILABLE IN TABLE A-13.

3-60

$$F = 125 \times 10^3 \text{ N} / 2 = 6250 \text{ N PER ROD.}$$

SPECIFY AISI 1040 H.R.  $S_u = 621 \text{ MPa}$ , 25% ELONGATION  
GOOD DUCTILITY

CASE: C DETERMINE DIAMETER

$$\sigma_b = S_u/12 \text{ SHOCK. } \sigma = F/A. \text{ REQ'D } A = F/\sigma_b$$

$$\sigma_b = 621 \text{ MPa} / 12 = 51.75 \text{ MPa} = 51.75 \text{ N/mm}^2$$

$$A = \frac{F}{\sigma_b} = \frac{6250 \text{ N}}{51.75 \text{ N/mm}^2} = 120.8 \text{ mm}^2 = \pi D^2/4$$

$$\text{REQ'D } D = \sqrt{4A/\pi} = \sqrt{4(120.8)/\pi} = 12.4 \text{ mm} = D_{\text{MIN}}$$

SPECIFY  $D = 14.0 \text{ mm}$  - PREFERRED BASIC SIZE

3-61

$F = 4.0 \text{ kN}$ . REPEATED. CASE C.  $\sigma_b = S_u/8$

AISI 1040 CD.  $S_u = 669 \text{ MPa}$ .  $\sigma_b = 669/8 = 83.6 \text{ MPa}$

$$\text{REQ'D } A = \frac{F}{\sigma_b} = \frac{4.0 \times 10^3 \text{ N}}{83.6 \text{ N/mm}^2} = 47.8 \text{ mm}^2 = b^2 \text{ [SQUARE]}$$

$$b_{\text{MIN}} = \sqrt{A} = \sqrt{47.8 \text{ mm}^2} = 6.9 \text{ mm} - \text{SPECIFY } b = 7.0 \text{ mm}$$

3-62

$$A = (30)(20) = 600 \text{ mm}^2, \sigma = F/A. F_{\text{MAX}} = \sigma_b A \text{ (REPEATED)}$$

AISI 4140 OQT 1300.  $S_u = 814 \text{ MPa}$ , 23% ELONGATION - DUCTILE

$$\sigma_b = S_u/8 = 814 \text{ MPa} / 8 = 102 \text{ MPa}$$

$$F_{\text{MAX}} = \sigma_b A = \frac{102 \text{ N}}{\text{mm}^2} \cdot 600 \text{ mm}^2 = 61,200 \text{ N} = 61.2 \text{ kN} = F_{\text{MAX}}$$

3-63

$F = 8.25 \text{ kN}$  REPEATED. CASE B - SPECIFY MATERIAL.

$$\sigma_{\text{MAX}} = K_t \sigma_{\text{MIN}} = K_t \cdot \frac{F}{\pi d_g^2/4} = K_t \frac{8.25 \times 10^3 \text{ N}}{\pi (22 \text{ mm})^2/4} = K_t (21.7 \text{ MPa})$$

$$\text{APP A21-1: } r/d_g = 4.0 \text{ mm} / 22 \text{ mm} = 0.182; \quad d/d_g = 30/22 = 1.36; \quad K_t = 1.65$$

$$\sigma_{\text{MAX}} = K_t \sigma_{\text{MIN}} = 1.65 (21.7 \text{ MPa}) = 35.8 \text{ MPa}$$

$$\text{LET } \sigma_{\text{MAX}} = \sigma_b = S_u/8, \text{ REQ'D } S_u = 8 \sigma_b = 8 (35.8 \text{ MPa}) = 286 \text{ MPa}$$

SPECIFY ALUMINUM 6061-T6.  $S_u = 310 \text{ MPa}$ , 17% ELONG.

OR ANY STEEL FROM TABLE A-13 COULD BE USED.

3-64

$$M = 2800 \text{ kg}, W = mg = 2800 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 27468 \text{ N} \cdot \text{s}^2/\text{m} = 27.5 \text{ kN}$$

$$\sigma = F/A. \text{ REQ'D } A = F/\sigma_s. \sigma_s = S_u/12 \text{ IMPACT}$$

$$6061\text{-T4 ALUMINUM. } S_u = 241 \text{ MPa. } \sigma_s = 241/12 = 20.08 \text{ MPa}$$

$$A_{\min} = \frac{F}{\sigma_s} = \frac{27.5 \times 10^3 \text{ N}}{20.08 \text{ N/mm}^2} = 1369 \text{ mm}^2 = \pi D^2/4$$

$$D_{\min} = \sqrt{4A/\pi} = \sqrt{4(1369 \text{ mm}^2)/\pi} = 41.7 \text{ mm. SPECIFY } D = 45 \text{ mm}$$

3-65

$$\text{HEMLOCK, GRADE 3 WOOD. } \sigma_{\text{ALLOW.}} = 500 \text{ psi COMP. PARALLEL TO GRAIN}$$

$$\sigma = F/A. F_{\max} = \sigma_{\text{ALL.}} \cdot A = 500 \text{ lb/in}^2 \cdot (3.50)(7.50) \text{ in}^2 = 13,125 \text{ LB} = F_{\max}$$

3-66

$$\text{AISI 1141 Q&T 900. } S_u = 1007 \text{ MPa. } \sigma_s = S_u/12 \text{ SHOCK. } F = 45.0 \text{ kN}$$

$$\sigma_s = 1007 \text{ MPa}/12 = 83.9 \text{ MPa. } \sigma = F/A. A_{\min} = F/\sigma_s$$

$$A_{\min} = \frac{45.0 \times 10^3 \text{ N}}{83.9 \text{ N/mm}^2} = 536 \text{ mm}^2 = \pi D^2/4. D = \sqrt{4A/\pi} = \sqrt{4(536 \text{ mm}^2)/\pi}$$

$$D_{\min} = 26.1 \text{ mm. SPECIFY } D = 28.0 \text{ mm BASIC SIZE}$$

3-67

$$\text{FIND REPEATED TENSILE LOAD. AISI 4140 Q&T 1300, } S_u = 118 \text{ ksi}$$

$$\sigma_s = S_u/8 = 118 \text{ ksi}/8 = 14.75 \text{ ksi. 1-IN PIPE: } A = 0.494 \text{ in}^2$$

$$\sigma = \frac{F}{A}. F_{\max} = \sigma_s A = (14750 \text{ lb/in}^2)(0.494 \text{ in}^2) = 7287 \text{ LB} = F_{\max}$$

3-68

$$F = 550 \text{ kN SHOCK. (a) SPECIFY DUCTILE IRON 60-40-18 FOR DUCTILITY.}$$

$$S_u = 414 \text{ MPa, 18 \% ELONGATION. } \sigma_s = S_u/12 = 414 \text{ MPa}/12 = 34.5 \text{ MPa}$$

$$\sigma = F/A. \text{ REQ'D } A = \frac{F}{\sigma_s} = \frac{550 \times 10^3 \text{ N}}{34.5 \text{ N/mm}^2} = 15,942 \text{ mm}^2 = \pi D^2/4$$

$$D_{\min} = \sqrt{4A/\pi} = \sqrt{4(15942 \text{ mm}^2)/\pi} = 142 \text{ mm. SPECIFY } D = 160 \text{ mm}$$

3-69

$$F = 16.5 \text{ kN. ASTM A242. } S_u = 483 \text{ MPa. } \sigma_s = S_u/12 = 483 \text{ MPa}/12 = 40.25 \text{ MPa}$$

$$\text{REPEATED FIND A. } \sigma_{\max} = K_t(F/A). A_{\min} = \frac{F(K_t)}{\sigma_s} = \frac{16.5 \times 10^3 \text{ N}(K_t)}{40.25 \text{ N/mm}^2} = 410 \text{ mm}^2(K_t)$$

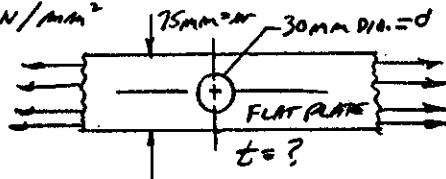
$$d/w = 30/75 = 0.40; K_t = 2.23$$

$$\text{APP A21-4 CURVE A.}$$

$$A_{\min} = 410 \text{ mm}^2(2.23) = 914 \text{ mm}^2 = w t$$

$$t_{\min} = \frac{A_{\min}}{w} = \frac{914 \text{ mm}^2}{75 \text{ mm}} = 12.2 \text{ mm}$$

$$\text{SPECIFY } t = 14 \text{ mm PREFERRED BASIC SIZE}$$



3-70 FIGURE P3-70. AISI 1141 OQT 1100.  $S_u = 800 \text{ MPa}$ ,  $\sigma_d = S_u/8$  REPEATED LOAD.  
 $\sigma_{\max} = K_t F/A$ .  $F_{\text{allow}} = \sigma_d A/K_t$   $\sigma_d = 100 \text{ MPa}$

MIDDLE SECTION:  $A = (10 \text{ mm})(6 \text{ mm}) = 60 \text{ mm}^2$ .  $r/h = \frac{1.5 \text{ mm}}{10 \text{ mm}} = 0.15$

APP. A21-3.  $h/h = 16 \text{ mm}/10 \text{ mm} = 1.60$ .  $K_t = 1.90$

$$F_{\text{allow}} = \frac{\sigma_d \cdot A}{K_t} = \frac{(100 \text{ N/mm}^2)(60 \text{ mm}^2)}{1.90} = 3158 \text{ N}$$

AT PIN: APP. A21-4: CURVE B.  $w = 16 \text{ mm}$ ,  $d = 6 \text{ mm}$ ,  $\frac{d}{w} = \frac{6}{16} = 0.375$

$$K_t = 3.15. A = (w-d)t = (16-6)(6) = 60 \text{ mm}^2$$

$$F_{\text{allow}} = \frac{\sigma_d \cdot A}{K_t} = \frac{(100 \text{ N/mm}^2)(60 \text{ mm}^2)}{3.15} = 1967 \text{ N} = F_{\text{allow}}$$

3-71 SPECIFY MATERIAL.  $\sigma_{\max} = K_t F/A$ .  $\sigma_d = S_u/12$  SHOCK

AT HOLE: APP. A21-5 CURVE A.  $d/D = 12/30 = 0.40$ .  $K_t = 5.0$

$$A = \frac{\pi D^2}{4} = \frac{\pi (30 \text{ mm})^2}{4} = 707 \text{ mm}^2 \text{ GROSS AREA}$$

$$\text{LET } \sigma_{\max} = \sigma_d = \frac{S_u}{12} = \frac{K_t F}{A}, \text{ REQ'D } S_u = \frac{12 K_t F}{A} = \frac{12(5.0)(12.6 \times 10^3 \text{ N})}{707 \text{ mm}^2}$$

REQ'D.  $S_u = 1070 \text{ MPa}$

AT FILLET: APP. A21-2.  $r/d = \frac{12 \text{ mm}}{18 \text{ mm}} = 0.667$ .  $d/D = \frac{30}{18} = 1.67$

$$K_t = 2.10. A = \pi d^2/4 = \frac{\pi (18 \text{ mm})^2}{4} = 254 \text{ mm}^2$$

$$\text{REQ'D. } S_u = \frac{12 K_t F}{A} = \frac{12(2.10)(12.6 \times 10^3 \text{ N})}{254 \text{ mm}^2} = 1248 \text{ MPa}$$

STRESS AT FILLET GOVERNS:  $S_{u \min} = 1248 \text{ MPa}$  WITH GOOD DUCTILITY

SPECIFY: AISI 4140 OQT 900.  $S_u = 1289 \text{ MPa}$ ; 15% ELONGATION

### DIRECT SHEAR STRESS

3-72 COMPUTE FORCE REQUIRED TO PUNCH OUT THE SHAPE IN

FIGURE P3-72.  $T = F/A$ ;  $F = TA$ . LET  $T = S_{us}$

AISI 1020 CD.  $S_u = 75 \text{ ksi}$ .  $S_{us} = 0.82 S_u = 0.82(75) = 61.5 \text{ ksi}$

$A = \text{SHEAR AREA} = \text{PERIMETER} \times \text{THICKNESS} = p \cdot t$ ;  $t = 0.085 \text{ IN}$

$$p = 1.0 + 1.0 + 2.0 + \sqrt{1^2 + 1^2} + 0.50 + \pi(0.5)/2 = 6.70 \text{ IN}$$

$$A = p \cdot t = (6.70)(0.085) = 0.569 \text{ IN}^2$$

$$F = S_{us} \cdot A = (61.5 \times 10^3 \text{ lb/in}^2)(0.569 \text{ IN}^2) = 35020 \text{ lb} = F$$

3-73

$F = TA = S_{us} \cdot A$ . 6061-T4.  $S_{us} = 24 \text{ ksi}$  - APP. A-17.  $t = 0.10 \text{ IN}$

$$p = 4(1.25) + 3(0.5) + \pi(1.50)/2 = 8.86 \text{ IN}^2; A = p \cdot t = 0.886 \text{ IN}^2$$

$$F = S_{us} \cdot A = (24 \times 10^3 \text{ lb/in}^2)(0.886 \text{ IN}^2) = 21260 \text{ lb} = F$$

3-74  $F = \tau \cdot A = S_{MS} \cdot A$  ALUM. 3003-H18  $S_{MS} = 110 \text{ MPa}$  ,  $A = p \cdot t$  ,  $t = 3.0 \text{ mm}$   
 $p = 30 + 60 + \sqrt{20^2 + 40^2} + \sqrt{10^2 + 40^2} = 176 \text{ mm}$  ,  $A = 527.9 \text{ mm}^2$   
 $F = S_{MS} \cdot A = (110 \text{ N/mm}^2)(527.9 \text{ mm}^2) = \underline{58.06 \text{ kN} = F}$

3-75  $F = \tau \cdot A = S_{MS} \cdot A$  . AISI 1040 CD.  $S_u = 669 \text{ MPa}$  .  $S_{MS} = 0.82 S_u = 549 \text{ MPa}$   
 $A = p \cdot t$  ,  $t = 1.60 \text{ mm}$  .  $p = 50 + 30 + 2(20) + \pi(20)/2 = 151.4 \text{ mm}$   
 $A = (151.4 \text{ mm})(1.60 \text{ mm}) = 242 \text{ mm}^2$  .  $F = 549 \text{ N/mm}^2 \cdot 242 \text{ mm}^2 = \underline{133 \text{ kN}}$

3-76  $F = \tau \cdot A = S_{MS} \cdot A$  . AISI 1080 OQT 900.  $S_u = 1234 \text{ MPa}$   
 $S_{MS} = 0.82 S_u = 0.82(1234 \text{ MPa}) = 1012 \text{ MPa}$   
 $A = p \cdot t$  .  $t = 0.80 \text{ mm}$  .  $p = 60 + 2\sqrt{15^2 + 15^2} + 22 + \pi(4) = 137 \text{ mm}$   
 $A = (137 \text{ mm})(0.80 \text{ mm}) = 109.6 \text{ mm}^2$  .  $F = (1012 \text{ N/mm}^2)(109.6 \text{ mm}^2)$   
 $F = \underline{110.9 \text{ kN}}$

3-77  $F = \tau \cdot A = S_{MS} \cdot A$  . ALUM. 3003-H12.  $S_{MS} = 83 \text{ MPa}$   
 $A = p \cdot t$  ,  $t = 1.40 \text{ mm}$  .  
 $p = 2(90) + 2(18) + 3[\pi(6)] + 2[2(12) + \pi(6)] = 358 \text{ mm}$   
 $A = p \cdot t = (358)(1.40) = 502 \text{ mm}^2$   
 $F = S_{MS} \cdot A = (83 \text{ N/mm}^2)(502 \text{ mm}^2) = \underline{41.6 \text{ kN} = F}$

3-78  $F = \tau \cdot A = S_{MS} \cdot A$  . ALUM 5052-H38.  $S_{MS} = 193 \text{ MPa}$   
 $A = p \cdot t$  .  $t = 2.00 \text{ mm}$  .  $p_{\text{OUTSIDE}} = 2(128) + 2(50) = 356 \text{ mm}$   
 $p_{\text{INSIDE}} = \pi(15) + 2[2(15) + 2(10)] = 147.1 \text{ mm}$   
 $\text{TOTAL } p = p_o + p_i = 503.1 \text{ mm}$  .  $A = p \cdot t = (503.1)(2.0) = 1006 \text{ mm}^2$   
 $F = S_{MS} \cdot A = (193 \text{ N/mm}^2)(1006 \text{ mm}^2) = \underline{194.2 \text{ kN} = F}$

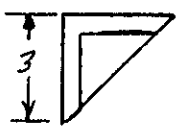
3-79 DATA FROM PROB. 3-78  
FIRST STEP:  $p = p_{\text{INSIDE}} = 147.1 \text{ mm}$  .  $A_1 = p \cdot t = (147.1)(2.0) = 294.2 \text{ mm}^2$   
 $F_1 = S_{MS} \cdot A_1 = (193 \text{ N/mm}^2)(294.2 \text{ mm}^2) = \underline{56.8 \text{ kN} = F_1}$   
2ND STEP:  $p = p_{\text{OUTSIDE}} = 356 \text{ mm}$  .  $A_2 = (356)(2.0) = 712 \text{ mm}^2$   
 $F_2 = S_{MS} \cdot A_2 = (193 \text{ N/mm}^2)(712 \text{ mm}^2) = \underline{137.4 \text{ kN} = F_2 = \text{ANSWER}}$



# BEARING STRESS

3-80  $\sigma_b = \frac{F_b}{A_b}$  .  $F_b = 28,500 \text{ lb} / 4 \text{ LEGS} = 7125 \text{ lb} / \text{LEG}$  .  $A = 1.44 \text{ in}^2$   
 $\sigma_b = 7125 \text{ lb} / 1.44 \text{ in}^2 = \underline{4948 \text{ psi} = \sigma_b}$

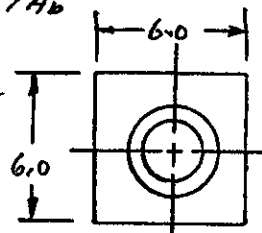
3-81 CONCRETE FLOOR.  $\sigma_c = 3000 \text{ psi}$  ,  $\sigma_{bd} = 0.35 \sigma_c \sqrt{A_2/A_1}$   
 BUT  $\sigma_{bd \text{ MAX}} = 0.70 \sigma_c$  BECAUSE  $A_2/A_1 > 4$  .  $F_b = 7125 \text{ lb}$   
 $\sigma_{bd} = 0.70 (3000) = 2100 \text{ psi} = F_b / A_b$  . REQ'D  $A_b = F_b / \sigma_{bd}$   
 $A_{b \text{ MIN}} = 7125 \text{ lb} / 2100 \text{ lb/in}^2 = 3.39 \text{ in}^2$   
 TRIANGULAR AREA:  $A = \frac{1}{2} (3) (3) = 4.50 \text{ in}^2$   
 WELD PAD TO BOTTOM OF EACH LEG.



3-82  $A_b = (3.50) (7.50) = 26.25 \text{ in}^2$  ,  $F_b = \sigma_{bd} A_b$  .  $\sigma_{bd} = 55 \text{ psi}$   
 $F_b = (55 \text{ lb/in}^2) (26.25 \text{ in}^2) = \underline{1444 \text{ lb} = F_b \text{ ALLOWABLE}}$

3-83 4-IN SCH. 40 PIPE .  $A_b = 3.174 \text{ in}^2$  .  $\sigma_{bd} = 0.70 \sigma_c$  ON LARGE FLOOR  
 $\sigma_{bd} = 0.70 (4000 \text{ psi}) = 2800 \text{ psi}$   
 $F_b = \sigma_{bd} A_b = (2800 \text{ lb/in}^2) (3.174 \text{ in}^2) = \underline{8887 \text{ lb} = F_b \text{ ALLOWABLE}}$

3-84 DATA FROM PROB 3-83.  $\sigma_{bd} = 2800 \text{ psi} = F_b / A_b$   
 $F_b = 10 (8887 \text{ lb}) = 88,870 \text{ lb}$   
 REQ'D.  $A_b = F_b / \sigma_{bd} = \frac{88,870 \text{ lb}}{2800 \text{ lb/in}^2} = 31.74 \text{ in}^2$   
 TRY SQUARE PLATE:  $A_b = b^2$   
 $b_{\text{MIN}} = \sqrt{A_{b \text{ MIN}}} = \sqrt{31.74 \text{ in}^2} = 5.63 \text{ in}$   
 USE  $b = 6.0 \text{ in}$  SQUARE PLATE.



# PROBLEMS WITH TWO OR MORE KINDS OF DIRECT STRESS

3-85 FIG. P3-85. RIVETED PLATES. ASSUME STATIC LOAD  
 6061-T6 PLATES:  $S_y = 40 \text{ ksi}$ ,  $S_u = 45 \text{ ksi}$ , 17% ELONGATION  
 2014-T4 RIVETS:  $S_{us} = 38 \text{ ksi}$ ,  $S_y = 42 \text{ ksi}$

a) SHEAR OF RIVETS:  $T_d = S_{us}/4 = 38 \text{ ksi}/4 = 9.50 \text{ ksi} = F/A_s$   
 $A_s = 2(\pi(0.5 \text{ in})^2/4) = 0.393 \text{ in}^2$  TWO CROSS SECTIONS  
 $F = T_d \cdot A_s = (9.5 \times 10^3 \frac{\text{lb}}{\text{in}^2})(0.393 \text{ in}^2) = \underline{3730 \text{ lb}}$

b) TENSILE STRESS ON PLATE:  $\sigma_s = S_y/3 = 40 \text{ ksi}/3 = 13.3 \text{ ksi}$   
 $\sigma = F/A$ ,  $A = [3.0 \text{ in} - 2(0.5 \text{ in})](0.375 \text{ in}) = 0.75 \text{ in}^2$   
 $F = \sigma_s \cdot A = (13333 \frac{\text{lb}}{\text{in}^2})(0.75 \text{ in}^2) = \underline{10000 \text{ lb}}$

c) BEARING AT RIVETS/HOLES:  $\sigma_b = \frac{F}{A_b} = \sigma_{bd} = 0.65 S_y$   
 $\sigma_{bd} = 0.65(40 \text{ ksi}) = 26 \text{ ksi}$  ON PLATE  
 $A_b = 2(D)(t) = (0.50)(0.375)(2) = 0.375 \text{ in}^2$  - PROJECTED AREA  
 $F = \sigma_{bd} \cdot A_b = (26000 \frac{\text{lb}}{\text{in}^2})(0.375 \text{ in}^2) = \underline{9750 \text{ lb}}$

SHEAR STRESS GOVERNS:  $F_{\text{ALLOW}} = 3730 \text{ lb}$

3-86 DATA FROM PROBLEM 3-85.

a) SHEAR OF RIVETS:  $T_d = 9500 \text{ lb/in}^2 = F/A_s$   
 $A_s = 3(\pi(0.375)^2/4) = 0.3313 \text{ in}^2$   
 $F = T_d \cdot A_s = (9500 \text{ lb/in}^2)(0.3313 \text{ in}^2) = \underline{3148 \text{ lb}}$

b) TENSILE STRESS IN PLATE:  $\sigma_s = 13333 \text{ psi} = F/A$   
 $A = [3.0 - 3(0.375)](0.375) = 0.703 \text{ in}^2$   
 $F = \sigma_s \cdot A = (13333 \text{ lb/in}^2)(0.703 \text{ in}^2) = \underline{9375 \text{ lb}}$

c) BEARING:  $\sigma_{bd} = 26000 \text{ lb/in}^2$  ON PLATE  
 $A_b = 3[D(t)] = 3[(0.375)(0.375)] = 0.422 \text{ in}^2$   
 $F = \sigma_{bd} \cdot A_b = (26000 \text{ lb/in}^2)(0.422 \text{ in}^2) = \underline{10969 \text{ lb}}$

SHEAR STRESS GOVERNS:  $F_{\text{ALLOW}} = 3148 \text{ lb}$

3-87 DATA FROM PROBLEM 3-85.

a) SHEAR OF RIVETS:  $T_d = 9500 \text{ lb/in}^2 = F/A_s$

$A_s = 4[\pi(0.375)^2/4] = 0.4418 \text{ in}^2$

$F = T_d \cdot A_s = (9500 \text{ lb/in}^2)(0.4418 \text{ in}^2) = \underline{4197 \text{ lb}}$

b) TENSION ON PLATE:  $\sigma_d = 13333 \text{ lb/in}^2 = F/A$

$A = [3.0 - 2(0.375)](0.375) = 0.844 \text{ in}^2$

$F = \sigma_d \cdot A = (13333 \text{ lb/in}^2)(0.844 \text{ in}^2) = \underline{11250 \text{ lb}}$

c) BEARING AT RIVETS:

ON PLATES:  $\sigma_{bd} = 0.65(40 \text{ ksi}) = 26 \text{ ksi}$

$A_b = 2(0.25)(0.375)(2) = 0.375 \text{ in}^2$

$F_b = \sigma_{bd} \cdot A_b = (26000 \text{ lb/in}^2)(0.375 \text{ in}^2) = \underline{9750 \text{ lb}}$

ON RIVETS:  $\sigma_{bd} = 0.65(42 \text{ ksi}) = 27.3 \text{ ksi}$

$A_b = 2(0.375)(0.375) = 0.281 \text{ in}^2$

$F_b = (27300 \text{ lb/in}^2)(0.281 \text{ in}^2) = \underline{7678 \text{ lb}}$

SHEAR OF RIVETS GOVERNS:  $F_{\text{ALLOW}} = \underline{4197 \text{ lb}}$

3-88

REPEATED FORCE.

TENSION IN LINK A:  $\sigma_d = S_u/8 = \frac{147 \text{ ksi}}{8} = 18.375 \text{ ksi}$  (AISI 4140 OQT 1100)

$\sigma = \frac{K_t F}{A_{\text{NET}}}$ ;  $F = \frac{\sigma_d A_{\text{NET}}}{K_t} = \frac{(18375 \text{ lb/in}^2)(1.5 - 0.75)(1.25) \text{ in}^2}{2.60}$

APP A 21-4, CURVES:  $d/N = 0.75/1.5 = 0.50$ ,  $K_t = 2.60$

$F_{\text{ALLOW}} = \underline{6626 \text{ lb}}$

SHEAR STRESS IN PIN:  $T_d = S_u/8 = \frac{97 \text{ ksi}}{8} = 12.125 \text{ ksi}$  (AISI 1141 OQT 1100)

$T = \frac{F}{A_s}$ ;  $F = T_d \cdot A_s = (12125 \text{ lb/in}^2)(2[\pi(0.75)^2/4]) = \underline{10713 \text{ lb}}$

BEARING STRESS AT PIN:  $\sigma_{bd} = 0.90 S_y = 0.90(97 \text{ ksi}) = 87.3 \text{ ksi}$

$A_b = (1.25 \text{ in})(0.75 \text{ in}) = 0.9375 \text{ in}^2$

$\sigma_b = F/A_b$ ;  $F = \sigma_{bd} \cdot A_b = (87300 \text{ lb/in}^2)(0.9375 \text{ in}^2) = \underline{81843 \text{ lb}}$

TENSION IN LINK GOVERNS:  $F_{\text{ALLOW}} = \underline{6626 \text{ lb}}$

3-89

FORCES IN MEMBERS:  $AB = 2465 \text{ LB (T)}$ ,  $AC = 1925 \text{ LB (C)}$   
 $BC = 1375 \text{ LB (T)}$ ,  $BD = 1200 \text{ LB (T)}$ ,  $CE = 650 \text{ LB (C)}$ ,  
 $CD = 750 \text{ LB (C)}$ ,  $DE = 961 \text{ LB (T)}$ .

SUPPORT FORCES:  $A_y = 1540 \text{ LB } \downarrow$ ,  $B_y = 2640 \text{ LB } \uparrow$ ,  $B_x = 100 \text{ LB } \leftarrow$

MATERIAL: ASTM A36 STRUCTURAL STEEL,  $S_y = 36 \text{ KSI}$   
 ASSUME STATIC LOAD.  $\sigma_b = S_y/2 = 18 \text{ KSI}$

RED'D AREA:  $A_{min} = F/\sigma_b$

MEMBER	F (LB)	$\sigma_b$ (KSI)	$A_{min}$ (IN <sup>2</sup> )	SQUARE $b_{min}$	ROUND $d_{min}$	THREAD
AB	2465	18	0.137	.374	.418	1/2-13
BC	1375	18	0.0764	.276	.312	3/8-16
BD	1200	18	0.0667	.258	.291	3/8-16
DE	961	18	0.0534	.231	.261	3/8-16

ALTERNATIVE DESIGNS:

SQUARE ROD:  $A = b^2$ ;  $b_{min} = \sqrt{A}$

ROUND ROD:  $A = \pi d^2/4$ ;  $d_{min} = \sqrt{4A/\pi}$

THREADED ROD: LET  $A_{min}$  = TENSILE STRESS AREA OF THREAD  
 FROM APP A-3, COARSE THREADS

FOR THREADED ROD, ATTACH TO CLEVIS AT EACH END. DESIGN  
 PIN FOR CLEVIS FOR SAFE SHEAR STRESS.

AT B: PIN JOINT ATTACHED TO FRAME

AT A: PROVIDE ROLLER ON PIN THAT BEARS ON FRAME.

AT C AND E: PROVIDE AN ADDITIONAL CLEVIS FROM WHICH  
 TO ATTACH LOADS.

NOTE: COMPRESSION MEMBERS MUST BE DESIGNED WITH  
 COLUMN BUCKLING ANALYSIS. SEE CHAPTER 14.

3-90

FORCES IN MEMBERS:

$AB = 4687 \text{ N (T)}$   $BE = 0$   $BF = 2241 \text{ N (T)}$   $CE = 1077 \text{ N (T)}$   
 $AD = 1400 \text{ N (T)}$   $DE = 2300 \text{ N (C)}$   $CF = 800 \text{ N (C)}$   $FG = 500 \text{ N (C)}$   
 $BD = 2597 \text{ N (C)}$   $BC = 750 \text{ N (T)}$   $EF = 2300 \text{ N (C)}$

SUPPORT FORCES:  $A_y = 1400 \text{ N } \uparrow$ ,  $A_x = 4683 \text{ N } \leftarrow$ ,  $D_x = 4488 \text{ N } \rightarrow$

DESIGNS COULD BE SIMILAR TO PROBLEM 3-89.

FORCES ARE GENERALLY SMALLER. SMALL WIRES MAY BE USED  
 FOR TENSION MEMBERS. COMPRESSION MEMBERS MUST BE  
 DESIGNED FOR BUCKLING. SEE CHAPTER 14.

3-91

FORCE ANALYSIS:

USING FBD OF ENTIRE STRUCTURE:

$$\sum M_c = 0 = (34.0 \text{ kN})(1.8 \text{ m}) - B_y(.90)$$

$$B_y = (34)(1.8)/.9 = 68.0 \text{ kN} \downarrow$$

$$\sum F_y = 0 = 34.0 + 68.0 - C_y$$

$$C_y = 102 \text{ kN} \uparrow$$

AB IS A TWO-FORCE MEMBER

$$AB_y = B_y = 68.0 \text{ kN} = AB \cos 21.8^\circ$$

$$AB = 68 / \cos 21.8^\circ = 73.2 \text{ kN TENSION}$$

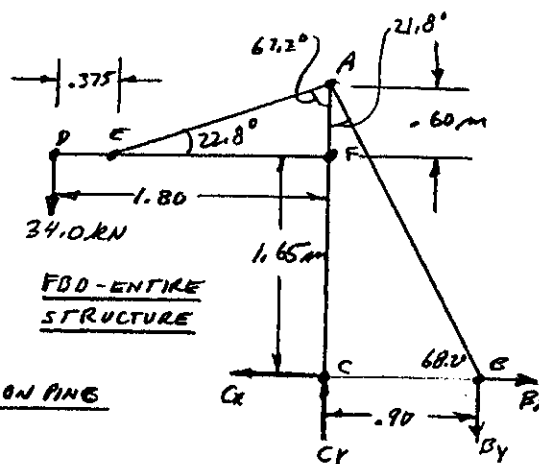
$$AB_x = AB \sin 21.8^\circ = 27.2 \text{ kN} \quad \text{--- FORCE ON PIN B}$$

$$B_x = AB_x = 27.2 \text{ kN}$$

$$\sum F_x = 0 = B_x - C_x$$

$$C_x = B_x = 27.2 \text{ kN}$$

$$\text{FORCE ON PIN C} = \sqrt{C_x^2 + C_y^2} = \sqrt{27.2^2 + 102^2} = 105.6 \text{ kN}$$

FBD - ENTIRE STRUCTURE

FBD OF BOOM:

$$\sum M_F = 0 = (34.0 \text{ kN})(1.80 \text{ m}) - AE_y(1.425)$$

$$AE_y = (34)(1.8)/1.425 = 42.95 \text{ kN}$$

$$AE = AE_y / \sin 22.8^\circ = 110.8 \text{ kN TENSION}$$

$$AE_x = AE \cos 22.8^\circ = 102.2 \text{ kN}$$

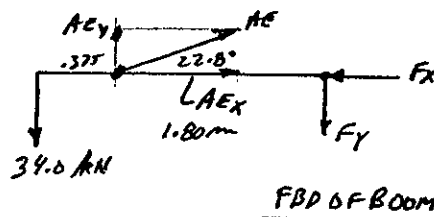
$$\sum F_x = 0 = AE_x - F_x$$

$$F_x = AE_x = 102.2 \text{ kN}$$

$$\sum M_C = 0 = (34.0 \text{ kN})(.375) - F_y(1.425)$$

$$F_y = (34)(.375)/1.425 = 8.95 \text{ kN}$$

$$\text{FORCE ON PIN F} = \sqrt{F_x^2 + F_y^2} = \sqrt{102.2^2 + 8.95^2} = 102.6 \text{ kN}$$

FBD OF BOOM

PIN A:

$$A_y = AE_y + AB_y = (110.8) \cos 67.2^\circ + (73.2) \cos 21.8^\circ$$

$$A_y = 42.95 + 68.0 = 110.9 \text{ kN} \downarrow$$

$$A_x = AB_x - AE_x = 73.2 \sin 21.8^\circ - (110.8) \sin 67.2^\circ$$

$$A_x = 27.18 - 102.1 = -74.9 \text{ kN} \leftarrow$$

$$AE = 110.8 \text{ kN}$$

SUMMARY OF RESULTS:a) FORCES IN WIRES:  $AE = 110.8 \text{ kN}$ ;  $AB = 73.2 \text{ kN}$ 

c) SHEARING FORCE IN EACH PIN:

$$\text{PIN A: } F_A = \sqrt{74.9^2 + 110.9^2} = 134 \text{ kN}$$

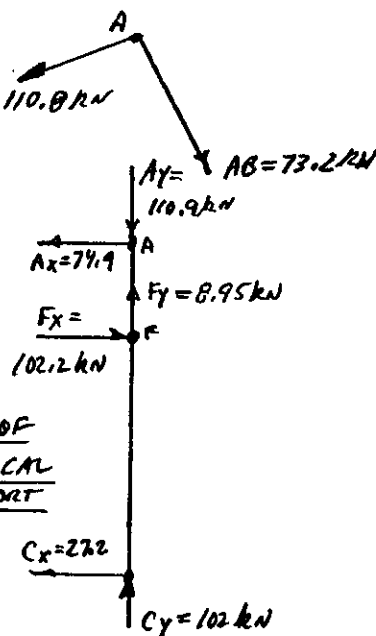
$$\text{PIN F: } F_F = \sqrt{102.2^2 + 8.95^2} = 102.6 \text{ kN}$$

$$\text{PIN C: } F_C = \sqrt{27.2^2 + 102^2} = 105.6 \text{ kN}$$

$$\text{PIN E: } F_E = AE = 110.8 \text{ kN}$$

$$\text{PIN D: } F_D = 34.0 \text{ kN}$$

[NEXT PAGE FOR PARTS b, d, e.]

FBD OF VERTICAL SUPPORT

### 3-91 (CONTINUED)

b) DESIGN OF RODS:  $A_E = 111.8 \text{ kN}$ ,  $A_B = 73.2 \text{ kN}$ , MODERATE SHOCK.  
 $\sigma_d = S_u/12$ . SPECIFY AISI 4140 OQT 900.  $S_u = 1289 \text{ MPa}$ , 15% ELONG.  
 HIGH STRENGTH, GOOD DUCTILITY.

$$\sigma_d = 1289 \text{ MPa}/12 = 107.4 \text{ MPa} = 107.4 \text{ N/mm}^2$$

$$\text{LET } \sigma_d = \sigma_{\max} = F/A. \text{ THEN REQ'D } A = \frac{F}{\sigma_d}$$

$$\text{FOR } A_E: A = \frac{F}{\sigma_d} = \frac{110.8 \times 10^3 \text{ N}}{107.4 \text{ N/mm}^2} = 1031 \text{ mm}^2 = \pi D^2/4$$

$$\text{REQ'D } D = \sqrt{4A/\pi} = \sqrt{4(1031 \text{ mm}^2)/\pi} = 36.2 \text{ mm}$$

SPECIFY  $D = 40 \text{ mm}$  - PREFERRED BASIC SIZE, APP. 2

$$\text{FOR } A_B: A = \frac{F}{\sigma_d} = \frac{73.2 \times 10^3 \text{ N}}{107.4 \text{ N/mm}^2} = 681.6 \text{ mm}^2 = \pi D^2/4$$

$$\text{REQ'D } D = \sqrt{4A/\pi} = \sqrt{4(681.6 \text{ mm}^2)/\pi} = 29.5 \text{ mm}$$

SPECIFY  $D = 30.0 \text{ mm}$  - PREFERRED BASIC SIZE

d) DESIGN OF PINS: ALL PINS TO BE IN DOUBLE SHEAR USING  
 A CLEVIS-TYPE CONNECTION. [SEE FIG. 3-7]

FROM TABLE 3-4:  $T_d = S_y/6 = S_y/12$

SPECIFY AISI 4140 OQT 900.  $S_y = 1193 \text{ MPa}$ , 15% ELONG.

$$T_d = S_y/12 = 1193 \text{ MPa}/12 = 99.4 \text{ MPa} = 99.3 \text{ N/mm}^2$$

$$\text{LET } T_d = T_{\max} = F/A_s. \text{ THEN REQ'D } A_s = F/T_d$$

$$\text{PIN A: } F_A = 134 \text{ kN}. \text{ REQ'D } A_s = \frac{F}{T_d} = \frac{134 \times 10^3 \text{ N}}{99.3 \text{ N/mm}^2} = 1348 \text{ mm}^2$$

$$A_s = 2A = 2\pi D^2/4 = \pi D^2/2. \text{ REQ'D } D_A = \sqrt{2A_s/\pi}$$

$$D_{\min} = \sqrt{\frac{2(1348 \text{ mm}^2)}{\pi}} = 29.3 \text{ mm}. \text{ SPECIFY } D = 30.0 \text{ mm}$$

$$\text{PIN F: } F = 102.6 \text{ kN}. A_s = \frac{F}{T_d} = \frac{102.6 \times 10^3}{99.3} = 1032 \text{ mm}^2$$

$$D_{\min} = \sqrt{\frac{2A_s}{\pi}} = \sqrt{\frac{2(1032 \text{ mm}^2)}{\pi}} = 25.6 \text{ mm}; D_F = 28 \text{ mm}$$

$$\text{PIN C: } F = 105.6 \text{ kN}. A_s = \frac{105.6 \times 10^3}{99.3} = 1062 \text{ mm}^2$$

$$D_{\min} = \sqrt{\frac{2(1062)}{\pi}} = 26.0 \text{ mm}. \text{ SPECIFY } D_C = 28.0 \text{ mm}$$

$$\text{PIN E: } F = 110.8 \text{ kN}. A_s = \frac{110.8 \times 10^3}{99.3} = 1115 \text{ mm}^2$$

$$D_{\min} = \sqrt{\frac{2(1115)}{\pi}} = 26.6 \text{ mm}. \text{ SPECIFY } D_E = 28.0 \text{ mm}$$

$$\text{PIN D: } F = 34.0 \text{ kN}. A_s = \frac{34 \times 10^3}{99.3} = 342.4 \text{ mm}^2$$

$$D_{\min} = \sqrt{\frac{2(342.4)}{\pi}} = 14.8 \text{ mm}. \text{ SPECIFY } D = 16 \text{ mm}$$

[NEXT PAGE FOR BEARING STRESS.]

3-9/ (CONTINUED)

BEARING STRESS ON PINS: [SEE FIG 3-7 FOR DESIGN OF JOINT]

$$\sigma_b = \frac{F}{A_b} = \frac{F}{D \cdot t_1} \text{ AND } t_2 \geq t_1/2$$

$$\text{REQ'D. } t_1 = \frac{F}{D \sigma_{bd}}$$

$$\sigma_{bd} = 0.90 S_y \text{ EQ. 3-8 FOR STEEL.}$$

$$S_y = 1193 \text{ MPa} - \text{AISI 4140 Q\&T 900 FOR PINS}$$

MATERIAL FOR MATING PARTS MUST BE AT LEAST AS STRONG.

$$\sigma_{bd} = 0.90(1193 \text{ MPa}) = 1074 \text{ MPa} = 1074 \text{ N/mm}^2$$

$$\text{PIN A: } D = 30.0 \text{ mm. } F = 134 \text{ kN}$$

$$t_{1 \min} = \frac{134 \times 10^3 \text{ N}}{(30 \text{ mm})(1074 \text{ N/mm}^2)} = \underline{4.16 \text{ mm}}$$

THE REQ'D THICKNESS IS QUITE SMALL. IT IS HIGHLY LIKELY THAT ACTUAL DIMENSIONS FOR  $t_1$  AND  $t_2$  ARE MUCH LARGER FOR OTHER STRESS CONDITIONS.

$$\text{PINS F, C, AND E: ALL HAVE } D = 28 \text{ mm. } F_E = 110.8 \text{ kN}$$

$$t_{1 \min} = \frac{110 \times 10^3 \text{ N}}{(28 \text{ mm})(1074 \text{ N/mm}^2)} = \underline{3.61 \text{ mm PIN E}}$$

THIS IS ALSO VERY SMALL. PINS F AND C HAVE SLIGHTLY LOWER FORCES, SO REQ'D  $t$  IS SIMILAR.

NOTE: IF BOOM OR COLUMN ARE MADE FROM A MATERIAL WITH LOWER STRENGTH (SUCH AS STRUCTURAL STEEL), BEARING STRESS CALCULATIONS MUST BE REDONE.

$$\text{PIN D: } F_D = 34.0 \text{ kN. } D = 16 \text{ mm}$$

$$t_{1 \min} = \frac{34.0 \times 10^3 \text{ N}}{(16 \text{ mm})(1074 \text{ N/mm}^2)} = \underline{1.98 \text{ mm SMALL}}$$

## CHAPTER 4 Deformation and Thermal Stress

4-1  $\sigma = 800 \text{ psi}$ ;  $E = 1.40 \times 10^6 \text{ psi}$ ;  $L = 6.0 \text{ ft} = 12 \text{ in/ft} = 72 \text{ in}$

$$\delta = \frac{\sigma L}{E} = \frac{800 \text{ psi} (72 \text{ in})}{1.40 \times 10^6 \text{ psi}} = 0.041 \text{ in}$$

4-2  $A = (0.75)(12) = 9 \text{ mm}^2$

a) ABS;  $E = 2.5 \text{ GPa} = 2500 \text{ MPa} = 2500 \text{ N/mm}^2$

$$\delta = \frac{FL}{EA} = \frac{(90 \text{ N})(375 \text{ mm})}{(2500 \text{ N/mm}^2)(9 \text{ mm}^2)} = 1.50 \text{ mm}$$

b) PHENOLIC;  $E = 17.2 \text{ GPa} = 17200 \text{ MPa} = 17200 \text{ N/mm}^2$

$$\delta = \frac{FL}{EA} = \frac{(90)(375)}{(17200)(9)} = 0.218 \text{ mm}$$

4-3  $D_o = 2.50 \text{ in}$ ;  $D_i = 2.50 - 2(0.085) = 2.33 \text{ in}$ ;  $E = 10.6 \times 10^6 \text{ psi}$

$$A = \pi(D_o^2 - D_i^2)/4 = 0.645 \text{ in}^2$$

$$\delta = \frac{FL}{EA}; F = \frac{\delta EA}{L} = \frac{(0.005 \text{ in})(10.6 \times 10^6 \text{ psi})(0.645 \text{ in}^2)}{14.5 \text{ in}} = 2357 \text{ lb}$$

$$\sigma = \frac{F}{A} = \frac{2357 \text{ lb}}{0.645 \text{ in}^2} = 3665 \text{ psi} \quad \text{OK}; S_y = 42000 \text{ psi}$$

4-4 1) WEIGH IT - DENSITY OF ALUMINUM -  $0.10 \text{ lb/in}^3$  (WT. DENSITY)  
DENSITY OF MAGNESIUM -  $0.066 \text{ lb/in}^3$

2) EXERT A KNOWN LOAD AND MEASURE DEFORMATION.

$E$  FOR ALUMINUM -  $10 \times 10^6 \text{ psi}$

$E$  FOR MAGNESIUM -  $6.5 \times 10^6 \text{ psi}$

4-5 a) AISI 1020 HR STEEL: STRENGTH:  $\sigma_s = \frac{S_u}{S} = \frac{448}{8} = 56 \text{ MPa}$

$$\text{REQ'D. } A = \frac{F}{\sigma_s} = \frac{2500 \text{ N}}{36.0 \text{ N/mm}^2} = 69.4 \text{ mm}^2 = \pi D^2/4$$

$$\text{REQ'D. } D = \sqrt{4A/\pi} = \sqrt{4(69.4)/\pi} = 8.92 \text{ mm}$$

DEFORMATION:  $E = 207 \text{ GPa} = 207000 \text{ MPa} = 207000 \text{ N/mm}^2$

$$\text{REQ'D. } A = \frac{FL}{E\delta} = \frac{(3500 \text{ N})(630 \text{ mm})}{(207000 \text{ N/mm}^2)(0.12 \text{ mm})} = 88.8 \text{ mm}^2 = \frac{\pi D^2}{4}$$

$$\text{REQ'D. } D = \sqrt{4A/\pi} = 10.63 \text{ mm} \quad \text{GOVERNS}$$

b) AISI 4140 Q&T 700 STEEL IS MUCH STRONGER THAN AISI 1020 HR.

THEREFORE: DEFORMATION GOVERNS.  $E$  IS SAME.

$$\text{REQ'D. } D = 10.63 \text{ mm}$$

c) ALUM. 6061-T6: STRENGTH:  $\sigma_s = \frac{S_u}{S} = \frac{310 \text{ MPa}}{8} = 38.75 \text{ MPa}$

$$\text{REQ'D. } A = F/\sigma_s = 3500 \text{ N}/38.75 \text{ N/mm}^2 = 90.32 \text{ mm}^2$$

$$\text{REQ'D. } D = \sqrt{4A/\pi} = 10.7 \text{ mm}$$

DEFORMATION:  $E = 69 \text{ GPa} = 69000 \text{ MPa} = 69000 \text{ N/mm}^2$

$$\text{REQ'D. } A = \frac{FL}{E\delta} = \frac{(3500)(630)}{(69000)(0.12)} = 266.3 \text{ mm}^2 \quad \text{GOVERNS}$$

$$\text{REQ'D. } D = \sqrt{4A/\pi} = 18.4 \text{ mm} \quad (\text{CONT. NEXT PAGE})$$



4-5 (CONTINUED)  $MASS = VOL \times DENSITY = A \times L \times DENSITY$   
 STEEL:  $MASS = (88.8 \text{ mm}^2)(630 \text{ mm})(7680 \text{ kg/m}^3) \frac{1 \text{ m}^3}{(10^3 \text{ mm})^3} = 0.430 \text{ kg}$

$ALUMI \text{ MASS} = (266.3)(630)(2770)/10^9 = 0.465 \text{ kg}$

4-6  $A = \pi D^2/4 = \pi(12)^2/4 = 113.1 \text{ mm}^2$   
 $\delta = \frac{FL}{EA} = \frac{(17.0 \times 10^3 \text{ N})(220 \text{ mm})}{(207000 \text{ N/mm}^2)(113.1 \text{ mm}^2)} = 0.160 \text{ mm}$

4-7 a) Ti-6AL-4V:  $E = 114 \text{ GPa} = 114000 \text{ MPa} = 114000 \text{ N/mm}^2$   
 $\delta = \frac{FL}{EA} = \frac{(6000 \text{ N})(1250 \text{ mm})}{(114000 \text{ N/mm}^2)(64 \text{ mm}^2)} = 0.857 \text{ mm}$

b) AISI 501 OQT 1800 STEEL:  $E = 200 \text{ GPa} = 200000 \text{ N/mm}^2$   
 $\delta = \frac{FL}{EA} = \frac{(5000 \text{ N})(1250)}{(200000)(64)} = 0.488 \text{ mm}$

4-8 Req'd.  $A = \frac{F}{\sigma} = \frac{35000 \text{ LB}}{21600 \text{ LB/in}^2} = 1.620 \text{ in}^2$ ; Use  $L \times W \times \frac{1}{4}$ ;  $A = 1.94 \text{ in}^2$   
 $\delta = \frac{FL}{AE} = \frac{(25000 \text{ LB})(13.0 \text{ FT})(12 \text{ in/FT})}{(1.94 \text{ in}^2)(29 \times 10^6 \text{ LB/in}^2)} = 0.097 \text{ in}$

4-9 ELONGATION:  $\delta = \frac{FL}{EA} = \frac{(450 \text{ LB})(8.40 \text{ in})}{(30 \times 10^6 \text{ LB/in}^2)(.125)(.125) \text{ in}^2} = 0.0040 \text{ in}$   
 COMPRESSION:  $\delta = \frac{FL}{EA} = \frac{(50)(8.40)}{(30 \times 10^6)(.125)(.125)} = 0.00045 \text{ in}$

4-10 LOWER SECTION:  $\delta_1 = \frac{F_1 L_1}{EA} = \frac{(5000)(10)}{(80 \times 10^6)(.50)} = 0.0031 \text{ in}$

MID SECTION:  $\delta_2 = \frac{F_2 L_2}{EA} = \frac{(10000)(15)}{(30 \times 10^6)(.50)} = 0.0070 \text{ in}$

TOP SECTION:  $\delta_3 = \frac{F_3 L_3}{EA} = \frac{(10500)(25)}{(30 \times 10^6)(0.50)} = 0.0175 \text{ in}$

TOTAL:  $\delta_T = \delta_1 + \delta_2 + \delta_3 = 0.0278 \text{ in}$

4-11  $A = \pi(D_o^2 - D_i^2)/4 = \pi(1.25^2 - 1.126^2)/4 = 0.2314 \text{ in}^2$   
 $F = \frac{\sigma_c A}{L} = \frac{(0.050 \text{ in})(10 \times 10^6 \text{ LB/in}^2)(0.2314 \text{ in}^2)}{36.0 \text{ in}} = 3214 \text{ LB}$

$\sigma = F/A = 3214 \text{ LB} / 0.2314 \text{ in}^2 = 13900 \text{ PSI}$  700 H114

$\sigma_B = \frac{S_M}{B} = \frac{45000 \text{ PSI}}{8} = 5625 \text{ PSI}$

4-12  $A = \pi(D_o^2 - D_i^2)/4 = \pi(0.75^2 - 0.563^2)/4 = 0.1928 \text{ in}^2$   
 $\sigma = F/A = 2500 \text{ LB} / 0.1928 \text{ in}^2 = 12964 \text{ PSI} = \sigma_2 = S_1/2; S_1 = 25930 \text{ PSI}$   
 ALUMINUM 3003-H18 HAS  $S_y = 27000 \text{ PSI}$   
 $\delta = \frac{FL}{EA} = \frac{(2500 \text{ LB})(8.75 \text{ FT})(12 \text{ in/FT})}{(10 \times 10^6 \text{ LB/in}^2)(0.1928 \text{ in}^2)} = 0.136 \text{ in}$

$$\begin{aligned} \underline{4-13} \quad A &= \frac{\pi(D_o^2 - D_i^2)}{4} = \frac{\pi(56^2 - 48^2)}{4} = 653 \text{ mm}^2 \\ \delta &= \frac{FL}{EA} = \frac{(18.2 \times 10^3 \text{ N})(40 \text{ mm})}{(69000 \text{ N/mm}^2)(653 \text{ mm}^2)} = 0.016 \text{ mm} \\ \sigma &= \frac{F}{A} = \frac{18.2 \times 10^3 \text{ N}}{653 \text{ mm}^2} = 27.9 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \underline{4-14} \quad A &= \pi D^2/4 = \pi(0.375)^2/4 = 0.1104 \text{ in}^2 \\ \sigma &= \frac{F}{A} = \frac{1600 \text{ LB}}{0.1104 \text{ in}^2} = 14500 \text{ PSI} \\ \delta &= \frac{FL}{EA} = \frac{(1600 \text{ LB})(135 \text{ FT})(12 \text{ in/FT})}{(30 \times 10^6 \text{ LB/in}^2)(0.1104 \text{ in}^2)} = 0.782 \text{ in} \end{aligned}$$

$$\begin{aligned} \underline{4-15} \quad A &= (30 \text{ mm})^2 = 900 \text{ mm}^2; E = 114 \text{ GPa} = 114000 \text{ N/mm}^2 \\ F_{AB} &= 110 + 80 - 40 = 150 \text{ kN TENSION} \\ F_{BC} &= 110 - 40 = 70 \text{ kN TENSION} \\ F_{CD} &= 110 \text{ kN TENSION} \\ \delta_r &= \delta_{AB} + \delta_{BC} + \delta_{CD} = \frac{F_{AB} L}{EA} + \frac{F_{BC} L}{EA} + \frac{F_{CD} L}{EA} \\ \delta_r &= \frac{(150 \times 10^3 \text{ N})(250 \text{ mm})}{(114000 \text{ N/mm}^2)(900 \text{ mm}^2)} + \frac{(70 \times 10^3)(250)}{(114000)(900)} + \frac{(110 \times 10^3)(250)}{(114000)(900)} \\ \delta_r &= 0.365 \text{ mm} + 0.171 \text{ mm} + 0.268 \text{ mm} = 0.804 \text{ mm} \end{aligned}$$

$$\begin{aligned} \underline{4-16} \quad E &= \frac{FL}{\delta A} = \frac{(10000 \text{ LB})(10.0 \text{ in})}{(0.023 \text{ in})(\pi(0.75)^2/4) \text{ in}^2} = 9.84 \times 10^6 \text{ PSI} \\ \text{PROBABLY ALUMINUM: } E &\approx 10.0 \times 10^6 \text{ PSI} \end{aligned}$$

$$\begin{aligned} \underline{4-17} \quad A_{AB} &= A_{BC} = \frac{\pi(25)^2}{4} = 491 \text{ mm}^2; A_{CD} = \frac{\pi(16)^2}{4} = 201 \text{ mm}^2 \\ F_{AB} &= -9.65 - 12.32 + 4.45 = -17.52 \text{ kN (COMP.)}; L_{AB} = 80 \text{ mm} \\ F_{BC} &= -9.65 - 12.32 = -21.97 \text{ kN (COMP.)}; L_{BC} = 100 \text{ mm} \\ F_{CD} &= -9.65 \text{ kN (COMP.)}; L_{CD} = 120 \text{ mm} \\ E &= 59 \text{ GPa} = 5900 \text{ N/mm}^2 \\ \delta_{AD} &= \frac{F_{AB} L_{AB}}{E A_{AB}} + \frac{F_{BC} L_{BC}}{E A_{BC}} + \frac{F_{CD} L_{CD}}{E A_{CD}} \\ \delta_{AD} &= \frac{(-17.52 \times 10^3 \text{ N})(80 \text{ mm})}{(5900 \text{ N/mm}^2)(491 \text{ mm}^2)} + \frac{(-21.97 \times 10^3)(100)}{(5900)(491)} + \frac{(-9.65 \times 10^3)(120)}{(5900)(201)} \\ \delta_{AD} &= -0.4838 \text{ mm} - 0.7584 \text{ mm} - 0.9765 \text{ mm} = -2.22 \text{ mm} \\ &\quad \underline{\text{SHORTER}} \end{aligned}$$

4-18  $\sigma_c = F/A_c = 64000 \text{ lb} / 50.27 \text{ in}^2 = 1273 \text{ psi}$ ; LET  $\sigma_c > 4\sigma_c = 5092 \text{ psi}$

USE  $\sigma_s = 6000 \text{ psi}$  RATED: THEN  $E = 4.7 \times 10^6 \text{ psi}$  (SEC 2-10)  
 $A_c = \pi(8)^2/4 = 50.27 \text{ in}^2$ ;  $A_T = 6.36 \text{ in}^2$  (APP. A-9)

$$\delta = \delta_c + \delta_T = \frac{(64000 \text{ lb})(3.0 \text{ ft})(12 \text{ in/ft})}{(4.7 \times 10^6 \text{ lb/in}^2)(50.27 \text{ in}^2)} + \frac{(64000)(8.6)(12)}{(30 \times 10^6)(6.36)}$$

$$\delta = 0.0098 \text{ in} + 0.0346 \text{ in} = \underline{0.0444 \text{ in}}$$

4-19 a)  $\delta = \frac{FL}{EA} = \frac{(120 \text{ lb})(10.5 \text{ ft})(12 \text{ in/ft})}{(17 \times 10^6 \text{ lb/in}^2)(0.00322 \text{ in}^2)} = \underline{0.276 \text{ in.}}$

$$A = \pi D^2/4 = \pi(0.064)^2/4 = 0.00322 \text{ in}^2$$

$$\sigma = F/A = 120 \text{ lb} / 0.00322 \text{ in}^2 = 37300 \text{ psi}; \text{ CLOSE TO } S_y = 44000 \text{ psi}$$

b)  $\sigma = \frac{F}{A} = 200 \text{ lb} / 0.00322 \text{ in}^2 = 62200 \text{ psi}$ ; THIS EXCEEDS THE ULTIMATE STRENGTH OF THE WIRE. IT SHOULD BREAK.

4-20  $\sigma = F/A = 25 \text{ lb} / (0.006)(0.75 \text{ in}^2) = \underline{5556 \text{ psi}}$

$$\delta = \frac{FL}{EA} = \frac{(25 \text{ lb})(2.5 \text{ ft})(12 \text{ in/ft})}{(30 \times 10^6 \text{ lb/in}^2)(0.0045 \text{ in}^2)} = \underline{0.0556 \text{ in}}$$

4-21  $F = \sigma_{\text{allow}} \cdot A = (550 \text{ lb/in}^2)(12.25 \text{ in}^2) = \underline{6737 \text{ lb}}$

$$\delta = \frac{FL}{EA} = \frac{(6737 \text{ lb})(10.75 \text{ ft})(12 \text{ in/ft})}{(1300000 \text{ lb/in}^2)(12.25 \text{ in}^2)} = \underline{0.055 \text{ in}}$$

4-22  $A = (200^2 - 150^2) = 17500 \text{ mm}^2$

$$F = \sigma \cdot A = (-200 \text{ N/mm}^2)(17500 \text{ mm}^2) = -3.5 \times 10^6 \text{ N} = \underline{-3.50 \text{ MN}}$$

$$\delta = \frac{\sigma L}{E} = \frac{(200 \text{ N/mm}^2)(1800 \text{ mm})}{(165000 \text{ N/mm}^2)} = \underline{-2.18 \text{ mm}}$$

4-23  $A = \pi D^2/4 = \pi(3.0)^2/4 = 7.07 \text{ mm}^2$ ; (ASSUME MASS OF PLATE IS SMALL)

$$F = \frac{\delta EA}{L} = \frac{(6.0 \text{ mm})(110000 \text{ N/mm}^2)(7.07 \text{ mm}^2)}{3600 \text{ mm}} = \underline{1296 \text{ N}}$$

$$\sigma = F/A = 1296 \text{ N} / 7.07 \text{ mm}^2 = \underline{183 \text{ MPa}} \text{ (LESS THAN } S_y)$$

$$m = W/g = 1296 \text{ N} / 9.81 \text{ m/s}^2 = 132 \text{ N} \cdot \text{s}^2/\text{m} = \underline{132 \text{ kg}}$$

4-24  $\sigma = 50 \text{ MPa}$  (PROB 1-53);  $\delta = \sigma L/E = \frac{(50 \text{ N/mm}^2)(120 \text{ mm})}{69000 \text{ N/mm}^2} = \underline{0.086 \text{ mm}}$

4-25  $\delta = \alpha L(\Delta T) = (6.0 \times 10^{-6} \text{ in/in}^\circ \text{F})(80 \text{ ft})(140^\circ \text{F})(12 \text{ in/ft}) = \underline{0.806 \text{ in}}$

4-26  $\delta = \alpha L(\Delta T) = (11.3 \times 10^{-6} \text{ in/in}^\circ \text{C})(12000 \text{ mm})(77^\circ \text{C}) = \underline{10.4 \text{ mm}}$

4-27  $\sigma = E \alpha(\Delta T) = (207 \times 10^9 \text{ Pa})(11.3 \times 10^{-6} \text{ in/in}^\circ \text{C})(77^\circ \text{C}) = \underline{180 \text{ MPa}} \text{ HIGH!}$

4-28  $\delta = \alpha L(\Delta T) = (11.3 \times 10^{-6} \text{ in/in}^\circ \text{C})(625 \text{ mm})(156^\circ \text{C}) = \underline{1.10 \text{ mm}}$

4-29 a)  $\delta = \alpha L(\Delta T) = (11.3 \times 10^{-6} \text{ in/in}^\circ \text{C})(625 \text{ mm})(65^\circ \text{C}) = \underline{0.459 \text{ mm}}$

b)  $\sigma = E \alpha(\Delta T) = (207 \times 10^9 \text{ Pa})(11.3 \times 10^{-6} \text{ in/in}^\circ \text{C})(91^\circ \text{C}) = \underline{213 \text{ MPa}}$

4-30  $\delta = \alpha L(\Delta T) = (6.0 \times 10^{-6} \text{ in/in}^\circ \text{F})(140 \text{ ft})(80^\circ \text{F})(12 \text{ in/ft}) = \underline{0.806 \text{ in.}}$   
 THIS IS THE TOTAL WIDTH. EACH END COULD BE 0.41 in.

4-31 DELK COULD EXPAND BY A TOTAL OF 0.50 IN WITHOUT STRESS.

$$\text{REQ'D } \Delta t = \frac{\delta}{\alpha L} = \frac{0.50 \text{ IN}}{(6.0 \times 10^{-6} \text{ } ^\circ\text{F}^{-1})(140 \text{ FT})(12 \text{ IN/FT})} = 49.6 \text{ } ^\circ\text{F}$$

$$t_2 = t_1 + \Delta t = 30 + 49.6 = 79.6 \text{ } ^\circ\text{F}$$

$$\text{REMAINING } \Delta t = 110 \text{ } ^\circ\text{F} - 79.6 \text{ } ^\circ\text{F} = 30.4 \text{ } ^\circ\text{F}$$

$$\sigma = E \alpha (\Delta t) = (3.8 \times 10^6 \text{ PSI})(6.0 \times 10^{-6} \text{ } ^\circ\text{F}^{-1})(30.4 \text{ } ^\circ\text{F}) = \underline{693 \text{ PSI}}$$

4-32  $\Delta t = 110 \text{ } ^\circ\text{F} - 60 \text{ } ^\circ\text{F} = 50 \text{ } ^\circ\text{F}$

$$\delta = \alpha L (\Delta t) = (6.0 \times 10^{-6} \text{ } ^\circ\text{F}^{-1})(140 \times 12 \text{ IN})(50 \text{ } ^\circ\text{F}) = \underline{0.504 \text{ IN}}$$

4-33  $\delta = \pi(55.300) - \pi(55.100) = 0.2 \pi \text{ mm}$  (CHANGE IN CIRCUMFERENCE)

$$\Delta t = \frac{\delta}{\alpha L} = \frac{0.2 \pi \text{ mm}}{(16.9 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(\pi)(55.100 \text{ mm})} = 214.8 \text{ } ^\circ\text{C}$$

$$t_2 = t_1 + \Delta t = 20 + 214.8 = \underline{234.8 \text{ } ^\circ\text{C}}$$

4-34 FOR FIRST PART OF COOLING, RING IS UNRESTRAINED UNTIL ITS DIAMETER GETS TO 55.200 mm.  $\delta = 55.300 - 55.200 = 0.100 \text{ mm}$

$$\Delta t = \frac{\delta}{\alpha L} = \frac{0.100 \text{ mm}}{(16.9 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(55.200 \text{ mm})} = 107.2 \text{ } ^\circ\text{C}$$

$$t_2 = t_1 - \Delta t = 234.8 \text{ } ^\circ\text{C} - 107.2 \text{ } ^\circ\text{C} = 127.6 \text{ } ^\circ\text{C}$$

$$\text{ADDITIONAL } \Delta t = 127.6 - 20 = 107.6 \text{ } ^\circ\text{C}$$

$$\sigma = E \alpha (\Delta t) = (193 \times 10^9 \text{ Pa})(16.9 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(107.6 \text{ } ^\circ\text{C}) = \underline{351 \text{ MPa}}$$

4-35  $\delta_{\text{LOSS}} = \alpha L (\Delta t) = (20.5 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(4200 \text{ mm})(75 \text{ } ^\circ\text{C}) = \underline{6.46 \text{ mm}}$

$$\delta_{\text{CS}} = \alpha L (\Delta t) = (10.4 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(4500 \text{ mm})(75 \text{ } ^\circ\text{C}) = \underline{3.51 \text{ mm}}$$

4-36  $\delta = \alpha L (\Delta t) = (6.5 \times 10^{-6} \text{ } ^\circ\text{F}^{-1})(40 \times 12 \text{ IN})(190 \text{ } ^\circ\text{F}) = \underline{0.593 \text{ IN}}$

4-37 INITIAL EXPANSION OF 0.10 mm IS UNRESTRAINED.

$$\text{REQ'D } \Delta t = \frac{\delta}{\alpha L} = \frac{0.10 \text{ mm}}{(25.2 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(250 \text{ mm})} = 15.9 \text{ } ^\circ\text{C}$$

$$t_2 = t_1 + \Delta t = 20 \text{ } ^\circ\text{C} + 15.9 \text{ } ^\circ\text{C} = 35.9 \text{ } ^\circ\text{C}$$

$$\text{ADD. } \Delta t = 70 \text{ } ^\circ\text{C} - 35.9 \text{ } ^\circ\text{C} = 34.1 \text{ } ^\circ\text{C} - \text{RESTRAINED}$$

$$\sigma = E \alpha \Delta t = (45 \times 10^9 \text{ Pa})(25.2 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(34.1 \text{ } ^\circ\text{C}) = \underline{38.7 \text{ MPa}}$$

4-38  $\sigma = E \alpha \Delta t = (207 \times 10^9 \text{ Pa})(11.3 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(90 \text{ } ^\circ\text{C}) = \underline{211 \text{ MPa}}$

4-39  $S_1 = 10.505 - 10.500 = 0.005 \text{ IN}$  UNRESTRAINED

$$\Delta t_1 = \frac{\delta}{\alpha L} = \frac{0.005 \text{ IN}}{(13.0 \times 10^{-6} \text{ } ^\circ\text{F}^{-1})(10.500 \text{ IN})} = 36.6 \text{ } ^\circ\text{F}$$

$$t_2 = t_1 + \Delta t_1 = 75 + 36.6 = 111.6 \text{ } ^\circ\text{F}$$

$$\text{ADD. } \Delta t = 400 - 111.6 = 288.4 \text{ } ^\circ\text{F} - \text{RESTRAINED}$$

$$\sigma = E \alpha \Delta t = (10 \times 10^6 \text{ PSI})(13.0 \times 10^{-6} \text{ } ^\circ\text{F}^{-1})(288.4 \text{ } ^\circ\text{F}) = \underline{37500 \text{ PSI}}$$

FOR 6061-TY,  $S_u = 35000 \text{ PSI}$ , BAR SHOULD FAIL.

ALSO, BECAUSE BARS IN COMPRESSION IT MAY BULGE.

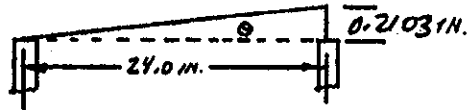
$$\underline{4-40} \quad \delta_p = \alpha L (\Delta t) = (53.0 \times 10^{-6} \text{ } ^\circ\text{F}^{-1})(30.0 \text{ m})(212 - 65)^\circ\text{F} = 0.2337 \text{ m.}$$

$$\delta_{Ti} = \alpha L (\Delta t) = (5.3 \times 10^{-6} \text{ } ^\circ\text{F}^{-1})(30.0 \text{ m})(117^\circ\text{F}) = 0.0234 \text{ m}$$

$$\delta_p - \delta_{Ti} = 0.2103 \text{ m}$$

$$\tan \theta = \frac{0.2103}{24} = 0.00876$$

$$\theta = 0.502 \text{ deg.}$$



$$\underline{4-41} \quad \delta = \alpha L (\Delta t) = (6.3 \times 10^{-6} \text{ } ^\circ\text{F}^{-1})(25 \text{ FT})(12 \text{ in/FT})(68 - (-15))^\circ\text{F} = 0.157 \text{ in}$$

$$\underline{4-42} \quad \text{TOTAL } \delta_T = 0.50 \text{ mm} = 0.00050 \text{ m.} = \delta_S + \delta_B$$

$$\delta_T = \alpha_S L_S (\Delta t) + \alpha_B L_B (\Delta t) = \Delta t (\alpha_S L_S + \alpha_B L_B)$$

$$\Delta t = \frac{\delta_T}{\alpha_S L_S + \alpha_B L_B} = \frac{0.0005 \text{ m}}{(10.4 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(2.800 \text{ m}) + (20.5 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(1.900 \text{ m})}$$

$$\Delta t = 7.35^\circ\text{C}; \quad t_2 = t_1 + \Delta t = 20 + 7.35 = 27.35^\circ\text{C}$$

$$\underline{4-43} \quad \text{ADDED } \sigma = E \alpha \Delta t = (193 \times 10^9 \text{ Pa})(16.9 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(35^\circ\text{C}) = 114 \text{ MPa}$$

$$\text{FINAL } \sigma = 40 + 114 = 154 \text{ MPa}$$

$$\underline{4-44} \quad \text{WHEN HEATED, WIRE WOULD RELAX.}$$

$$\Delta t = \frac{\sigma}{E \alpha} = \frac{40 \times 10^6 \text{ Pa}}{(193 \times 10^9 \text{ Pa})(16.9 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})} = 12.3^\circ\text{C}$$

$$t_2 = t_1 + \Delta t = 20^\circ\text{C} + 12.3^\circ\text{C} = 32.3^\circ\text{C}$$

$$\underline{4-45} \quad A_s = 150^2 - 130^2 = 5600 \text{ mm}^2; \quad A_c = 130^2 = 16900 \text{ mm}^2$$

$$\sigma_c = \frac{F_{EC}}{A_s E_s + A_c E_c} = \frac{(900 \times 10^3 \text{ N})(32.4 \text{ GPa})}{(5600 \text{ mm}^2)(207 \text{ GPa}) + (16900 \text{ mm}^2)(32.4 \text{ GPa})} = 17.1 \text{ MPa}$$

$$\sigma_s = \sigma_c E_s / E_c = (17.1 \text{ MPa})(207 \text{ GPa}) / 32.4 \text{ GPa} = 109 \text{ MPa}$$

$$\underline{4-46} \quad A_s = 10.4 \text{ in}^2 \text{ (APR A-9)}; \quad A_c = (5.0 \text{ in})^2 = 25.0 \text{ in}^2$$

$$E_s = 30 \times 10^6 \text{ psi}; \quad E_c = 4.7 \times 10^6 \text{ psi (SEC 2-10)}$$

$$\text{LET } \sigma_c = \sigma_{sc} = 1500 \text{ psi}; \quad \text{THEN } \sigma_s < \sigma_{ss} = 21600 \text{ psi}$$

$$F = \frac{\sigma_c [A_s E_s + A_c E_c]}{E_c} = \frac{1500 \text{ psi} [(10.4)(30 \times 10^6) + 25(4.7 \times 10^6)]}{4.7 \times 10^6}$$

$$F = 137,000 \text{ LB}; \quad \text{CHECK } \sigma_s = \sigma_c \frac{E_s}{E_c} = 1500 \left( \frac{30}{4.7} \right) = 9575 \text{ psi}$$

$$\underline{4-47} \quad \text{LET } \sigma_c = 1500 \text{ psi}; \quad \sigma_s = \sigma_c \frac{E_s}{E_c} = 1500 \frac{30}{4.7} = 9575 \text{ psi} - \text{OK}$$

$$\frac{F}{\sigma_c} = \frac{A_s E_s + A_c E_c}{E_c} = \frac{A_s E_s}{E_c} + A_c = A_s \frac{30}{4.7} + A_c = 6.38 A_s + A_c$$

$$A_s = b^2 - (b - 2t)^2 = b^2 - [b - 2(0.5)]^2 = b^2 - (b - 1)^2 = b^2 - b^2 + 2b - 1 = 2b - 1$$

$$A_c = (b - 1)^2 = b^2 - 2b + 1$$

$$\frac{F}{\sigma_c} = 6.38(2b - 1) + b^2 - 2b + 1 = b^2 + 10.776b - 5.38$$

$$\frac{F}{\sigma_c} = \frac{500,000}{1500} = 333.3 = b^2 + 10.776b - 5.38$$

$$b^2 + 10.776b - 338.7 = 0$$

$$\text{BY QUADRATIC EQN., } b = 13.8 \text{ in}$$

4-48  $A_s = A_A = 2 \pi (L)^2 / 4 = 56.55 \text{ mm}^2$   
 $\sigma_A = \frac{P E_A}{A_s E_s + A_A E_A} = \frac{(11.3 \times 10^3 \text{ N}) / (69 \text{ GPa})}{(56.55 \text{ mm}^2)(207 \text{ GPa}) + (56.55 \text{ mm}^2)(69 \text{ GPa})} = 49.5 \text{ MPa}$   
 $\sigma_s = \sigma_A \cdot E_s / E_A = 49.5 \text{ MPa} \cdot 207 / 69 = 148.5 \text{ MPa}$

4-49  $W = m \cdot g = 22.65 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 22.22 \times 10^3 \text{ N}$ ;  $A_s = 2 A_c$   
 IF  $\sigma_s = 552 \text{ MPa} / 2 = 276 \text{ MPa}$ ; THEN  $\sigma_c = \sigma_s \cdot \frac{E_s}{E_c} = 276 \cdot \frac{131}{200} = 181 \text{ MPa}$   
 $\sigma_{\text{CALLON}} = S_y / 2 = 1379 / 2 = 689 \text{ MPa OK}$   
 $\sigma_c = \frac{P E_c}{A_s E_s + A_c E_c} = \frac{P E_c}{2 A_c E_s + A_c E_c} = \frac{P E_c}{A_c [2 E_s + E_c]}$   
 $A_c = \frac{P E_c}{\sigma_c [2 E_s + E_c]} = \frac{(22.22 \times 10^3 \text{ N}) / (131 \text{ GPa})}{(181 \text{ N/mm}^2) [2(200) + 131] \text{ GPa}} = 30.3 \text{ mm}^2$   
 $D = \sqrt{4 A_c / \pi} = \sqrt{4(30.3) / \pi} = 6.20 \text{ mm} = \text{WIRE DIA.}$

4-50 1) COMPUTE FORCE REQ'D. TO DEFLECT ALUM. ROD 0.12 mm  
 $P_1 = \frac{\delta E A}{L} = \frac{(0.12 \text{ mm}) (69000 \text{ N/mm}^2) (15394 \text{ mm}^2)}{1650.12 \text{ mm}} = 77.24 \text{ kN}$   
 2) ADDITIONAL LOAD AVAILABLE:  $P_2 = 350 \text{ kN} - 77.24 \text{ kN} = 272.76 \text{ kN}$   
 3) BOTH MEMBERS DEFLECT THE SAME AMOUNT UNDER  $P_2$   
 $\delta_A = \delta_s : \frac{P_A L_A}{E_A A_A} = \frac{P_s L_s}{E_s A_s} \quad [L_A = L_s]$   
 $P_A = P_s \frac{E_A A_A}{E_s A_s} = P_s \frac{(69 \text{ GPa}) (15394 \text{ mm}^2)}{(207 \text{ GPa}) (3600 \text{ mm}^2)} = 1.425 P_s$   
 6 IN SCH 40 PIPE:  $A_s = 5.581 \text{ in}^2 (25.4 \text{ mm}^2 / \text{in}^2) = 3600 \text{ mm}^2$   
 4)  $P_s + P_A = P_2 = 272.76 \text{ kN}$   
 $P_s + 1.425 P_s = 2.425 P_s = 272.76 \text{ kN}$   
 $P_s = 191.39 \text{ kN}$   
 $P_A = 272.76 - 191.39 = 81.37 \text{ kN}$   
 5) TOTAL LOAD ON ALUM:  $P_{AT} = P_1 + P_A = 77.24 + 81.37 = 158.61 \text{ kN}$   
 6) STRESSES:  
 $\sigma_s = \frac{P_s}{A_s} = \frac{191.39 \times 10^3 \text{ N}}{3600 \text{ mm}^2} = 53.2 \text{ MPa}$   
 $\sigma_A = \frac{P_{AT}}{A_A} = \frac{158.61 \times 10^3 \text{ N}}{15394 \text{ mm}^2} = 10.3 \text{ MPa}$

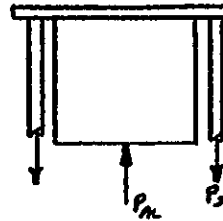
4-51

- 1) THE NUT MOVES 1.25 mm IN ONE TURN
- 2) THE FORCE CREATED CAUSES THE TUBE TO SHORTEN AND THE BOLTS TO GET LONGER.  $\delta_{AL} + \delta_S = 1.25 \text{ mm}$ ; AND  $L_{AL} = L_S$
- 3) THE COMPRESSIVE FORCE IN THE TUBE  
EQUALS THE TENSILE FORCE IN THE BOLTS

$$P_{AL} = P_S \text{ (ON ALL FOUR BOLTS)}$$

$$4) A_{AL} = \frac{\pi}{4} (150^2 - 138^2) = 2714 \text{ mm}^2$$

$$A_S = 4\pi(10^2)/4 = 314 \text{ mm}^2$$



$$5) \frac{P_{AL} L_{AL}}{E_{AL} A_{AL}} + \frac{P_S L_S}{E_S A_S} = 1.25 \text{ mm}$$

$$\frac{P_S (450) \text{ mm}}{(69 \times 10^9 \text{ N/mm}^2)(2714 \text{ mm}^2)} + \frac{P_S (450)}{(207 \times 10^9)(314)} = 1.25$$

$$P_S [2.40 \times 10^{-6} + 6.92 \times 10^{-6}] = P_S [9.32 \times 10^{-6}] = 1.25$$

$$P_S = 134 \text{ kN} = P_{AL}$$

6) STRESSES:

$$\sigma_S = \frac{P_S}{A_S} = \frac{134000 \text{ N}}{314 \text{ mm}^2} = 427 \text{ MPa}$$

$$\sigma_{AL} = \frac{P_{AL}}{A_{AL}} = \frac{134000 \text{ N}}{2714 \text{ mm}^2} = 49.4 \text{ MPa}$$

4-52

$$\sigma_c = \frac{P E_c}{A_S E_S + A_C E_C} = \frac{(50000 \text{ Lb})(2.7 \times 10^6 \text{ psi})}{(4.43 \text{ in}^2)(29 \times 10^6 \text{ psi}) + (16.7 \text{ in}^2)(2.7 \times 10^6 \text{ psi})} = 320 \text{ psi}$$

$$A_S = 4.43 \text{ in}^2; A_C = \pi(12)^2/4 - 4.43 = 108.7 \text{ in}^2$$

$$\sigma_S = \sigma_c \cdot E_c / E_S = (320 \text{ psi})(29/2.7) = 3436 \text{ psi}$$

NOTE:  $E_S = 29 \times 10^6 \text{ psi}$  FOR STRUCTURAL STEEL

$E_C = 2.7 \times 10^6 \text{ psi}$  FOR CONCRETE WITH  $\sigma_c = 200 \text{ psi}$   
RATED

(SEE SEC. 2-10)

4-53

N & 2 SOUTHERN PINE.  $\sigma_{\text{ALLOWABLE}} = 1.59 \text{ MPa}$ ;  $E = 9.0 \text{ GPa}$

4x4 POST.  $A = 7.96 \times 10^3 \text{ mm}^2$ ;  $L = 4.25 \text{ m}$

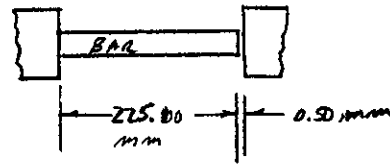
$$\sigma = \frac{F}{A}; F_{\text{MAX}} = \sigma_{\text{ALL}} \cdot A = \frac{1.59 \text{ N}}{\text{mm}^2} \times 7.96 \times 10^3 \text{ mm}^2 = \frac{12560 \text{ N}}{12.56 \text{ kN}}$$

$$\delta = \frac{F L}{E A} = \frac{(12560 \text{ N})(4.25 \text{ m})}{(9.0 \times 10^9 \text{ N/m}^2)(7.96 \times 10^3 \text{ mm}^2)} \times \frac{(10^3 \text{ mm})^2}{\text{m}^2}$$

$$\delta = 7.51 \times 10^{-4} \text{ m} = 0.751 \text{ mm}$$

4-54

$t_1 = 20^\circ\text{C}$   $L_1 = 225.0 \text{ mm}$   
 $t_{\text{FINAL}} = 205^\circ\text{C}$ . ALUM 6061-T4,  $S_y = 110 \text{ MPa}$



a) TEMP. AT WHICH BAR TOUCHES PLATE

$$\delta = 0.50 \text{ mm} = \alpha L (\Delta t)$$

$$\Delta t_1 = \frac{\delta}{\alpha L} = \frac{0.50 \text{ mm}}{(23.4 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(225.0 \text{ mm})} = 95.0^\circ\text{C}$$

$$t_2 = t_1 + \Delta t = 20 + 95 = 115^\circ\text{C} \text{ NO STRAIN AT THIS TEMP.}$$

b) ADDITIONAL  $\Delta t_2 = 205^\circ\text{C} - 115^\circ\text{C} = 90.0^\circ\text{C}$  RESTRAINED.

$$\sigma = E \alpha (\Delta t) = (69 \times 10^9 \text{ Pa})(23.4 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(90.0^\circ\text{C})$$

$$\sigma = 145 \times 10^6 \text{ Pa} = 145 \text{ MPa} > S_y \text{ MATERIAL WOULD YIELD-FAILURE COMPRESSION}$$

4-55

(a)  $t_1 = 20^\circ\text{C}$ .  $L_1 = 2.400 \text{ m}$ . ALUM. 2014-T4,  $L_2 = 2.405 \text{ m}$

$$\delta = L_2 - L_1 = 2.405 - 2.400 = 0.005 \text{ m} = 5.00 \text{ mm} = \alpha L (\Delta t)$$

$$\Delta t_1 = \frac{\delta}{\alpha L_1} = \frac{0.005 \text{ m}}{(23.0 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(2.400 \text{ m})} = 90.6^\circ\text{C}$$

$$t_2 = t_1 + \Delta t_1 = 20 + 90.6 = 110.6^\circ\text{C}$$

(b) INCREASE  $30^\circ\text{C}$ .  $t_3 = 110.6^\circ\text{C} + 30 = 140.6^\circ\text{C}$  RESTRAINED.

$$\sigma = E \alpha \Delta t_2 = (73 \times 10^9 \text{ Pa})(23.0 \times 10^{-6} \text{ } ^\circ\text{C}^{-1})(30^\circ\text{C})$$

$$\sigma = 50.4 \times 10^6 \text{ Pa} = 50.4 \text{ MPa} \text{ COMPRESSION}$$

(c)  $S_y = 290 \text{ MPa}$  FOR 2014-T4. SAFE AGAINST YIELDING. BUT BUCKLING SHOULD BE CHECKED.

4-56

$\delta_{\text{MAX}} = 0.50 \text{ mm}$ . REPEATED AXIAL TENSILE LOAD.

SPECIFY MAX. PERMISSIBLE LOAD. 4140 DOT 1300.  $E = 207 \text{ GPa}$

a) DEFORMATION:  $\delta = \frac{FL}{EA}$ . THEN  $F_{\text{MAX}} = \frac{\delta EA}{L}$

$$A = 30 \times 20 = 600 \text{ mm}^2, L = 700 \text{ mm}$$

$$F_{\text{MAX}} = \frac{(0.50 \text{ mm})(207 \times 10^9 \text{ N})}{(700 \text{ mm})} \times \frac{600 \text{ mm}^2}{1 \text{ m}^2} \times \frac{1 \text{ m}^2}{(10^3 \text{ mm}^2)}$$

$$F_{\text{MAX}} = 88.7 \times 10^3 \text{ N} = 88.7 \text{ kN}$$

(b) STRESS:  $\sigma = S_y / \delta = 814 \text{ MPa} / \delta = 101.8 \text{ MPa} = F/A$

$$F_{\text{MAX}} = \sigma \cdot A = (101.8 \text{ N/mm}^2)(600 \text{ mm}^2) = 61000 \text{ N} = 61.0 \text{ kN}$$

STRESS GOVERNS THE DESIGN.  $F_{\text{MAX}} = 61.0 \text{ kN}$



4-57 FIGURE P4-57. COMPUTE TOTAL ELONGATION.

$$\textcircled{1} \delta_1 = \frac{F_1 L_1}{E A_1} = \frac{(40 \times 10^3 \text{ N})(30 \text{ mm})}{(73 \times 10^9 \text{ N/mm}^2)(100 \text{ mm}^2)}$$

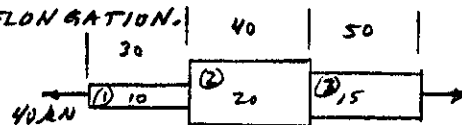
$$\delta_1 = 0.164 \text{ mm}$$

$$\textcircled{2} \delta_2 = \frac{F_2 L_2}{E A_2} = \frac{(40 \times 10^3 \text{ N})(40 \text{ mm})}{(73 \times 10^9 \text{ N/mm}^2)(400 \text{ mm}^2)} = 0.0548 \text{ mm}$$

$$\textcircled{3} \delta_3 = \frac{F_3 L_3}{E A_3} = \frac{(40 \times 10^3 \text{ N})(50 \text{ mm})}{(73 \times 10^9 \text{ N/mm}^2)(225 \text{ mm}^2)} = 0.1218 \text{ mm}$$

$$\delta_{\text{TOTAL}} = \delta_1 + \delta_2 + \delta_3 = 0.341 \text{ mm}$$

CHECK STRESS IN  $\textcircled{1}$ :  $\sigma_1 = \frac{F_1}{A_1} = \frac{40 \times 10^3 \text{ N}}{100 \text{ mm}^2} = 400 \text{ MPa}$  - CLOSE TO  $S_y$   
NOT SAFE.



2014-T6 ALUM.  
 $E = 73 \text{ GPa} = 73 \times 10^9 \text{ Pa}$   
 $E = 73 \times 10^3 \text{ MPa}$   
 $E = 73 \times 10^3 \text{ N/mm}^2$   
 $S_y = 414 \text{ MPa}$

4-58 AISI 1040 CD,  $S_y = 82 \text{ ksi}$ . LET  $\sigma = 0.9 S_y = 73.8 \text{ ksi} = F/A$

$$L = 2.00 \text{ IN}; A = \frac{\pi D^2}{4} = \frac{\pi (0.505 \text{ IN})^2}{4} = 0.200 \text{ IN}^2$$

$$E = 30 \times 10^6 \text{ psi} = 30 \times 10^3 \text{ ksi}$$

$$\delta = \frac{FL}{EA} = \frac{\sigma L}{E} = \frac{(73.8 \text{ ksi})(2.00 \text{ IN})}{30 \times 10^3 \text{ ksi}} = 0.00492 \text{ IN} = \delta$$

$$\text{STRAIN} = \delta/L = 0.00492 \text{ IN} / 2.00 \text{ IN} = 0.00246 \text{ IN/IN} = \epsilon$$

$$F = \sigma \cdot A = (73.8 \times 10^3 \text{ LB/IN}^2)(0.200 \text{ IN}^2) = 14760 \text{ LB} = F$$

$$\text{DISTANCE BETWEEN GAGE MARKS} = 2.00 \text{ IN} + \delta = 2.0049 \text{ IN}$$

PROBLEMS 59-66 FOLLOW SIMILAR SOLUTION PROCEDURE.

RESULTS SUMMARIZED ON FOLLOWING PAGE.

4-67

NYLON 6/6 PLASTIC. TENSILE STRENGTH =  $S_u = 179 \text{ MPa}$

$$\text{LET } \sigma = 0.5 S_u = 0.5(179 \text{ MPa}) = 89.5 \text{ MPa}$$

$$L = 50 \text{ mm}, A = t \cdot w = (12.5 \text{ mm})(16.0 \text{ mm}) = 200 \text{ mm}^2$$

$$E = 9.0 \text{ GPa} = 9.0 \times 10^9 \text{ Pa} \times \frac{1 \text{ MPa}}{10^6 \text{ Pa}} = 9000 \text{ MPa} = 9000 \text{ N/mm}^2$$

$$\delta = \frac{FL}{EA} = \frac{\sigma L}{E} = \frac{(89.5 \text{ MPa})(50 \text{ mm})}{9000 \text{ MPa}} = 0.497 \text{ mm} = \delta$$

$$\text{STRAIN} = \epsilon = \delta/L = 0.497 \text{ mm} / 50 \text{ mm} = 0.00994 \text{ mm/mm} = \epsilon$$

$$F = \sigma \cdot A = (89.5 \text{ N/mm}^2)(200 \text{ mm}^2) = 17900 \text{ N} = 17.90 \text{ kN} = F$$

$$\text{DISTANCE BETWEEN GAGE MARKS} = 50 \text{ mm} + \delta = 50.497 \text{ mm}$$

PROBLEMS 68-77 FOLLOW SIMILAR SOLUTION PROCEDURE.

RESULTS SUMMARIZED ON FOLLOWING PAGE.

NOTE! DATA FROM TABLE 2-6 FOR PROBLEMS 4-71 TO 4-73 REQUIRE CONVERSIONS AS SHOWN IN LOWER TABLE ON FOLLOWING PAGE.

ALSO NOTE!  $E = \text{SPECIFIC MODULUS} \times \text{SPECIFIC WT.}$

SOLUTIONS TO PROBLEMS 4-58 TO 4-66.										
Prob. No.	Material	$s_y$	$0.9*s_y$	$E$	$L$	$A$	$Elong.$	Length		Force
		(ksi)	(ksi)	(ksi)	(in)	(in <sup>2</sup> )	(in)	Bet. gage marks (in)	Strain (in/in)	
4-58	AISI 1040 CD	82	73.8	30000	2.000	0.200	0.00492	2.0049	0.00246	14760
4-59	AISI 5160 OQT 700	238	214.2	30000	2.000	0.200	0.01428	2.0143	0.00714	42840
4-60	AISI 501 OQT 1000	135	121.5	29000	2.000	0.200	0.00838	2.0084	0.00419	24300
4-61	C17000 Copper, hard	200	180.0	19000	2.000	0.200	0.01895	2.0189	0.00947	36000
4-62	AZ 63A-T6 Magn.	19	17.1	6500	2.000	0.200	0.00526	2.0053	0.00263	3420
4-63	ZA 12 Zinc	47	42.3	12000	2.000	0.200	0.00705	2.0071	0.00353	8460
4-64	ASTM A572 Gr 65	65	58.5	29000	2.000	0.200	0.00403	2.0040	0.00202	11700
4-65	Grade 4 ADI	140	126.0	24000	2.000	0.200	0.01050	2.0105	0.00525	25200
4-66	5154-H38 Alum.	39	35.1	10200	2.000	0.200	0.00688	2.0069	0.00344	7020

SOLUTIONS TO PROBLEMS 4-67 TO 4-77.										
Prob. No.	Material	$s_u$	$0.5*s_u$	$E$	$L$	$A$	$Elong.$	Length		Force
		MPa	MPa	MPa	mm	mm <sup>2</sup>	mm	Bet. gage marks (mm)	Strain mm/mm	
4-67	Nylon 6/6	179	89.5	9000	50.0	200	0.49722	50.4972	0.00994	17.9
4-68	Polyester PET	152	76.0	11700	50.0	200	0.32479	50.3248	0.0065	15.2
4-69	Polyimide	186	93.0	22100	50.0	200	0.21041	50.2104	0.00421	18.6
4-70	Polystyrene	83	41.5	5500	50.0	200	0.37727	50.3773	0.00755	8.3
4-71	Glass/Epoxy Comp*	786	393.0	27759	50.0	200	0.7079	50.7079	0.01416	78.6
4-72	Aramid/Epoxy*	1379	689.5	75845	50.0	200	0.45455	50.4545	0.00909	137.9
4-73	Graphite/Epoxy*	1917	958.4	135590	50.0	200	0.35342	50.3534	0.00707	191.7
4-74	Unidir Lam-Long.	1380	690.0	145000	50.0	200	0.23793	50.2379	0.00476	138.0
4-75	Unidir Lam-Transv.	34	17.0	11000	50.0	200	0.07727	50.0773	0.00155	3.4
4-76	Quasi-isotropic-Long	552	276.0	55000	50.0	200	0.25091	50.2509	0.00502	55.2
4-77	Quasi-isotropic-Transv	552	276.0	55000	50.0	200	0.25091	50.2509	0.00502	55.2

* Problems 71-73: Data conversion of units							
Prob. No.	Material From Table 2-6	$s_u$	$s_u$	Specific Modulus	Specific Weight	Modulus E	Modulus E
		ksi	MPa	in	lb/in <sup>3</sup>	psi	MPa
4-71	Glass/Epoxy Comp	114	786	6.60E+07	0.061	4.03E+06	27759
4-72	Aramid/Epoxy	200	1379	2.20E+08	0.050	1.10E+07	75845
4-73	Graphite/Epoxy	278	1917	3.45E+08	0.057	1.97E+07	135590

4-78 FORCE ANALYSIS:  $m = 680 \text{ kg}$ .

$W = m \cdot g = 680 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 6671 \text{ N} = \text{DOWNWARD FORCE AT C.}$

SINE LAW:  $\frac{6671 \text{ N}}{\sin 75^\circ} = \frac{AC}{\sin 35^\circ} = \frac{BC}{\sin 70^\circ}$

$AC = \frac{6671 \text{ N} (\sin 35^\circ)}{\sin 75^\circ} = 3961 \text{ N}$

$BC = \frac{6671 \text{ N} (\sin 70^\circ)}{\sin 75^\circ} = 6490 \text{ N}$

STRESS: ROD AREA  $= A = \pi (8.00 \text{ mm})^2 / 4 = 50.27 \text{ mm}^2$

$\sigma_{BC} = BC/A = 6490 \text{ N} / 50.27 \text{ mm}^2 = 129 \text{ MPa}$

$\sigma_{AC} = AC/A = 3961 \text{ N} / 50.27 \text{ mm}^2 = 78.8 \text{ MPa}$

ASSUME STATIC LOAD: SAFE FOR ANY STEEL

DEF.  $\delta_{AC} = \frac{AC(L_1)}{EA}$ ,  $\delta_{BC} = \frac{BC(L_2)}{EA}$

$L_1 = 14.000 \text{ m} / \cos 20^\circ = 14.898 \text{ m}$

$L_2 = 7.000 \text{ m} / \cos 55^\circ = 12.204 \text{ m}$

ORIGINAL VERTICAL DISTANCE FROM CEILING TO C:

$\tan 55^\circ = BD/CD$ ,  $BD = CD \tan 55^\circ = (7.111) (\tan 55^\circ)$

$BD = 9.997 \text{ m}$

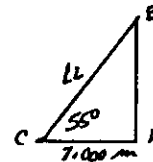
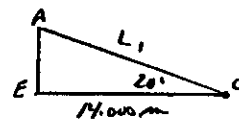
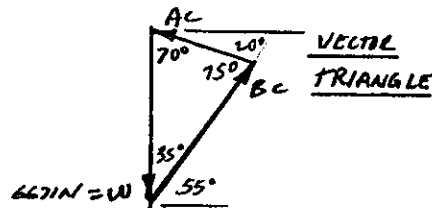
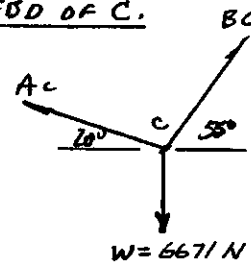
$\delta_{AC} = \frac{AC L_1}{EA} = \frac{(3961 \text{ N})(14.898 \text{ m})}{(207 \times 10^9 \text{ N/m}^2)(50.27 \text{ mm}^2)} \times \frac{(10^3 \text{ mm})^2}{\text{m}^2}$

$\delta_{AC} = 5.67 \times 10^{-3} \text{ m} = 5.67 \text{ mm}$

$\delta_{BC} = \frac{BC L_2}{EA} = \frac{(6490 \text{ N})(12.204 \text{ m})}{(207 \times 10^9 \text{ N/m}^2)(50.27 \text{ mm}^2)} \times \frac{(10^3 \text{ mm})^2}{\text{m}^2} = 7.61 \times 10^{-3} \text{ m}$

$\delta_{BC} = 7.61 \text{ mm}$

FBD OF C.



4-79 FORCE ANALYSIS.  $m = 4200 \text{ kg}$ .

$$W = m g = (4200 \text{ kg})(9.81 \text{ m/s}^2) = 41200 \text{ N} = 41.2 \text{ kN}$$

VECTOR TRIANGLE IS A RIGHT TRIANGLE,

$$BC = W \sin 35^\circ = (41.2 \text{ kN})(\sin 35^\circ) = 23.63 \text{ kN}$$

$$AB = W \cos 35^\circ = (41.2 \text{ kN})(\cos 35^\circ) = 33.75 \text{ kN}$$

STRESS:  $D = 10.0 \text{ mm}; A = \pi D^2/4 = 78.54 \text{ mm}^2$

$$\sigma = \frac{F}{A} = \frac{AB}{A} = \frac{33.75 \times 10^3 \text{ N}}{78.54 \text{ mm}^2} = 429.7 \text{ MPa}$$

ASSUME STATIC LOAD,  $\sigma_d = S_y/2$

$$\text{REQ'D } S_y = 2\sigma = 2(429.7 \text{ MPa}) = 859 \text{ MPa}$$

SPECIFY AISI 4140 Q&T 1100,  $S_y = 903 \text{ MPa}$

DEFORMATION:

$$\delta_{AB} = \frac{(AB) L_1}{EA}; \quad \delta_{BC} = \frac{(BC) L_2}{EA}$$

$$L_1 = 6.00 \text{ m} / \cos 35^\circ = 7.32 \text{ m}$$

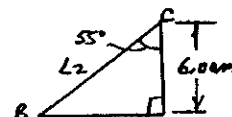
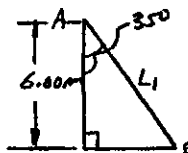
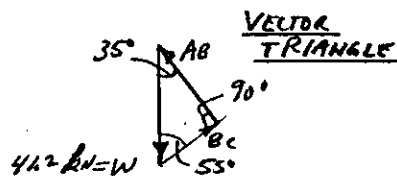
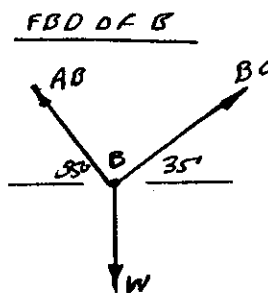
$$\delta_{AB} = \frac{(33.75 \times 10^3 \text{ N})(7.32 \text{ m})}{(207 \times 10^9 \text{ N/m}^2)(78.54 \text{ mm}^2)} \times (10^3 \text{ mm})^2$$

$$\delta_{AB} = 0.0152 \text{ m} = 15.2 \text{ mm}$$

$$\delta_{BC} = \frac{(BC) L_2}{EA} = \frac{(23.63 \times 10^3 \text{ N})(10.46 \text{ m})}{(207 \times 10^9 \text{ N/m}^2)(78.54 \text{ mm}^2)} \times (10^3 \text{ mm})^2$$

$$[L_2 = 6.00 \text{ m} / \cos 55^\circ = 10.46 \text{ m}]$$

$$\delta_{BC} = 0.0152 \text{ m} = 15.2 \text{ mm}$$



# CHAPTER 5 Torsional Shear Stress and Torsional Deflection

$$\underline{5-1} \quad T = \frac{T_c}{J} = \frac{(280 \text{ N}\cdot\text{m})(10 \text{ mm})}{\pi(20)^4/32 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = \underline{178 \text{ MPa}}$$

$$\underline{5-2} \quad J = \frac{\pi}{32} (D_o^4 - D_i^4) = \frac{\pi}{32} (35^4 - 25^4) = 109 \times 10^3 \text{ mm}^4$$

$$T = \frac{T_c}{J} = \frac{(560 \text{ N}\cdot\text{m})(35/2) \text{ mm}}{109 \times 10^3 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = \underline{89.9 \text{ MPa}}$$

$$\underline{5-3} \quad T = \frac{T_c}{J} = \frac{(530 \text{ LB}\cdot\text{in})(1.25/2) \text{ in}}{\pi(1.25)^4/32 \text{ in}^4} = \underline{4042 \text{ psi}}$$

$$\underline{5-4} \quad D_i = D_o - 2t = 1.75 - 2(0.125) = 1.50 \text{ in}$$

$$J = \frac{\pi}{32} (D_o^4 - D_i^4) = \frac{\pi}{32} (1.75^4 - 1.50^4) = 0.424 \text{ in}^4$$

$$T_o = \frac{T_c}{J} = \frac{(5500 \text{ LB}\cdot\text{in})(1.75/2) \text{ in}}{0.424 \text{ in}^4} = \underline{11360 \text{ psi}}$$

$$T_i = \frac{T_c}{J} = \frac{(5500 \text{ LB}\cdot\text{in})(1.50/2) \text{ in}}{0.424 \text{ in}^4} = \underline{9734 \text{ psi}}$$

$$\underline{5-5} \quad T = \frac{P}{m} = \frac{0.08 \times 10^3 \text{ N}\cdot\text{m/s}}{180 \text{ rad/s}} = 0.444 \text{ N}\cdot\text{m}$$

$$T = \frac{T_c}{J} = \frac{(0.440 \text{ N}\cdot\text{m})(1.50 \text{ mm})}{\pi(3.0)^4/32 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = \underline{83.8 \text{ MPa}}$$

$$\underline{5-6} \quad T = \frac{P}{m} = \frac{35 \times 10^3 \text{ N}\cdot\text{m/s}}{42 \text{ rad/s}} = 833 \text{ N}\cdot\text{m}$$

$$J = \frac{\pi}{32} (40^4 - 25^4) \text{ mm}^4 = 213 \times 10^3 \text{ mm}^4$$

$$T = \frac{T_c}{J} = \frac{(833 \text{ N}\cdot\text{m})(20 \text{ mm})}{213 \times 10^3 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = \underline{78.3 \text{ MPa}}$$

$$\underline{5-7} \quad T = \frac{63000 \text{ (P)}}{m} = \frac{(63000)(15.0 \text{ hp})}{240 \text{ RPM}} = 3938 \text{ LB}\cdot\text{in}$$

$$T = \frac{T_c}{J} = \frac{3938 \text{ LB}\cdot\text{in}(1.44/2) \text{ in}}{\pi(1.44)^4/32 \text{ in}^4} = \underline{6716 \text{ psi}}$$

$$T_o = \frac{3V}{2N} = \frac{101000 \text{ (psi)}}{2(6)} = \underline{8417 \text{ psi}} \quad \text{OK}$$

5-8

FROM PROBLEM 5-7,  $T = 3938 \text{ LB}\cdot\text{IN}$ ,  $\tau_s = 8417 \text{ PSI}$   
 $C = 1.44 \text{ IN} / 2 = 0.72 \text{ IN}$

$$J = \frac{\pi D^4}{32} = \frac{\pi (1.44 \text{ IN})^4}{32} = 0.422 \text{ IN}^4$$

FOR PROFILE KEYSEAT,  $K_t = 2.0$

$$\tau = \frac{K_t T C}{J} = \frac{(2.0)(3938 \text{ LB}\cdot\text{IN})(0.72 \text{ IN})}{0.422 \text{ IN}^4} = 13433 \text{ PSI}$$

BECAUSE  $\tau > \tau_s$  DESIGN IS NOT SAFE.

5-9

$$T = \frac{63000 (7.5 \text{ HP})}{2200 \text{ RPM}} = 215 \text{ LB}\cdot\text{IN}$$

AT FILLET:  $r/d = 0.05/0.75 = 0.066$ ;  $D/d = 1.25/0.75 = 1.67$ ;  $K_t = 1.55$

AT KEYSEAT:  $K_t = 2.0$

$$\tau_{\text{MAX}} = \frac{K_t T}{Z_P} = \frac{(2.0)(215 \text{ LB}\cdot\text{IN})}{\pi (0.75)^3 / 16 \text{ IN}^3} = 5190 \text{ PSI}$$

BLAD WOULD SEE STRESS:  $\tau_s = \frac{S_y}{2(6)}$

REQ'D.  $S_y = 2(6)(5190) = 62300 \text{ PSI}$ ; AISI 1040 NOT 1300  
 $S_y = 63 \text{ KSI}$ ; 32% BLW.

5-10

AT FILLET:  $r/d = 0.08/1.50 = 0.053$ ;  $D/d = 2.4/1.50 = 1.33$ ;  $K_t = 1.58$

$$a) \tau_{\text{MAX}} = \frac{K_t T}{Z_P} = \frac{(1.58)(7500 \text{ LB}\cdot\text{IN})}{\pi (1.50)^3 / 16 \text{ IN}^3} = 17880 \text{ PSI}$$

$$b) \text{ AT HOLE: } Z_{P_{\text{HOLE}}} = \pi (2.00)^3 / 16 = 1.57 \text{ IN}^3$$

$$\tau_{\text{HOLE}} = \frac{T}{Z_P} = \frac{7500}{1.57} = 4775 \text{ PSI}$$

$$K_{t_{\text{MAX}}} = \frac{\tau_{\text{MAX}}}{\tau_{\text{HOLE}}} = \frac{17880 \text{ PSI}}{4775 \text{ PSI}} = 3.74$$

FROM APP. A-21-5  $(d/D)_{\text{MAX}} = 0.18$ ; THEN  $d_{\text{MAX}} = 0.18(2.4) = 0.360 \text{ IN}$

5-11

$$J = \frac{\pi (D_o^4 - D_i^4)}{32} = \frac{\pi (80^4 - 60^4)}{32} = 2.75 \times 10^6 \text{ mm}^4$$

$$\tau_{\text{MAX}} = \frac{T C}{J} = \frac{(4500 \text{ N}\cdot\text{m})(40 \text{ mm})}{2.75 \times 10^6 \text{ mm}^4} \cdot \frac{10^3 \text{ mm}}{\text{m}} = 65.5 \text{ MPa}$$

$$\theta = \frac{T L}{G J} = \frac{(4500 \text{ N}\cdot\text{m})(600 \text{ mm})}{(26 \times 10^9 \text{ N/m}^2)(2.75 \times 10^6 \text{ mm}^4)} \cdot \frac{(10^3 \text{ mm})^2}{\text{m}^2} = 0.0378 \text{ rad}$$

ASSUME STEADY TORQUE: REQ'D.  $S_y = 2(2)(\tau_{\text{MAX}})$

$$S_y = 4(65.5) = 262 \text{ MPa}$$

6061-T6 HAS  $S_y = 276 \text{ MPa}$

5-12

$$\text{SOLID: } J = \pi D^4 / 32 = \pi (50)^4 / 32 \text{ mm}^4 = 613.6 \times 10^3 \text{ mm}^4$$

$$\tau = \frac{TC}{J} = \frac{(850 \text{ N}\cdot\text{m})(25 \text{ mm})}{613.6 \times 10^3 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = 34.6 \text{ MPa}$$

$$\theta = \frac{TL}{GJ} = \frac{(850 \text{ N}\cdot\text{m})(600 \text{ mm})}{(80 \times 10^9 \text{ N/m}^2)(613.6 \times 10^3 \text{ mm}^4)} \times \frac{(10^3 \text{ mm})^3}{\text{m}^3} = 0.004 \text{ rad}$$

$$\text{MASS} = (\text{VOL})(\text{DENS.}) = A \cdot L \cdot \text{DENS.} = \frac{\pi (50)^2}{4} \times 600 \text{ mm} \times \frac{7680 \text{ kg/m}^3 \cdot 1 \text{ m}^3}{(10^3 \text{ mm})^3}$$

$$M = 9.05 \text{ kg}$$

$$\text{HOLLOW: } J = \frac{\pi}{32} (50^4 - 40^4) = 362.3 \times 10^3 \text{ mm}^4$$

$$\tau = \frac{(850)(25)(10^3)}{362.3 \times 10^3} = 58.7 \text{ MPa} \quad (1.69 \text{ TIMES } \tau_{\text{SOLID}})$$

$$\theta = \frac{(850)(600)(10^3)}{(80 \times 10^9)(362.3 \times 10^3)} = 0.0176 \text{ rad} \quad [1.69 \text{ TIMES } \theta_{\text{SOLID}}]$$

$$M = \frac{\pi (50^4 - 40^4)}{4} \times (600) (7680) = 3.26 \text{ kg} \quad [\text{SOLID IS 2.78 TIMES HEAVIER}]$$

5-13

$$\text{REQ'D. } Z_p = \frac{T}{\tau_s} = \frac{1200 \text{ N}\cdot\text{m}}{45 \text{ N/mm}^2} \times \frac{10^3 \text{ mm}}{\text{m}} = 26667 \text{ mm}^3$$

$$Z_p = \frac{\pi}{16} \frac{D_o^4 - D_i^4}{D_o} = \frac{\pi}{16} \frac{(1.25 D_i)^4 - D_i^4}{1.25 D_i} = 0.226 D_i^3$$

$$\text{REQ'D. } D_i = \sqrt[3]{26667 / 0.226} = 49.0 \text{ mm}$$

$$D_o = 1.25 D_i = 61.3 \text{ mm}$$

5-14

$$T = 63000(7.5) / 240 = 1969 \text{ LB}\cdot\text{IN}$$

$$\tau = \frac{T}{Z_p} = \frac{1969 \text{ LB}\cdot\text{IN}}{\pi (0.46)^3 / 16 \text{ IN}^3} = 15764 \text{ PSI}$$

5-15

$$T = 63000(7.5) / 1140 = 414 \text{ LB}\cdot\text{IN}$$

$$\text{REQ'D. } Z_p = \frac{T}{\tau} = \frac{414 \text{ LB}\cdot\text{IN}}{15764 \text{ LB/IN}^2} = 0.0263 \text{ IN}^3 = \pi D^3 / 16$$

$$\text{REQ'D. } D = \sqrt[3]{16 Z_p / \pi} = \sqrt[3]{16(0.0263) / \pi} = 0.512 \text{ IN}$$

5-16

$$T = F \cdot d = (80 \text{ LB})(18 \text{ IN}) = 1440 \text{ LB}\cdot\text{IN}; Z_p = 0.6524 \text{ IN}^3 \quad (\text{APP. A-12})$$

$$\tau = T / Z_p = 1440 \text{ LB}\cdot\text{IN} / 0.6524 \text{ IN}^3 = 2207 \text{ PSI}$$

5-17

$$M = (1200 / 5 \text{ SEC}) (60560 / \text{MM}) = 120 \text{ RM}; T = 80 \text{ LB} \cdot \text{FT} (12 \text{ IN/FT}) = 960 \text{ LB}\cdot\text{IN}$$

$$\rho = \frac{T \cdot m}{63000} = \frac{(360)(12)}{63000} = 0.686 \text{ KP}$$

$$\tau = T / Z_p = 360 \text{ LB}\cdot\text{IN} / \pi (0.60)^3 / 16 \text{ IN}^3 = 8488 \text{ PSI} = 51/2 \text{ IN} = 51/8$$

$$\text{REQ'D } S_y = 8 T = 8(8488) = 67900 \text{ PSI}$$

$$\text{POSSIBLE STEEL: AISI 1040 Q\&T 1100, } S_y = 80 \text{ KSI, } 24\% \text{ ELONG.}$$

$$\begin{aligned} \underline{5-18} \quad T &= S_{YS} = S_Y/2 = 44 \text{ MPa}/2 = 220 \text{ MPa} \\ T &= T \cdot Z_p = \frac{220 \text{ N}}{\text{mm}^2} \times \frac{\pi (15)^3 \text{ mm}^3}{16} \cdot \frac{1 \text{ m}}{10^3 \text{ mm}} = 196 \text{ N}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} \underline{5-19} \quad T &= 63000(2500)/75 = 2.1 \times 10^6 \text{ LB}\cdot\text{IN} \\ T_0 &= \frac{S_Y}{2N} = \frac{63000 \text{ PSI}}{2(6)} = 5250 \text{ PSI} \\ \text{REQ'D. } Z_p &= \frac{T}{T_0} = \frac{2.1 \times 10^6 \text{ LB}\cdot\text{IN}}{5250 \text{ LB/IN}^2} = 400 \text{ IN}^3 = \frac{\pi}{16} \left( \frac{D_o^4 - D_i^4}{D_o} \right) \\ \text{BUT } D_i &= 0.8 D_o; D_i^4 = 0.4096 D_o^4 \\ Z_p &= \frac{\pi}{16} \frac{(D_o^4 - 0.4096 D_o^4)}{D_o} = \frac{\pi (0.5904 D_o^3)}{16 D_o} = 0.1159 D_o^2 \\ \text{REQ'D. } D_o &= \sqrt{\frac{Z_p}{0.1159}} = \sqrt{\frac{400}{0.1159}} = 15.11 \text{ IN} \\ D_i &= 0.8 D_o = 12.09 \text{ IN} \end{aligned}$$

$$\begin{aligned} \underline{5-20} \quad \text{REQ'D } Z_p &= 400 \text{ IN}^3 = \pi D^3/16; D = \sqrt[3]{16 Z_p/\pi} = 12.68 \text{ IN} \\ \frac{WTS}{WTH} &= \frac{A_{TL} \cdot D_{TL}}{A_{TL} \cdot D_{TH}} = \frac{A_L}{A_H} = \frac{\pi (12.68)^2/K}{\pi (15.11^2 - 12.09^2)/K} = 1.96 \end{aligned}$$

$$\underline{5-21} \quad T = T \cdot Z_p = \frac{80 \text{ N}}{\text{mm}^2} \cdot \frac{\pi (5.0)^3 \text{ mm}^3}{16} \cdot \frac{1 \text{ m}}{10^3 \text{ mm}} = 1.96 \text{ N}\cdot\text{m}$$

$$\begin{aligned} \underline{5-22} \quad J &= \pi D^4/32 = \pi (6.0)^4/32 = 127.2 \text{ mm}^4 \\ T &= \frac{TC}{J} = \frac{(5.5 \text{ N}\cdot\text{m})(2.0 \text{ mm})}{127.2 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{1 \text{ m}} = 130 \text{ MPa} \\ \theta &= \frac{TL}{GJ} = \frac{(5.5 \text{ N}\cdot\text{m})(250 \text{ mm})}{(80 \times 10^9 \text{ N/m}^2)(127.2 \text{ mm}^4)} \times \frac{(10^3 \text{ mm})^2}{\text{m}^2} = 0.135 \text{ RAD} \\ &\quad (7.74 \text{ deg}) \end{aligned}$$

$$\underline{5-23} \quad \theta = \frac{TL}{GJ} = \frac{(640 \text{ N}\cdot\text{m})(250 \text{ mm})}{(80 \times 10^9 \text{ N/m}^2)(\pi (15)^4/32) \text{ mm}^4} \times \frac{(10^3 \text{ mm})^2}{\text{m}^2} = 0.1509 \text{ RAD} \\ (8.65 \text{ deg})$$

$$\begin{aligned} \underline{5-24} \quad J &= \frac{\pi (80^4 - 60^4)}{32} = 2.75 \times 10^6 \text{ mm}^4 \\ \theta &= \frac{TL}{GJ} = \frac{(2250 \text{ N}\cdot\text{m})(1200 \text{ mm})}{(26 \times 10^9 \text{ N/m}^2)(2.75 \times 10^6 \text{ mm}^4)} \times \frac{(10^3 \text{ mm})^2}{\text{m}^2} = 0.0378 \text{ RAD} \\ &\quad (2.16 \text{ DEG}) \end{aligned}$$

$$\underline{5-25} \quad \theta = \frac{TL}{GJ} = \frac{(40 \text{ LB}\cdot\text{FT})(8 \text{ FT})}{(11.5 \times 10^6 \text{ LB/IN}^2)(\pi (0.625)^4/32) \text{ IN}^4} \times \frac{144 \text{ IN}^2}{\text{FT}^2} = 0.267 \text{ RAD} \\ (15.3 \text{ DEG})$$

$$\begin{aligned} \underline{5-26} \quad \theta &= (2.0 \text{ DEG})(\pi \text{ RAD}/180 \text{ DEG}) = 0.0349 \text{ RAD} \\ J &= \frac{TL}{G\theta} = \frac{(40 \text{ LB}\cdot\text{FT})(8 \text{ FT})}{(11.5 \times 10^6 \text{ LB/IN}^2)(0.0349 \text{ RAD})} \times \frac{144 \text{ IN}^2}{\text{FT}^2} = 0.1148 \text{ IN}^4 = \pi D^4/32 \\ \text{REQ'D. } D &= \sqrt[4]{32(0.1148)/\pi} = 1.04 \text{ IN} \end{aligned}$$



5-27  $\theta = \theta_1 + \theta_2 = \frac{(200 \text{ N}\cdot\text{m})(400 \text{ mm})}{(80 \times 10^9 \text{ N/m}^2)(5708 \text{ mm}^4)} \times \frac{(10^3 \text{ mm})^2}{\text{m}^2} + \frac{(200)(1200)(10^9)}{(80 \times 10^9)(251300)}$   
 $J_1 = \pi(20)^4/32 = 15708 \text{ mm}^4$  ;  $J_2 = \pi(40)^4/32 = 251300 \text{ mm}^4$   
 $\theta = 0.0071 + 0.0119 = 0.0190 \text{ RAD} \quad (4.83 \text{ DEG})$

5-28  $\theta = (10.0 \text{ DEG})(\pi \text{ RAD}/180 \text{ DEG}) = 0.1745 \text{ RAD}$   
 $\text{REQD } J = \frac{TL}{G\theta} = \frac{(500 \text{ N}\cdot\text{m})(150 \text{ mm})}{(26 \times 10^9 \text{ N/m}^2)(0.1745)} \times \frac{10^9 \text{ mm}^2}{\text{m}^2} = 165.3 \text{ mm}^4$   
 $\text{REQD } D = \sqrt[4]{32J/\pi} = 6.40 \text{ mm}$   
 $\tau = \frac{TC}{J} = \frac{(500 \text{ N}\cdot\text{m})(13.20 \text{ mm})}{165.3 \text{ mm}^4} = 96.8 \text{ MPa}$   
 $N = \frac{SY}{2(\tau)} = \frac{276 \text{ MPa}}{2(96.8 \text{ MPa})} = 1.43 \text{ LOW}$   
 COULD USE STRONGER ALUMINUM OR LONGER BAR

5-29  $T = T_1 + T_2 = \frac{250 \text{ N}}{\text{mm}^2} \times \frac{\pi(1.50)^3 \text{ mm}^3}{16} = 165.7 \text{ N}\cdot\text{mm}$   
 $J = \pi(1.50)^4/32 = 0.497 \text{ mm}^4$   
 $\theta = \frac{TL}{GJ} = \frac{(165.7 \text{ N}\cdot\text{mm})(40 \text{ mm})}{(48 \times 10^9 \text{ N/m}^2)(0.497 \text{ mm}^4)} \times \frac{10^6 \text{ mm}^2}{\text{m}^2} = 0.278 \text{ RAD} \quad (15.9 \text{ DEG})$

5-30  $J = \frac{\pi}{32}(18^4 - 16^4) = 3872 \text{ mm}^4$  ;  $\theta = 40 \text{ DEG} \times \pi \text{ RAD}/180 \text{ DEG} = 0.698 \text{ RAD}$   
 $T = \frac{GJ\theta}{L} = \frac{(60.42 \times 10^3 \text{ N/mm}^2)(3872 \text{ mm}^4)(0.698 \text{ RAD})}{1650 \text{ mm}} \times \frac{1 \text{ mm}^2}{10^6 \text{ mm}^2} = 70.4 \times 10^3 \text{ N}\cdot\text{mm}$   
 $\tau = \frac{TC}{J} = \frac{(60.42 \times 10^3 \text{ N}\cdot\text{mm})(9 \text{ mm})}{3872 \text{ mm}^4} = 164 \text{ MPa}$   
 $N = \frac{SY}{2\tau} = \frac{1070 \text{ MPa}}{2(164 \text{ MPa})} = 3.27$

5-31  $J = \pi(35)^4/32 = 147.3 \times 10^3 \text{ mm}^4$   
 $T_{BC} = T_3 = 500 \text{ N}\cdot\text{m} = 500 \times 10^3 \text{ N}\cdot\text{mm}$  ;  $T_{AB} = T_2 + T_3 = 1500 \text{ N}\cdot\text{m} = 1.5 \times 10^6 \text{ N}\cdot\text{mm}$   
 $G = 80 \text{ GPa} = (80 \times 10^9 \text{ N/m}^2) \left( \frac{1 \text{ m}^2}{10^6 \text{ mm}^2} \right) = 80 \times 10^3 \text{ N/mm}^2$   
 $\theta_{AC} = \theta_{AB} + \theta_{BC} = \frac{T_{AB} L_1}{GJ} + \frac{T_{BC} L_2}{GJ}$   
 $\theta_{AC} = \frac{(1.5 \times 10^6 \text{ N}\cdot\text{mm})(600 \text{ mm})}{(80 \times 10^3 \text{ N/mm}^2)(147.3 \times 10^3 \text{ mm}^4)} + \frac{(500 \times 10^3)(800)}{(80 \times 10^3)(147.3 \times 10^3)}$   
 $\theta_{AC} = 0.0636 + 0.0339 = 0.0976 \text{ RAD} \quad (5.59 \text{ DEG}) \quad \left| \begin{array}{l} \theta_{AB} = 0.0636 \text{ RAD} \\ (3.64 \text{ DEG}) \end{array} \right.$

5-32  $\theta = (2.2 \text{ DEG})(\pi \text{ RAD}/180 \text{ DEG}) = 0.0384 \text{ RAD}$   
 $\text{REQD } J = \frac{TL}{G\theta} = \frac{(1360 \text{ N}\cdot\text{m})(820 \text{ mm})}{(80 \times 10^9 \text{ N/m}^2)(0.0384)} \times \frac{(10^3 \text{ mm})^2}{\text{m}^2} = 363 \times 10^3 \text{ mm}^4 = \frac{\pi D^4}{32}$   
 $\text{REQD } D = \sqrt[4]{32J/\pi} = 43.9 \text{ mm}$   
 $\tau = \frac{TC}{J} = \frac{(1360 \text{ N}\cdot\text{m})(21.95 \text{ mm})}{363 \times 10^3 \text{ mm}^4} \times \frac{10^3 \text{ mm}^2}{\text{m}^2} = 82.1 \text{ MPa}$

5-33  $T = \frac{P}{\dot{\theta}} = \frac{120 \times 10^3 \text{ N}\cdot\text{m/s}}{225 \text{ RAD/s}} = 533 \text{ N}\cdot\text{m}$   
 $J = \frac{\pi}{32} (75^4 - 55^4) = 2.208 \times 10^6 \text{ mm}^4$   
 $\tau = \frac{Tc}{J} = \frac{(533 \text{ N}\cdot\text{m})(37.5 \text{ mm})}{2.208 \times 10^6 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = 9.06 \text{ MPa}$   
 $\theta = \frac{TL}{GJ} = \frac{(533 \text{ N}\cdot\text{m})(1.525 \text{ m})}{(80 \times 10^9 \text{ N/m}^2)(2.208 \times 10^6 \text{ mm}^4)} \times \frac{(10^3 \text{ mm})^2}{\text{m}^3} = \frac{0.0046 \text{ RAD}}{(0.264 \text{ DEG})}$

5-34  $T = \frac{P}{\dot{\theta}} = \frac{60 \times 10^3 \text{ N}\cdot\text{m/s}}{70 \text{ RAD/s}} = 857 \text{ N}\cdot\text{m}$   
 $Z_p = \pi d^3/16 = \pi (35)^3/16 = 8418 \text{ mm}^3$   
 $r/d = 4/35 = 0.114$ ;  $d/d = 50/35 = 1.43$ ;  $K_t = 1.35$  FROM APP. A-21-7  
 $\tau = \frac{TK_t}{Z_p} = \frac{(857 \text{ N}\cdot\text{m})(1.35)}{8418 \text{ mm}^3} \times \frac{10^3 \text{ mm}}{\text{m}} = 137 \text{ MPa}$

5-35  $T = \frac{P}{\dot{\theta}} = \frac{105 \times 10^3 \text{ N}\cdot\text{m/s}}{220 \text{ RAD/s}} = 477 \text{ N}\cdot\text{m}$   
 $Z_p = \pi d^3/16 = \pi (40)^3/16 = 12566 \text{ mm}^3$   
 $r/d = 6/40 = 0.150$ ;  $d/d = 70/40 = 1.75$ ;  $K_t = 1.29$  FROM APP. A-21-7  
 $\tau = \frac{TK_t}{Z_p} = \frac{(477 \text{ N}\cdot\text{m})(1.29)}{12566 \text{ mm}^3} \times \frac{10^3 \text{ mm}}{\text{m}} = 49.0 \text{ MPa}$

FOR PROBLEMS 5-36 THROUGH 5-39:  $T_0 = \frac{S_y}{2N} = \frac{669 \text{ MPa}}{2(4)} = 83.6 \text{ MPa}$

OR  $T_0 = \frac{97000 \text{ PSI}}{2(4)} = 12125 \text{ PSI}$

$T = TK_t/Z_p$ ; ALLOW.  $T = (T_0)(Z_p)/K_t$

5-36 LEFT END:  $Z_p = \pi (12)^3/16 = 339.3 \text{ mm}^3$   
 $r/d = 2/12 = 0.167$ ;  $d/d = 24/12 = 2.0$ ;  $K_t = 1.27$  (A-21-7)  
 $T = \frac{(83.6 \text{ N/mm}^2)(339.3 \text{ mm}^3)}{1.27} = 27.3 \text{ N}\cdot\text{m}$  CRITICAL VALUE  
 RIGHT END:  $Z_p = \pi (16)^3/16 = 804.2 \text{ mm}^3$   
 $r/d = 1/16 = 0.063$ ;  $d/d = 24/16 = 1.50$ ;  $K_t = 1.53$  (A-21-7)  
 $T = \frac{(83.6 \text{ N/mm}^2)(804.2 \text{ mm}^3)}{1.53} = 43.9 \times 10^3 \text{ N}\cdot\text{mm} = 43.9 \text{ N}\cdot\text{m}$

5-37 GROOVE  $Z_p = \pi (1.20)^3/16 = 0.339 \text{ in}^3$   
 LEFT GROOVE:  $r/d = 0.006/1.20 = 0.0067$ ;  $d/d = 1.50/1.20 = 1.25$ ;  $K_t = 3.0$  (A-21-7)  
 RIGHT GROOVE:  $r/d = 0.08/1.20 = 0.067$ ;  $d/d = 1.25$ ;  $K_t = 1.63$  (A-21-6)  
 LEFT GROOVE CRITICAL:  $T = \frac{(12125 \text{ LB/in}^2)(0.339 \text{ in}^3)}{3.0} = 1370 \text{ LB}\cdot\text{IN}$

5-38 GROOVE:  $Z_p = \pi(25)^3/16 = 3068 \text{ mm}^3$   
 $r/d = 1.5/25 = 0.060$ ;  $D/d = 30/25 = 1.20$ ;  $K_t = 1.66$  (A-21-6)  
 $T = T_d \cdot Z_p / K_t = (83.6 \text{ N/mm}^2)(3068 \text{ mm}^3) / 1.66 = 154.5 \times 10^3 \text{ N}\cdot\text{mm} = 154.5 \text{ N}\cdot\text{m}$   
FILLET:  $Z_p = \pi(20)^3/16 = 1571 \text{ mm}^3$   
 $r/d = 1.5/20 = 0.075$ ;  $D/d = 30/20 = 1.50$ ;  $K_t = 1.47$  (A-21-7)  
 $T = (83.6)(1571) / 1.47 = 89.3 \times 10^3 \text{ N}\cdot\text{mm} = 89.3 \text{ N}\cdot\text{m}$   
HOLE:  $Z_p = 1571 \text{ mm}^3$ ;  $d/D = 4/20 = 0.200$ ;  $K_{ts} = 3.8$  (A-21-5C)  
 $T = (83.6)(1571) / 3.8 = 34.5 \times 10^3 \text{ N}\cdot\text{mm} = 34.5 \text{ N}\cdot\text{m}$  CRITICAL

5-39 LEFT PART:  $Z_p = \pi(125)^3/16 = 0.383 \text{ IN}^3$   
FILLET:  $r/d = 0.188/1.25 = 0.150$ ;  $D/d = 2.0/1.25 = 1.60$ ;  $K_t = 1.60$  (A-21-7)  
KEYSEAT:  $K_t = 1.60$   
 $T = \frac{T_d \cdot Z_p}{K_t} = \frac{(21250 \text{ lb/in}^2)(0.383 \text{ IN}^3)}{1.60} = 2902 \text{ LB}\cdot\text{IN}$  CRITICAL  
 OTHER PARTS OBVIOUSLY STRONGER

5-40  $T = T/Q$ ;  $T = T Q = (50 \text{ N/mm}^2)(166 \text{ mm}^2) = 83.2 \times 10^3 \text{ N/mm} = 83.2 \text{ N.m}$   
 $Q = 0.208 a^3 = 0.208 (20)^3 = 1664 \text{ mm}^3$  (FIG 5-25)

5-41  $K = 0.141 a^4 = 0.141 (20)^4 = 22.56 \times 10^3 \text{ mm}^4$  (FIG 5-25)  
 $\theta = \frac{TL}{GK} = \frac{(83.2 \times 10^3 \text{ N/mm})(1800 \text{ mm})}{(80000 \text{ N/mm}^2)(22.56 \times 10^3 \text{ mm}^4)} = 0.083 \text{ RAD}$  (4.75 DEG)

5-42  $Q = 0.208 a^3 = 0.208 (1.25)^3 = 0.406 \text{ IN}^3$  (FIG 5-25)  
 $T = T Q = (7500 \text{ LB/IN}^2)(0.406 \text{ IN}^3) = 3047 \text{ LB.IN}$

5-43  $K = 0.141 a^4 = 0.141 (1.25)^4 = 0.344 \text{ IN}^4$  (FIG 5-25)  
 $\theta = \frac{TL}{GK} = \frac{(3047 \text{ LB.IN})(48 \text{ IN})}{(3.75 \times 10^6 \text{ LB/IN}^2)(0.344 \text{ IN}^4)} = 0.112 \text{ RAD}$  (6.42 DEG)

5-44  $Q = \frac{6A^2}{[3 + 1.8(4/3)]} = \frac{(3.0)(1.25)^2}{[3 + 1.8(4/3)]} = 1.25 \text{ IN}^3$  (FIG 5-25)  
 $T = T Q = (7500 \text{ LB/IN}^2)(1.25 \text{ IN}^3) = 9375 \text{ LB.IN}$

5-45  $K = (3.0)(1.25)^3 \left[ \frac{1}{3} - 0.21 \frac{1.25}{2.0} \left( 1 - \frac{(1.25/3.0)^4}{12} \right) \right] = 1.44 \text{ IN}^4$  (FIG 5-25)  
 $\theta = \frac{TL}{GK} = \frac{(7500)(48)}{(3.75 \times 10^6)(1.44)} = 0.0667 \text{ RAD}$  (3.82 DEG.)

5-46  $K = 0.0217 a^4 = 0.0217 (30)^4 = 17.58 \times 10^3 \text{ mm}^4$  (FIG 5-25)  
 $\theta = (0.80 \text{ DEG})(17 \text{ RAD}/180 \text{ DEG.}) = 0.0140 \text{ RAD}$   
 $T = \frac{GK}{L} = \frac{(0.0140 \text{ RAD})(26000 \text{ N/mm}^2)(17.58 \times 10^3 \text{ mm}^4)}{2600 \text{ mm}} = 246 \text{ N/mm}$   
 $= 2.46 \text{ N.in}$

5-47  $Q = 0.050 a^3 = 0.050 (30)^3 = 1350 \text{ mm}^3$  (FIG 5-25)  
 $T = \frac{T}{Q} = \frac{246 \text{ N/mm}}{1350 \text{ mm}^3} = 1.82 \text{ MPa}$

5-48 CIRCULAR PART:  $Z_p = \pi D^3/16 = \pi (1.75)^3/16 = 1.052 \text{ IN}^3$   
 $T = \frac{T}{Z_p} = \frac{850 \text{ LB.IN}}{1.052 \text{ IN}^3} = 808 \text{ PSI}$

SHAFT WITH FLAT:  $h = 1.50 - 0.875 = 0.625 \text{ IN}$ ;  $r = \frac{1.75}{2} = 0.875 \text{ IN}$   
 $h/r = 0.625/0.875 = 0.714$ ;  $C_2 = 1.069$  (INTERPOLATION-FIG 5-25)  
 $Q = C_2 h^3 = (1.069)(0.625)^3 = 0.716 \text{ IN}^3$   
 $T = T/Q = 850 \text{ LB.IN}/0.716 \text{ IN}^3 = 1187 \text{ PSI}$

5-49  $J = \pi (1.75)^4/32 = 0.9208 \text{ IN}^4$ ;  $C_1 = 1.24$  (INTERPOLATION-FIG 5-25)  
 $K = C_1 J = 1.24 (0.9208 \text{ IN}^4) = 0.727 \text{ IN}^4$   
 $\theta = \frac{TL}{GJ} + \frac{TL}{GK} = \frac{(850)(24)}{(11.5 \times 10^6)(0.9208)} + \frac{(850)(24)}{(11.5 \times 10^6)(0.727)} = 0.0018 + 0.00203$   
 $\theta = 0.00384 \text{ RAD} = 0.219 \text{ DEG.}$

5-50  $\lambda = 1.25/2 = 0.625 \text{ in} : \lambda/\alpha = 0.625/0.875 = 0.714 : C_1 = 0.839 \text{ (FIG. 5-25)}$   
 $Q = C_1 \lambda^3 = 0.839 (0.875)^3 = 0.562 \text{ in}^3$   
 $T = \frac{T}{Q} = \frac{850 \text{ LB} \cdot \text{in}}{0.562 \text{ in}^3} = 1512 \text{ PSI IN SHAFT WITH FLATS}$

5-51  $C_2 = 0.966 \text{ for } \lambda/\alpha = 0.714 \text{ IN FIG 5-25 BY INTERPOLATION}$   
 $K = C_2 \lambda^4 = 0.966 (0.875)^4 = 0.566 \text{ in}^4$   
 $\theta = \frac{T L}{G K} = \frac{(850)(20)}{(11.5 \times 10^6)(0.566)} = 0.0026 \text{ RAD IN SHAFT WITH FLATS}$   
 $\theta_{\text{TOT}} = \underbrace{0.0016}_{(\text{ROUND})} + \underbrace{0.0026}_{(\text{FLATS})} = 0.0042 \text{ RAD (0.24 DEG.)}$

5-52  $Q = 0.208 a^3 = 0.208 (8)^3 = 106.5 \text{ mm}^3 \text{ (FIG. 5-25)}$   
 $K = 0.141 a^4 = 0.141 (8)^4 = 577.5 \text{ mm}^4$   
 $T = S_{ys} = 0.554 = 0.5 (1070) = 535 \text{ MPa}$   
 $T = T' Q = (535 \text{ N/mm}^2) (106.5 \text{ mm}^3) = 57.0 \times 10^3 \text{ N} \cdot \text{mm} = 57.0 \text{ N} \cdot \text{m}$   
 $\theta = \frac{T L}{G K} = \frac{(57.0 \times 10^3 \text{ N} \cdot \text{mm})(200 \text{ mm})}{(43000 \text{ N/mm}^2)(577.5 \text{ mm}^4)} = 0.459 \text{ RAD (26.3 DEG.)}$

5-53  $\theta = 3.0 \text{ DEG (} \pi \text{ RAD/180 DEG)} = 0.0524 \text{ RAD.}$   
 $K = \frac{2t(a-t)^2(b-t)^2}{(a+b-2t)} = \frac{2(0.25)(3.75)^2(3.75)^2}{(4+4-0.5)} = 13.18 \text{ in}^4 \text{ (FIG. 5-25)}$   
 $T = \frac{\theta G K}{L} = \frac{(0.0524)(11.5 \times 10^6 \text{ LB/in}^2)(13.18 \text{ in}^4)}{(8 \times 12) \text{ in}} = 82750 \text{ LB} \cdot \text{in}$

5-54  $Q = 2t(a-t)(b-t) = 2(0.25)(3.75)(3.75) = 7.03 \text{ in}^3 \text{ (FIG. 5-25)}$   
 $T = \frac{T}{Q} = \frac{82750 \text{ LB} \cdot \text{in}}{7.03 \text{ in}^3} = 11770 \text{ PSI}$   
 FOR ASTM A501 STEEL,  $S_y = 36000 \text{ PSI}$   
 $T_d = \frac{S_y}{2(2)} = \frac{36000 \text{ PSI}}{4} = 9000 \text{ PSI NOT SAFE}$

5-55  $K = \frac{2(0.25)(3.75)^2(5.75)^2}{(4+6-2(0.25))} = 24.47 \text{ in}^4 \text{ (FIG. 5-25)}$   
 $T = \frac{\theta G K}{L} = \frac{(0.0524)(11.5 \times 10^6)(24.47)}{96} = 153600 \text{ LB} \cdot \text{in}$

5-56  $Q = 2(0.25)(3.75)(5.75) = 10.78 \text{ in}^3 \text{ (FIG. 5-25)}$   
 $T = T/Q = 153600/10.78 = 14250 \text{ PSI NOT SAFE}$

<u>5-57</u> <u>TUBE:</u> $Q = 2(0.25)(5.75)(5.75) = 16.53 \text{ in}^3$	<u>PIPE:</u> $Q_p = 16.99 \text{ in}^3$
$K = \frac{2(0.25)(5.75)^2(5.75)^2}{(6+6-0.5)} = 42.53 \text{ in}^4 \text{ (FIG. 5-25)}$	$J = 2I = 2(28 \text{ in}^4) = 56.28 \text{ in}^4$
$T = T/Q = 0.0605 T$	$T_p = T/Q_p = 0.0589 T$
$\theta_T = \frac{T L}{G K} = 0.0210 (T/6)$	$\theta_p = \frac{T L}{G J} = 0.0177 (T/6)$
$\frac{T_T}{T_P} = \frac{0.0605}{0.0589} = 1.028$	$\frac{\theta_T}{\theta_P} = \frac{0.0210}{0.0177} = 1.186$

5-58

$$P = T \cdot m \quad T = P/m \quad D = 25 \text{ mm}$$

$$m = \frac{1150 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ s}} = 120.4 \text{ rad/s}$$

$$T = \frac{P}{m} = \frac{125 \times 10^3 \text{ N}\cdot\text{m/s}}{120.4 \text{ rad/s}} = 1038 \text{ N}\cdot\text{m}$$

$$\tau_{\max} = \frac{T_c}{J} = \frac{T}{Z_p} = \frac{1038 \text{ N}\cdot\text{m}}{\pi (35 \text{ mm})^3 / 16} \cdot \frac{10^3 \text{ mm}}{\text{mm}} = \frac{123 \text{ N}}{\text{mm}^2} = 123 \text{ MPa}$$

5-59

SMOOTH POWER.  $T_d = S_y / N = S_y / 2N$ . LET  $N = 2$

$$T_d = S_y / 4. \quad \text{LET } T_{\max} = T_d = S_y / 4$$

$$\text{REQ'D } S_y = 4(T_d) = 4(123 \text{ MPa}) = 493 \text{ MPa}$$

POSSIBLE STEEL: AISI 1040 CP,  $S_y = 565 \text{ MPa}$

5-60

REPEATED POWER. LET  $N = 4$ .  $T_d = S_y / 2N = S_y / 8 = T_{\max}$

$$\text{REQ'D } S_y = 8 T_{\max} = 8(123 \text{ MPa}) = 986 \text{ MPa}$$

POSSIBLE STEEL: AISI 4140 OQT 900.  $S_y = 1193 \text{ MPa}$   
15% ELONGATION. 6.60 DUCTILITY.

5-61

SHOCK LOADING. LET  $N = 6$ .  $T_d = S_y / 2N = S_y / 12 = T_{\max}$

$$\text{REQ'D } S_y = 12 T_{\max} = 12(123 \text{ MPa}) = 1480 \text{ MPa}$$

POSSIBLE STEEL: AISI 4140 OQT 700.  $S_y = 1462 \text{ MPa}$   
12% ELONGATION.

MARGINAL STRENGTH, SATISFACTORY DUCTILITY.

5-62

$P = 12.0 \text{ HP}$  STEADY.  $m = 1150 \text{ rpm}$ .  $S_y = 80 \text{ ksi}$

$$T = 63000(P)/m = 63000(12)/1150 = 657 \text{ IN}\cdot\text{LB}$$

$$T_{\max} = T/Z_p. \quad \text{REQ'D } Z_p = \frac{T}{T_{\max}} = \frac{657 \text{ IN}\cdot\text{LB}}{20000 \text{ LB/IN}^2} = 0.0329 \text{ IN}^3 = \frac{\pi D^3}{16}$$

$$\text{LET } T_{\max} = T_d = S_y / 4 = 80 \text{ ksi} / 4 = 20 \text{ ksi} = 20000 \text{ LB/IN}^2$$

$$D_{\min} = \sqrt[3]{\frac{16 Z_p}{\pi}} = \sqrt[3]{\frac{16(0.0329 \text{ IN}^3)}{\pi}} = 0.551 \text{ IN}; \quad \text{SPECIFY } D = 0.60 \text{ IN}$$

5-63

$P = 20.0 \text{ HP}$  STEADY.  $m = 3450 \text{ rpm}$ .  $S_y = 101 \text{ ksi}$

$$T = 63000(P)/m = 63000(20)/3450 = 365 \text{ IN}\cdot\text{LB}$$

$$T_{\max} = T/Z_p. \quad \text{REQ'D } Z_p = \frac{T}{T_{\max}} = \frac{365 \text{ IN}\cdot\text{LB}}{25250 \text{ LB/IN}^2} = 0.0145 \text{ IN}^3 = \frac{\pi D^3}{16}$$

$$\text{LET } T_{\max} = T_d = S_y / 4 = 101 \text{ ksi} / 4 = 25.25 \text{ ksi} = 25250 \text{ PSI}$$

$$D_{\min} = \sqrt[3]{\frac{16 Z_p}{\pi}} = \sqrt[3]{\frac{16(0.0145 \text{ IN}^3)}{\pi}} = 0.419 \text{ IN}; \quad \text{SPECIFY } D = 0.50 \text{ IN}$$

5-64  $D_o = 100 \text{ mm}$ ;  $D_i = 60 \text{ mm}$ ; ALLOY STEEL

$$\tau_{\text{solid}} = 200 \text{ MPa} = \frac{T}{Z_p} \cdot T = \tau \cdot Z_p$$

$$Z_p = \pi D_o^3 / 16 = \pi (100 \text{ mm})^3 / 16 = 196350 \text{ mm}^3$$

$$T = \tau \cdot Z_p = (200 \text{ N/mm}^2)(196350 \text{ mm}^3) = 3.927 \times 10^7 \text{ N}\cdot\text{mm}$$

$$\tau_{\text{Hollow}} = \frac{T}{Z_p} = \frac{3.927 \times 10^7 \text{ N}\cdot\text{mm}}{1.709 \times 10^5 \text{ mm}^3} = 230 \text{ N/mm}^2 \approx 230 \text{ MPa} \approx \tau_H$$

$$Z_p = \frac{\pi}{16} \frac{D_o^4 - D_i^4}{D_o} = \frac{\pi [100^4 - 60^4]}{16 (100)} \text{ mm}^3 = 170900 \text{ mm}^3$$

5-65 FIND ANGLE OF TWIST FOR SHAFT OF PROB. 5-64.

$$T = 3.927 \times 10^7 \text{ N}\cdot\text{mm}$$

$$\text{SOLID SEGMENT: } J = \frac{\pi D_o^4}{32} = \frac{\pi (100 \text{ mm})^4}{32} = 9.817 \times 10^6 \text{ mm}^4$$

$$\theta_s = \frac{T L}{G J} = \frac{(3.927 \times 10^7 \text{ N}\cdot\text{mm})(300 \text{ mm})}{(80 \times 10^3 \text{ N/mm}^2)(9.817 \times 10^6 \text{ mm}^4)} = 0.0150 \text{ rad.}$$

$$G = 80 \text{ GPa} = \frac{80 \times 10^9 \text{ N}}{\text{m}^2} \times \frac{1 \text{ m}^2}{(10^3 \text{ mm})^2} = 80 \times 10^3 \text{ N/mm}^2$$

$$\text{HOLLOW SEGMENT: } J = \frac{\pi}{32} (100^4 - 60^4) \text{ mm}^4 = 8.545 \times 10^6 \text{ mm}^4$$

$$\theta_H = \frac{T L}{G J} = \frac{(3.927 \times 10^7)(300)}{(80 \times 10^3)(8.545 \times 10^6)} = 0.0172 \text{ rad}$$

$$\text{TOTAL } \theta_T = \theta_s + \theta_H = 0.0150 + 0.0172 = 0.0322 \text{ rad (1.850 deg)}$$

5-66 FIND  $T$  IN EACH PART OF SHAFT.

$$n = \frac{1750 \text{ rev}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = 183 \text{ rad/s}$$

$$T_A = \frac{P_A}{n} = \frac{15 \times 10^3 \text{ N}\cdot\text{m/s}}{183 \text{ rad/s}} = 81.85 \text{ N}\cdot\text{m}$$

$$T_C = \frac{P_C}{n} = \frac{20 \times 10^3}{183} = 109.1 \text{ N}\cdot\text{m}; T_B = \frac{P_B}{n} = \frac{35 \times 10^3}{183} = 191.0 \text{ N}\cdot\text{m}$$

$$T_{AB} = T_A = 81.85 \text{ N}\cdot\text{m} \quad T_{BC} = T_C = 109.1 \text{ N}\cdot\text{m}$$

T AT A:  $D = 9.50 \text{ mm}$ , RETAINING RING TO RIGHT OF PULLEY;  $K_t = 3.0$

$$\tau_A = T_{AB} \cdot K_t / Z_p = \frac{(81.85 \text{ N}\cdot\text{m})(3.0)}{\pi (9.5 \text{ mm})^3 / 16} \cdot \frac{10^3 \text{ mm}}{\text{m}} = 12.51 \text{ N/mm}^2 = 14.59 \text{ MPa}$$

VERY HIGH

T AT BEARING TO RIGHT OF PULLEY: STEPPED SHAFT.  $D = 10 \text{ mm}$

$$P/d = 15/10 = 1.50; r/d = 0.5 \text{ mm} / 10 \text{ mm} = 0.05; K_t = 1.60$$

$$\tau = T_{AB} \cdot K_t / Z_p = \frac{(81.85)(1.60)(10^3)}{\pi (10)^3 / 16} = 667 \text{ MPa}$$

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# 5-66 (CONTINUED)

$\tau$  TO LEFT OF B: RETAINING RING GROOVE;  $D=14\text{ mm}$  AT GROOVE

$$\tau = \frac{T_{AB} \cdot K_t}{Z_P} = \frac{(81.85)(3.0)(1000)}{\pi(14)^3/16} = \underline{456\text{ MPa}}$$

$\tau$  AT KEYSEAT AT B:  $D=15\text{ mm}$ ,  $K_t=2.0$  FOR KEYSEAT

$$T = T_{BC} = 109.1\text{ N}\cdot\text{m}$$

$$\tau = \frac{T_{BC} \cdot K_t}{Z_P} = \frac{(109.1)(2.0)(1000)}{\pi(15)^3/16} = \underline{329\text{ MPa}}$$

$\tau$  TO RIGHT OF B AT SHOULDER FILLET:  $D_1=15\text{ mm}$ ,  $D_2=20\text{ mm}$

$$D_2/D_1 = 20/15 = 1.33; r/D_1 = 0.50\text{ mm}/15\text{ mm} = 0.033; K_t = 1.75$$

$$\tau = \frac{T_{BC} \cdot K_t}{Z_P} = \frac{(109.1)(1.75)(1000)}{\pi(15)^3/16} = \underline{288\text{ MPa}}$$

$\tau$  AT RIGHT BEARING AT SHOULDER FILLET:  $D_1=15\text{ mm}$ ,  $D_2=20\text{ mm}$

$$\text{SAME CONDITIONS AS AT PULLEY: } \tau = \underline{288\text{ MPa}}$$

$\tau$  AT C AT RETAINING RING GROOVE:  $K_t=3.0$ ;  $D_g=14.0\text{ mm}$

$$\tau = \frac{T_{BC} \cdot K_t}{Z_P} = \frac{(109.1)(3.0)(1000)}{\pi(14.0)^3/16} = \underline{607\text{ MPa}}$$

$\tau$  AT C AT SLED RUNNER KEY SEAT:  $D=15\text{ mm}$ ;  $K_t=1.60$

$$\tau = \frac{T_{BC} \cdot K_t}{Z_P} = \frac{(109.1)(1.60)(1000)}{\pi(15)^3/16} = \underline{263\text{ MPa}}$$

SUMMARY SEVERAL STRESSES ARE QUITE HIGH. LARGER SHAFT DIAMETERS RECOMMENDED.

# 5-67 FIND $\tau$ IN EACH PART OF SHAFT.

$$M = \frac{1150\text{ rev}}{\text{min}} \cdot \frac{2\pi\text{ rad}}{\text{rev}} \cdot \frac{1\text{ min}}{60\text{ s}} = 120.4\text{ rad/s}$$

$$T_A = \frac{P_A}{\omega} = \frac{20 \times 10^3\text{ N}\cdot\text{m/s}}{120.4\text{ rad/s}} = 166\text{ N}\cdot\text{m} \times \frac{10^3\text{ mm}}{\text{m}} = 1.66 \times 10^5\text{ N}\cdot\text{mm}$$

$$T_C = \frac{P_C}{\omega} = \frac{12 \times 10^3\text{ N}\cdot\text{m/s}}{120.4\text{ rad/s}} = 99.6\text{ N}\cdot\text{m} \times \frac{10^3\text{ mm}}{\text{m}} = 9.96 \times 10^4\text{ N}\cdot\text{mm}$$

$$T_B = \frac{P_B}{\omega} = \frac{32 \times 10^3\text{ N}\cdot\text{m/s}}{120.4\text{ rad/s}} = 266\text{ N}\cdot\text{m} \times \frac{10^3\text{ mm}}{\text{m}} = 2.66 \times 10^5\text{ N}\cdot\text{mm}$$

$$T_{AB} = T_A = 1.66 \times 10^5\text{ N}\cdot\text{mm} \quad T_{BC} = T_C = 9.96 \times 10^4\text{ N}\cdot\text{mm}$$

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# 5-67 (CONTINUED)

$T_1$  AT A AT KEYSEAT:  $D=20.0\text{ mm}$ ;  $K_t=2.0$  - PROFILE KEYSEAT

$$T_1 = \frac{T_{AB} K_t}{Z_P} = \frac{1.66 \times 10^5 \text{ N/mm} (2.0)}{\pi (20 \text{ mm})^3 / 16} = 211 \text{ N/mm}^2 = \boxed{211 \text{ MPa}} = T_{MAX}$$

$T_2$  AT SHOULDER TO RIGHT OF A:  $D=20.0\text{ mm}$ ;  $D/d = \frac{30}{20} = 1.50$

$$r/d = 1.0/20 = 0.05; K_t = 1.62$$

$$T_2 = \frac{T_{AB} K_t}{Z_P} = \frac{(1.66 \times 10^5)(1.62)}{\pi (20)^3 / 16} = 171 \text{ MPa}$$

$T_3$  AT RIGHT OF BEARING SEAT:  $D=30\text{ mm}$ ;  $D/d = \frac{40}{30} = 1.33$

$$r/d = 1.0/30 = 0.033; K_t = 1.78$$

$$T_3 = \frac{T_{AB} K_t}{Z_P} = \frac{(1.66 \times 10^5)(1.78)}{\pi (30)^3 / 16} = 55.7 \text{ MPa}$$

$T_4$  AT RETAINING RING TO LEFT OF B:  $D=40.0\text{ mm}$ ;  $K_t=3.0$

$$T_4 = \frac{T_{AB} K_t}{Z_P} = \frac{(1.66 \times 10^5)(3.0)}{\pi (40)^3 / 16} = 39.6 \text{ MPa}$$

$T_5$  AT KEYSEAT AT B:  $K_t=2.0$ ,  $D=40\text{ mm}$

$$T_5 = \frac{T_{AB} K_t}{Z_P} = T_4 \cdot \frac{K_{t5}}{K_{t4}} = 39.6 \text{ MPa} \cdot \frac{2.0}{3.0} = 26.4 \text{ MPa}$$

$T_6$  AT STEP TO RIGHT OF B:  $D=40\text{ mm}$ ,  $D/d = \frac{50.0}{40.0} = 1.25$

$$r/d = 1.0/40 = 0.025; K_t = 1.85$$

$$T_6 = \frac{T_{BC} K_t}{Z_P} = \frac{(9.96 \times 10^4)(1.85)}{\pi (40)^3 / 16} = 14.7 \text{ MPa}$$

$T_7$  AT STEP FROM 50 TO 30 mm DIA.:  $D=30.0\text{ mm}$ ,  $D/d = \frac{50}{30} = 1.67$

$$r/d = 1.0/30 = 0.033; K_t = 1.82$$

$$T_7 = \frac{T_{BC} K_t}{Z_P} = \frac{(9.96 \times 10^4)(1.82)}{\pi (30)^3 / 16} = 10.7 \text{ MPa}$$

$T_8$  AT LEFT OF BEARING:  $D=20.0\text{ mm}$ ;  $D/d = \frac{30}{20} = 1.50$

$$r/d = 1.0/20 = 0.05; K_t = 1.62$$

$$T_8 = \frac{T_{BC} K_t}{Z_P} = \frac{(9.96 \times 10^4)(1.62)}{\pi (20)^3 / 16} = 10.3 \text{ MPa}$$

$T_9$  AT STEP TO LEFT OF C:  $D=15.0\text{ mm}$ ;  $D/d = \frac{20}{15} = 1.33$ ;  $r/d = \frac{1}{15} = 0.067$

$$K_t = 1.50; T_9 = \frac{T_{BC} K_t}{Z_P} = \frac{(9.96 \times 10^4)(1.50)}{\pi (15)^3 / 16} = 22.5 \text{ MPa}$$

$T_{10}$  AT KEYSEAT AT C:  $T_{10} = \frac{T_{BC} K_t}{Z_P} = T_9 \cdot \frac{K_{t10}}{K_{t9}} = (22.5) \cdot \frac{2.0}{1.5} = 30.1 \text{ MPa}$   
 $K_t=2.0$

5-68 DESIGN SHAFT  $P=225 \text{ kW}$ ;  $n=80 \text{ rpm}$ ;  $T_s=60 \text{ MPa}$ ;  $K_t=1.0$

$$T = \frac{P}{\omega} = \frac{225 \times 10^3 \text{ N}\cdot\text{m/s}}{8.38 \text{ rad/s}} = 26857 \text{ N}\cdot\text{m}$$

$$\omega = 80 \frac{\text{REV}}{\text{MIN}} \cdot \frac{2\pi \text{ RAD}}{\text{REV}} \cdot \frac{1 \text{ MIN}}{60 \text{ S}} = 8.38 \text{ rad/s}$$

$$T = \frac{T}{Z_P}; \text{REQ'D } Z_P = \frac{T}{T_s} = \frac{26857 \text{ N}\cdot\text{m}}{60 \text{ N/mm}^2} = 447.6 \text{ mm}^3$$

$$Z_P = 4.476 \times 10^5 \text{ mm}^3 = \frac{\pi}{16} \frac{D_o^4 - D_i^4}{D_o}$$

$$\text{BUT } Z_P = \frac{\pi [(1.25 D_i)^4 - D_i^4]}{(16)(1.25 D_i)} = \frac{\pi (1.44 D_i^4)}{(16)(1.25 D_i)} = 0.226 D_i^3$$

$$\text{THEN REQ'D } D_i = \sqrt[3]{\frac{Z_P}{0.226}} = \sqrt[3]{\frac{4.476 \times 10^5 \text{ mm}^3}{0.226}} = 125.5 \text{ mm}$$

$$D_o \approx 1.25 D_i = 1.25(125.5) = 157 \text{ mm}$$

$$\text{LET } D_o = 160 \text{ mm}; D_i \approx \frac{D_o}{1.25} = \frac{160}{1.25} = 128 \text{ mm}; \text{ USE } D_i = 125 \text{ mm}$$

$$\text{CHECK } Z_P = \frac{\pi}{16} \frac{D_o^4 - D_i^4}{D_o} = \frac{\pi [(160)^4 - (125)^4]}{(16)(160)} = 5.05 \times 10^5 \text{ mm}^3 \quad \text{OK BUT HIGH}$$

$$\text{TRY } D_o = 160 \text{ mm}, D_i = 130 \text{ mm}$$

$$Z_P = \frac{\pi [(160)^4 - (130)^4]}{16(160)} = 4.54 \times 10^5 \text{ mm}^3 \quad \text{OK}$$

$$\text{OR: LET } D_o = 160 \text{ mm}, \text{ SOLVE FOR REQ'D } D_i \text{ FOR } Z_P = 4.476 \times 10^5 \text{ mm}^3$$

$$Z_P = \frac{\pi [(160)^4 - D_i^4]}{16(160)}; (16)(160) Z_P = \pi [(160)^4 - D_i^4]$$

$$\frac{16(160) Z_P}{\pi} = 160^4 - D_i^4; D_i^4 = 160^4 - \frac{16(160)(4.476 \times 10^5)}{\pi}$$

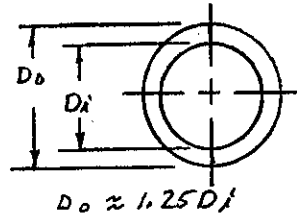
$$D_{i \text{ MAX}} = 130.6 \text{ mm}; \text{ USE } D_o = 160 \text{ mm}; D_i = 130 \text{ mm}$$

$$\text{CHECK FOR WALL THICKNESS: } t = \frac{D_o - D_i}{2} = \frac{160 - 130}{2} = 15 \text{ mm}$$

$$\text{MEAN RADIUS} = \frac{(D_o + D_i)/2}{2} = 72.5 \text{ mm}$$

$$r_{\text{MEAN}}/t = 72.5/15 = 4.83 < 10. \text{ SHAFT IS NOT THIN-WALLED}$$

BUCKLING NOT LIKELY.



5-69  $D = 4.0 \text{ mm}$ ;  $\theta = 180 \text{ DEG} \times \frac{\pi \text{ RAD}}{180^\circ} = \pi \text{ RAD}$ ;  $T_{\text{MAX}} = 150 \text{ MPa} = \frac{T_c}{J}$

$$J = \frac{\pi D^4}{32} = \frac{\pi (4.0 \text{ mm})^4}{32} = 25.1 \text{ mm}^4$$

$$T_{\text{MAX}} = \frac{T_{\text{MAX}} J}{c} = \frac{(150 \text{ N/mm}^2)(25.1 \text{ mm}^4)}{2.0 \text{ mm}} = 1885 \text{ N}\cdot\text{mm}$$

$$\theta = \frac{TL}{GJ}; L_{\text{MIN}} = \frac{\theta GJ}{T_{\text{MAX}}}$$

$$G = 26 \text{ GPa} = \frac{26 \times 10^9 \text{ N}}{\text{m}^2} \times \frac{1/\text{m}^2}{(10^3 \text{ mm})^2} = 26.0 \times 10^3 \text{ N/mm}^2$$

$$L_{\text{MIN}} = \frac{(\pi \text{ RAD})(26 \times 10^3 \text{ N/mm}^2)(25.1 \text{ mm}^4)}{1885 \text{ N}\cdot\text{mm}} = 1088 \text{ mm} \approx \underline{1.088 \text{ m}}$$

5-70 TORSION BAR:  $L = 200 \text{ mm} = 0.200 \text{ m}$ .  $D_o/D_i \approx 1.50$

TORSIONAL STIFFNESS =  $\frac{\theta}{T} = \frac{0.015 \text{ DEG}}{1.0 \text{ N}\cdot\text{m}} \times \frac{\pi \text{ RAD}}{180 \text{ DEG}} = \frac{0.2618 \times 10^{-3} \text{ RAD}}{1.0 \text{ N}\cdot\text{m}}$

$$\theta = \frac{TL}{GJ}; \text{REQ'D } J = \frac{TL}{\theta G}$$

$$G = 43 \text{ GPa} = (43 \times 10^9 \text{ N/m}^2) \left( \frac{1.0 \text{ m}^2}{10^6 \text{ mm}^2} \right) = 43 \times 10^3 \text{ N/mm}^2$$

$$J = \frac{(1.0 \text{ N}\cdot\text{m})(200 \text{ mm})}{(0.2618 \times 10^{-3} \text{ RAD})(43 \times 10^3 \text{ N/mm}^2)} \times \frac{10^3 \text{ mm}}{1 \text{ m}} = 17766 \text{ mm}^4$$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32} = \frac{\pi(1.50i)^4 - D_i^4}{32} = 0.3988 D_i^4$$

$$D_i = \frac{J}{0.3988} = \sqrt[4]{\frac{17766 \text{ mm}^4}{0.3988}} = 14.53 \text{ mm} = D_i$$

$$D_o = 1.50(D_i) = 1.50(14.53 \text{ mm}) = 21.79 \text{ mm} \approx D_o$$

ALTERNATE DESIGN: PREFERRED SIZE FOR  $D_o = 22.0 \text{ mm}$

FIND REQ'D  $D_i$  FOR  $J = 17766 \text{ mm}^4$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32}; D_o^4 - D_i^4 = \frac{32J}{\pi}; D_i^4 = D_o^4 - \frac{32J}{\pi}$$

$$D_i = \sqrt[4]{D_o^4 - \frac{32J}{\pi}} = \sqrt[4]{22.0^4 - \frac{32(17766)}{\pi}} = 15.19 \text{ mm} = D_i$$

FOR  $D_o = 22.0 \text{ mm}$

5-71 USE FIRST DESIGN FROM 5-70.  $D_o = 21.79 \text{ mm}$ ,  $D_i = 14.53 \text{ mm}$

$$T = \frac{T_c}{J} = \frac{T D_o}{J \cdot 2}; \theta = \frac{TL}{GJ} \text{ OR } T = \frac{\theta GJ}{L}$$

$$\text{THEN } T = \frac{\theta GJ}{L} \cdot \frac{D_o}{2J} = \frac{\theta G D_o}{2L} = \frac{(0.1745 \text{ RAD})(43 \times 10^3 \text{ N/mm}^2)(21.79 \text{ mm})}{2(200 \text{ mm})}$$

$$\theta = 10^\circ \times \frac{\pi \text{ RAD}}{180^\circ} = 0.1745 \text{ RAD}$$

$$T = 408.8 \text{ N/mm}^2 = \underline{408.8 \text{ MPa}}$$

5-72 FIND  $\theta$  WHEN  $T_d = \frac{S_{ys}}{3} = \frac{S_y}{2(3)} = \frac{S_y}{6} = \frac{903 \text{ MPa}}{6} = 150.5 \text{ MPa}$

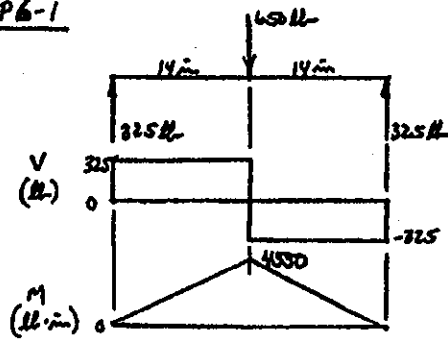
ALSI 4140 OQT 1100;  $S_y = 903 \text{ MPa}$ , 18% ELONGATION

$$T = \frac{TC}{J} ; T = \frac{T_d J}{C} = \frac{(150.5 \text{ N/mm}^2)(17766 \text{ mm}^4)}{21.79 \text{ mm} / 2} \times \frac{1 \text{ m}}{1000 \text{ mm}} = 245.4 \text{ N}\cdot\text{m}$$

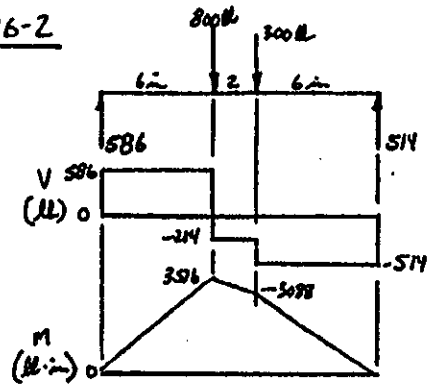
$$\theta = \frac{TL}{GJ} = \frac{(245.4 \text{ N}\cdot\text{m})(200 \text{ mm})}{(43000 \text{ N/mm}^2)(17766 \text{ mm}^4)} \times \frac{10^3 \text{ mm}}{\text{m}} = 0.061 \text{ RAD} \times \frac{180^\circ}{\pi \text{ RAD}} = 3.48 \text{ DEG}$$

# CHAPTER 6 Shearing Forces and Bending Moments in Beams

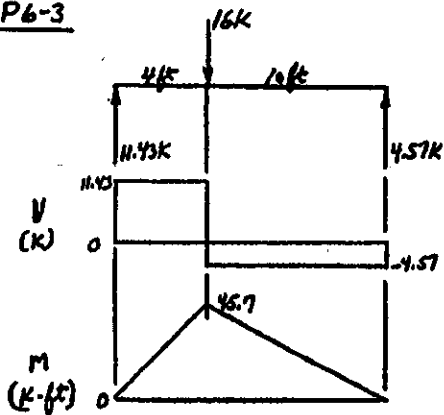
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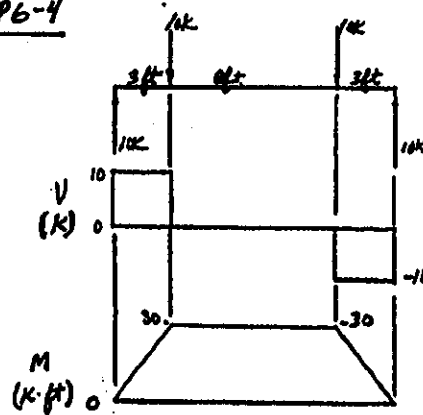
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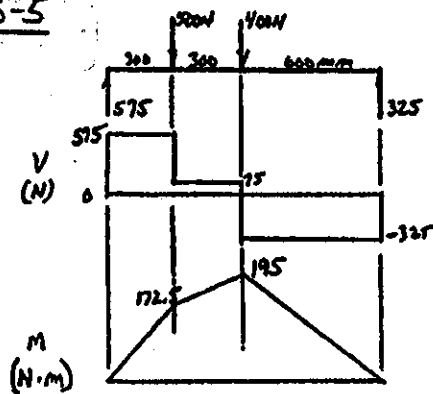
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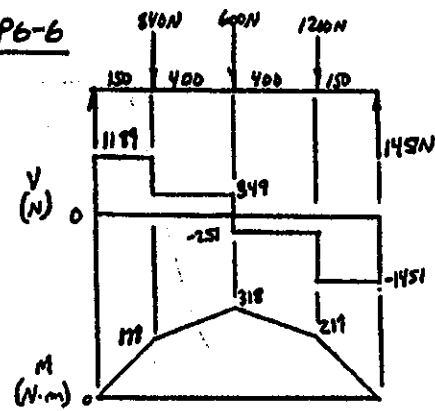
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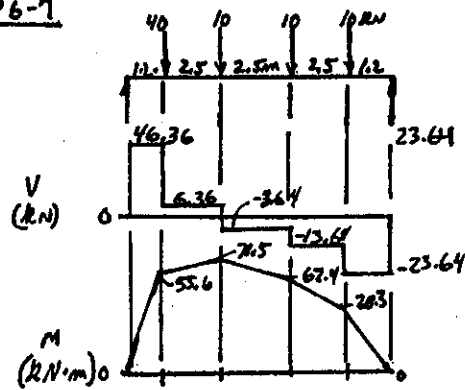
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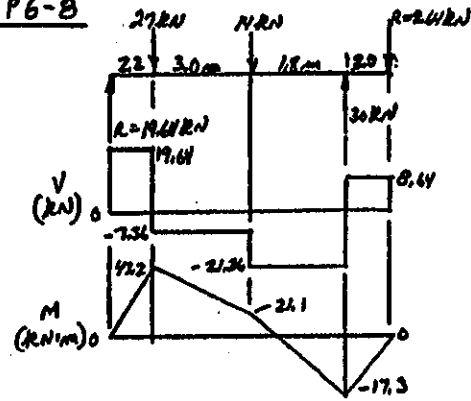
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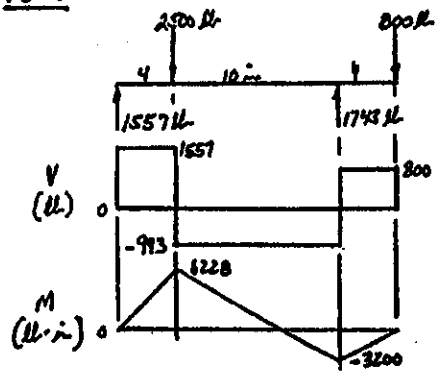
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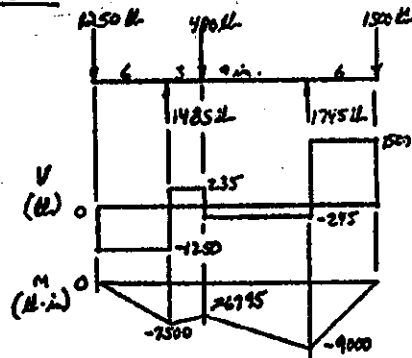
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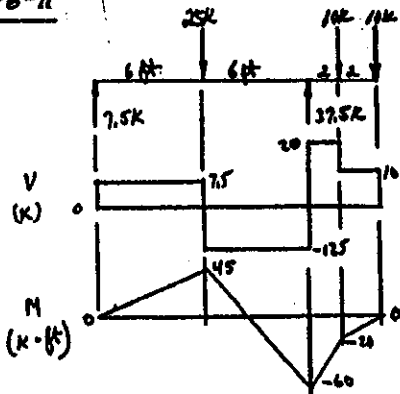
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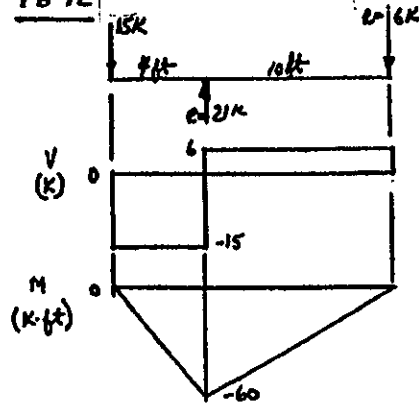
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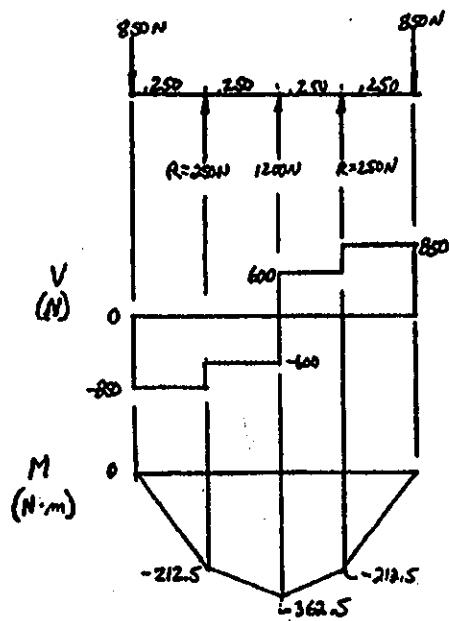
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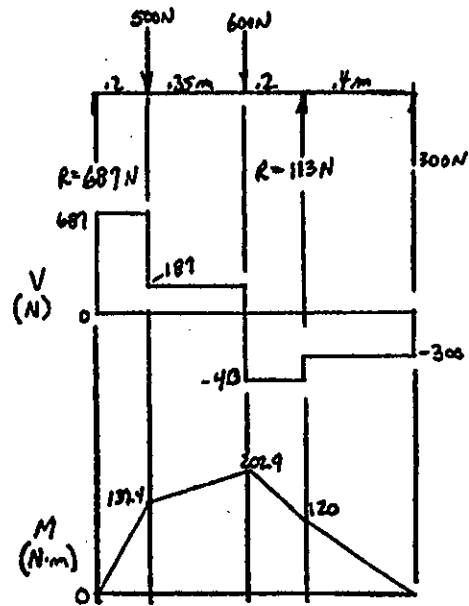
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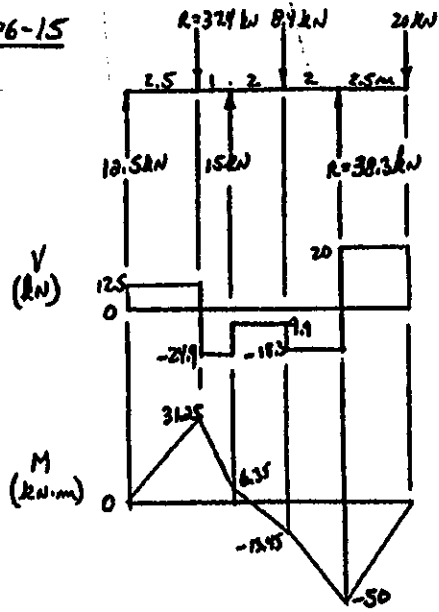
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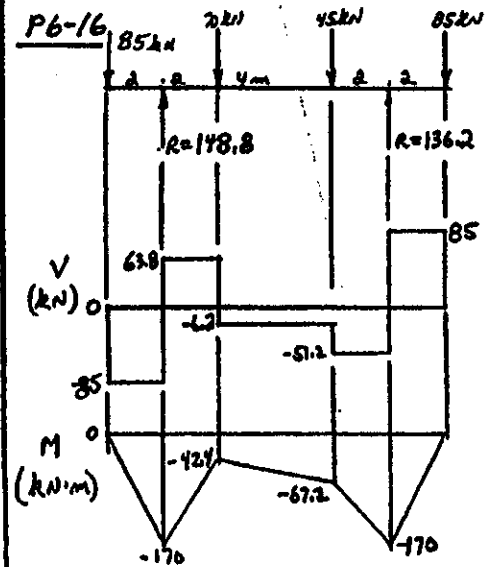
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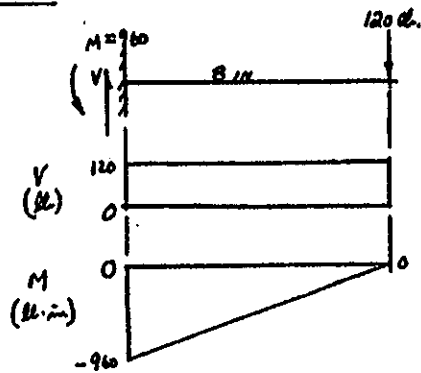
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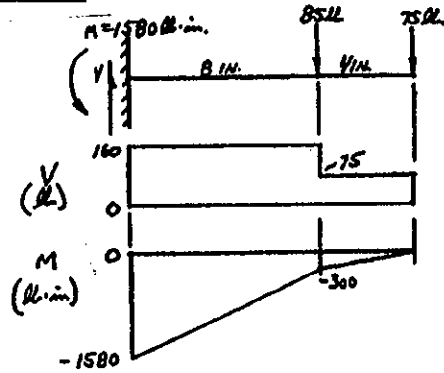
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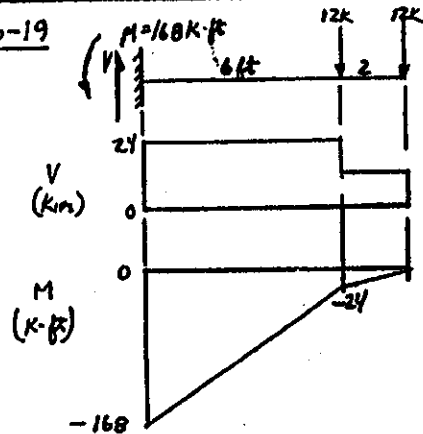
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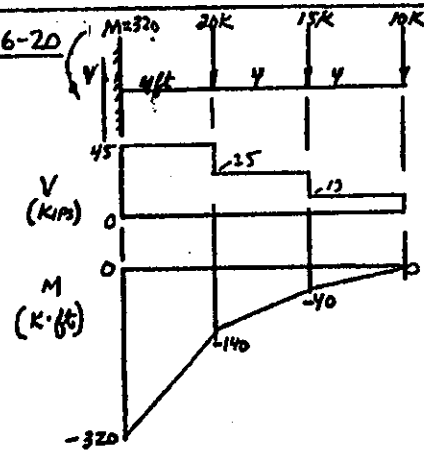
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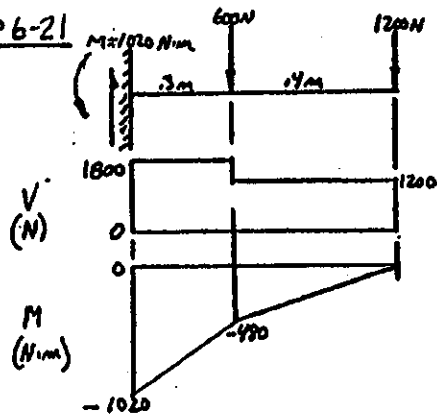
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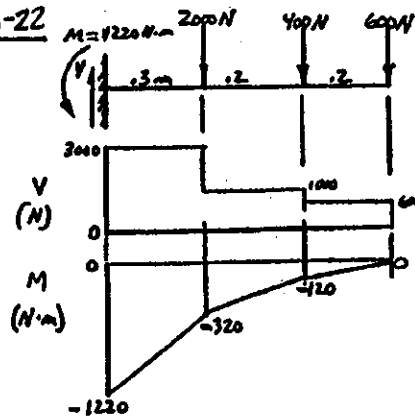
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P6-21

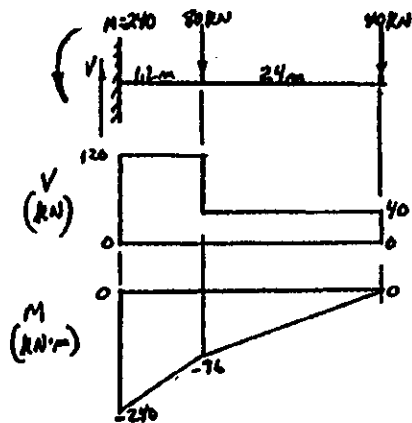


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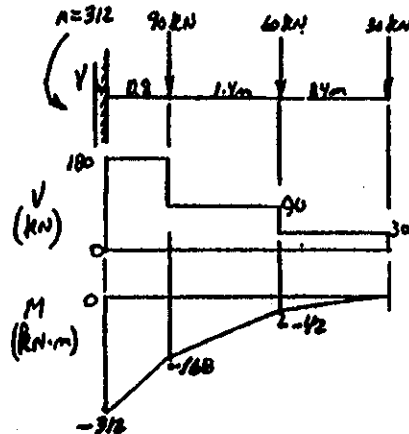




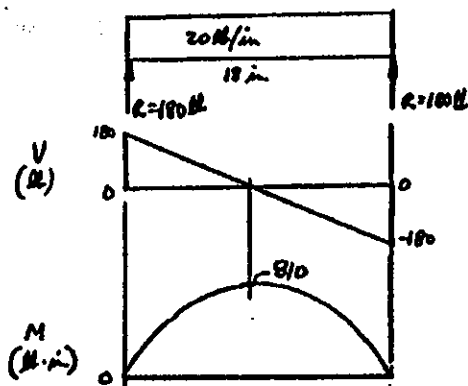
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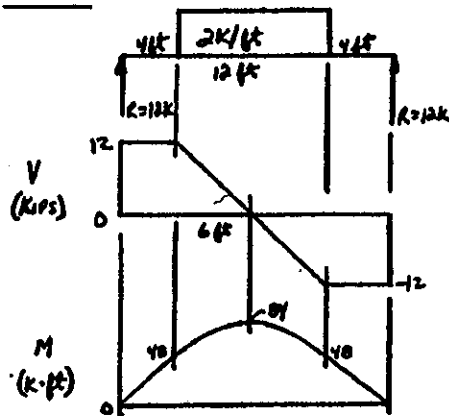
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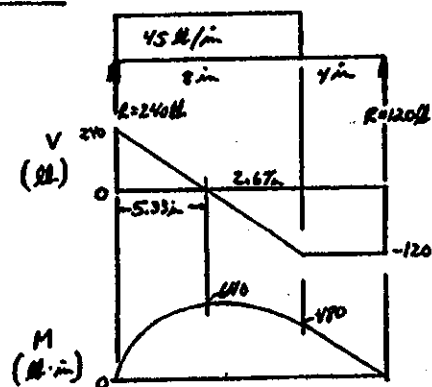
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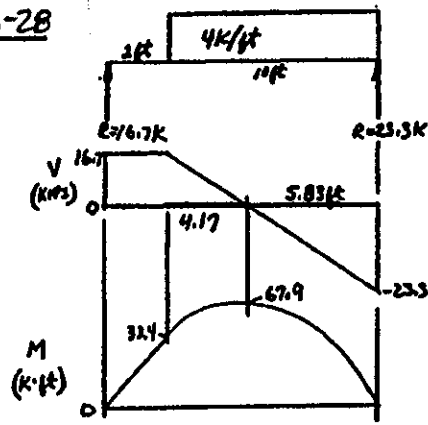
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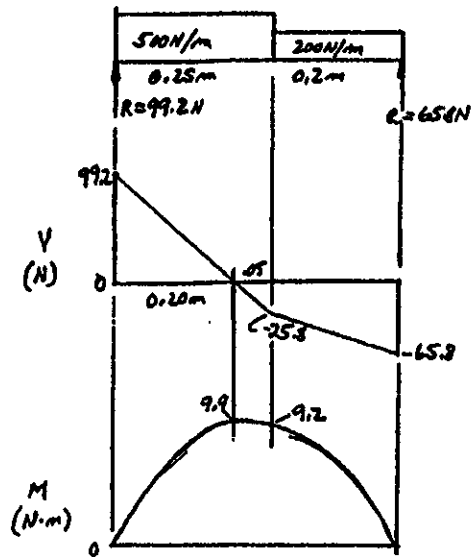
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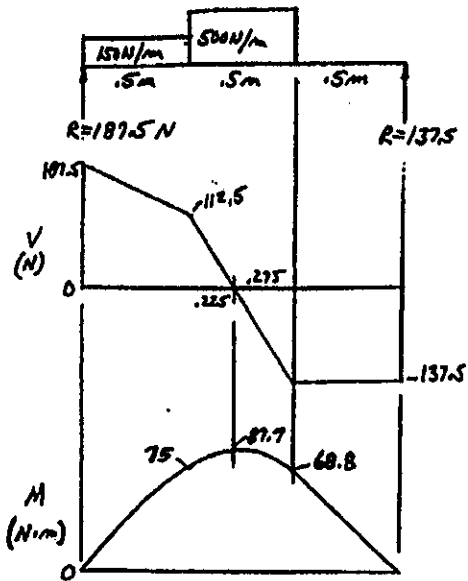
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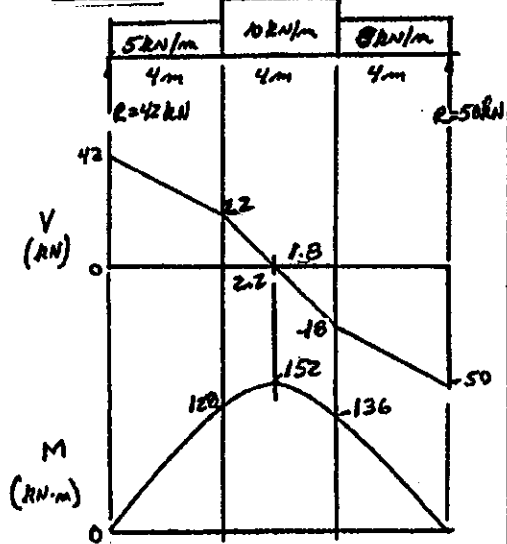
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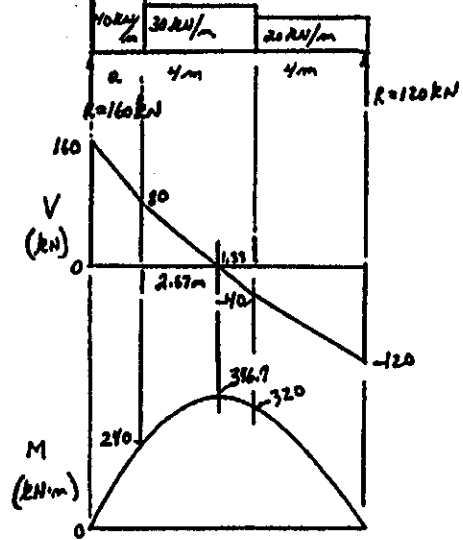
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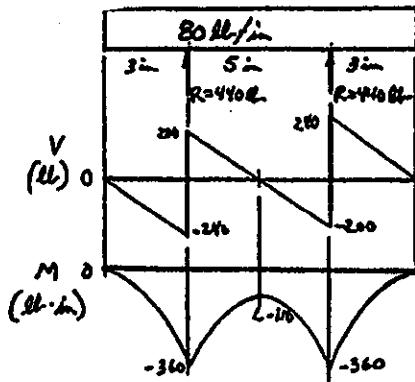
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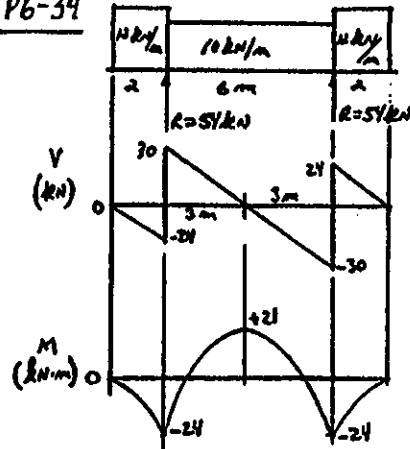
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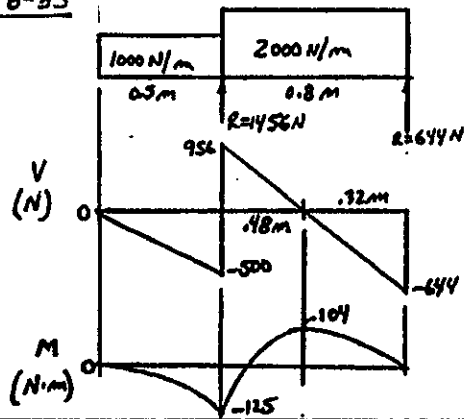
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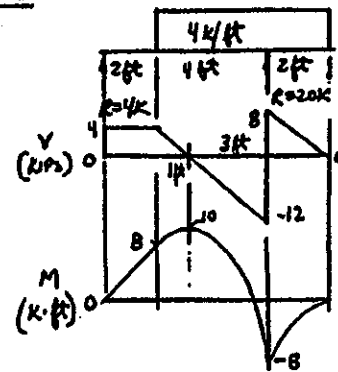
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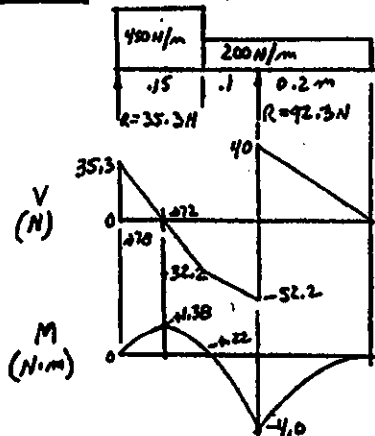
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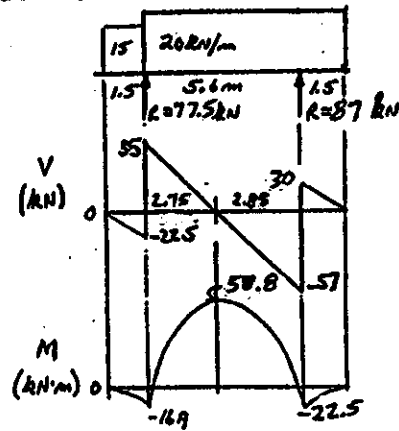
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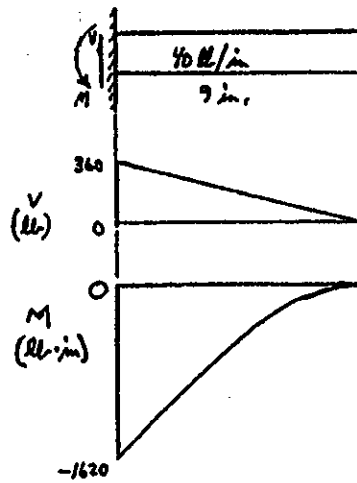
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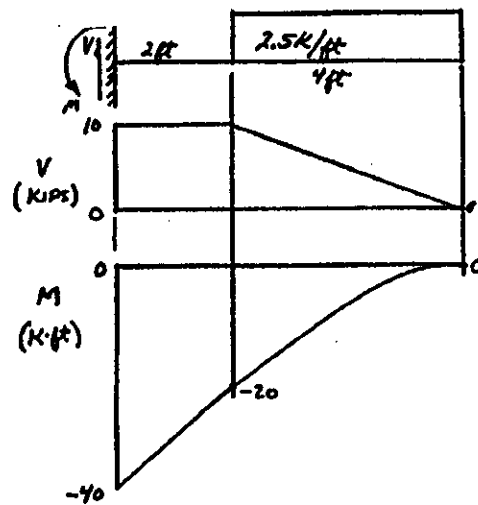
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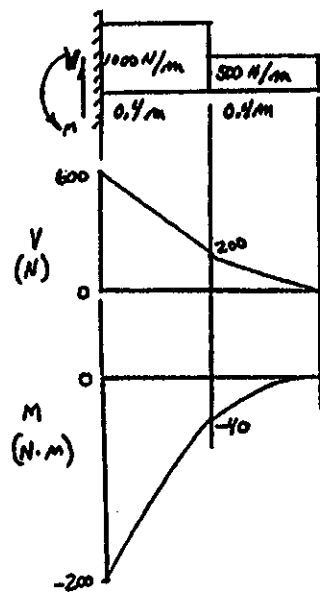
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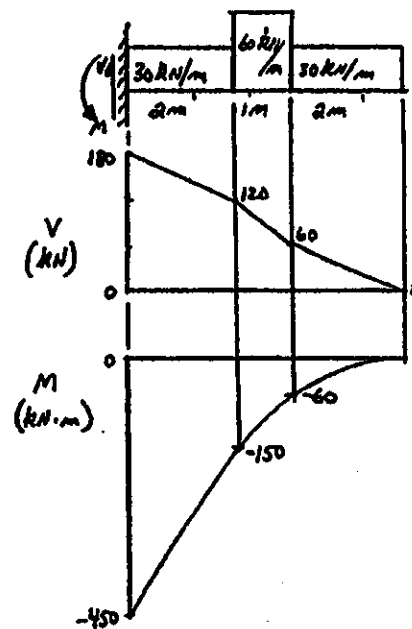
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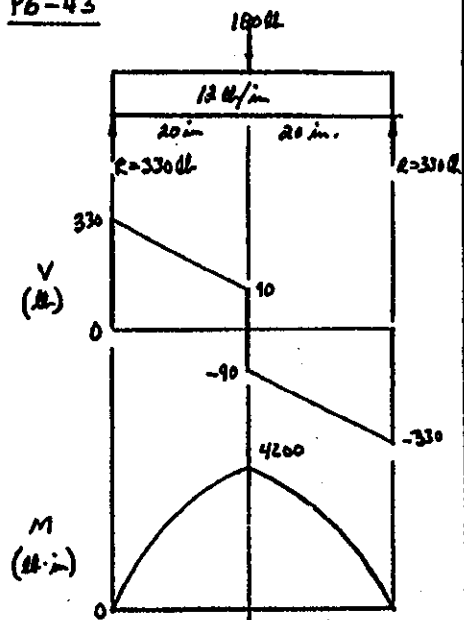
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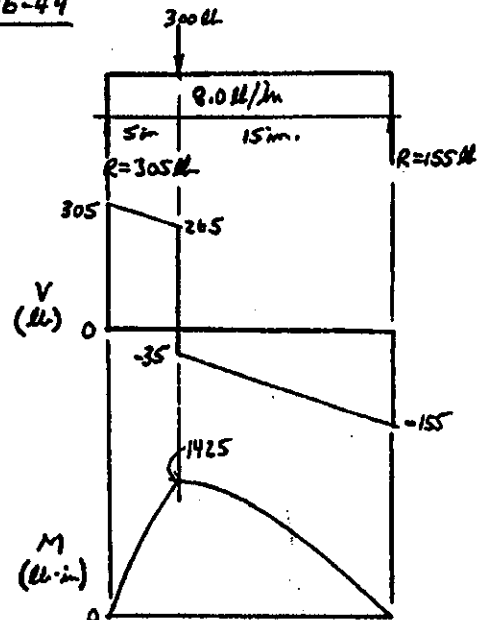
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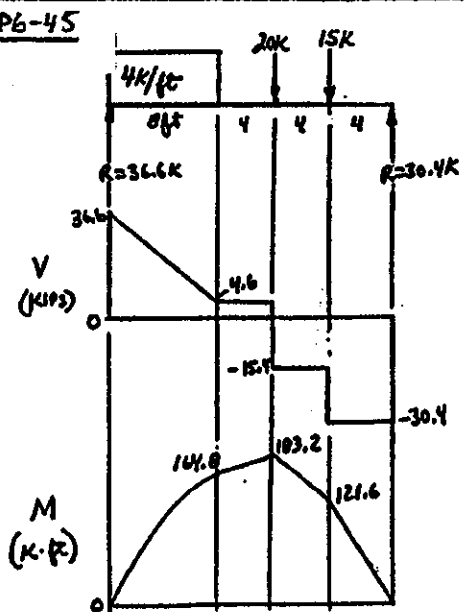
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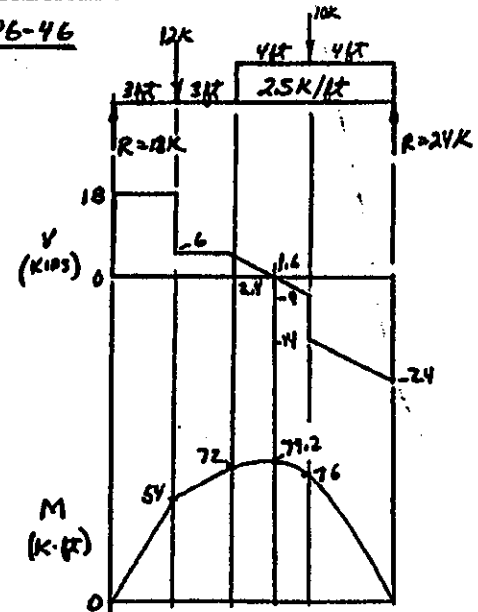
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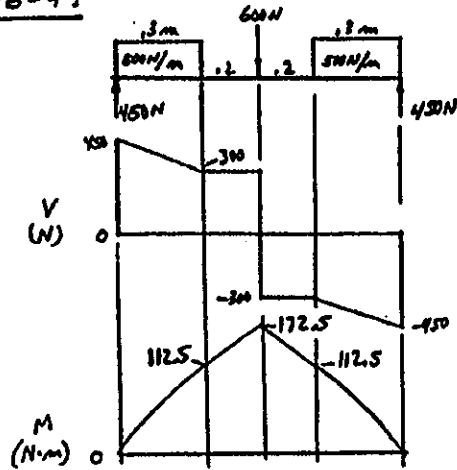
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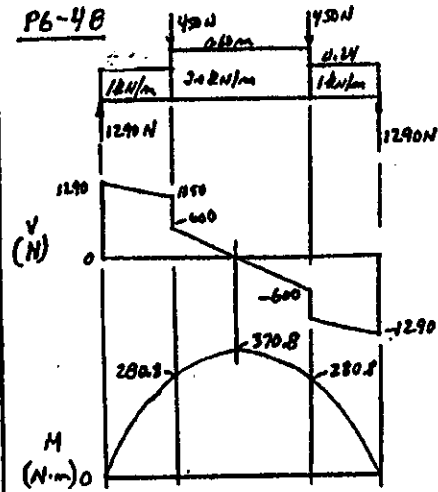
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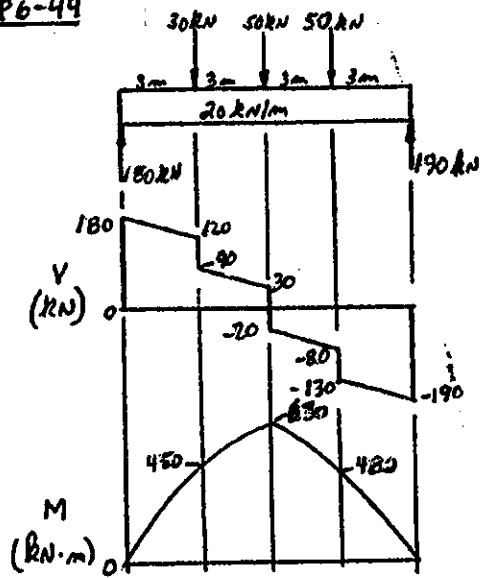
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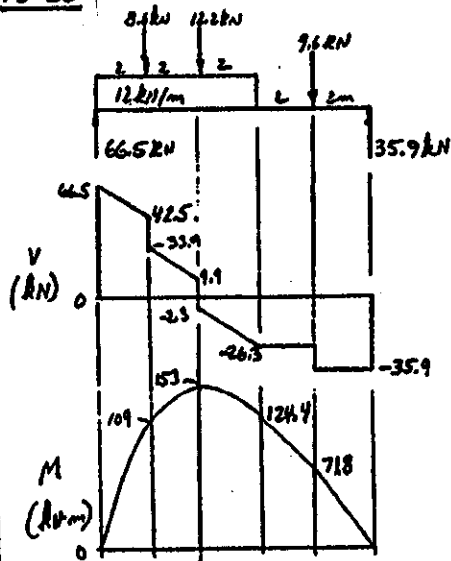
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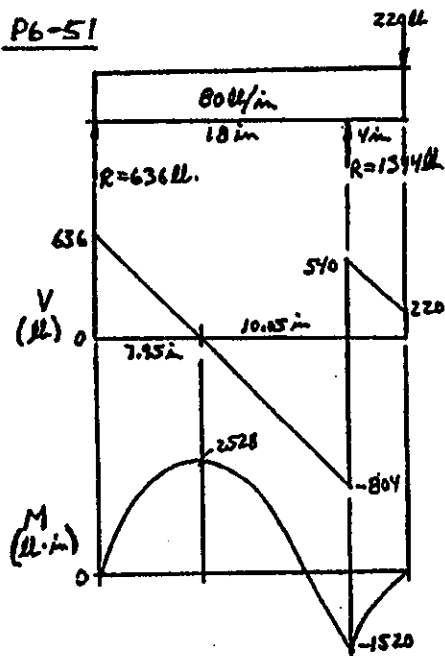
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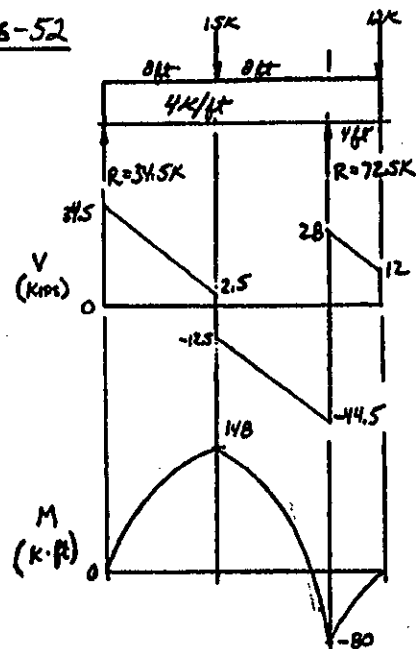
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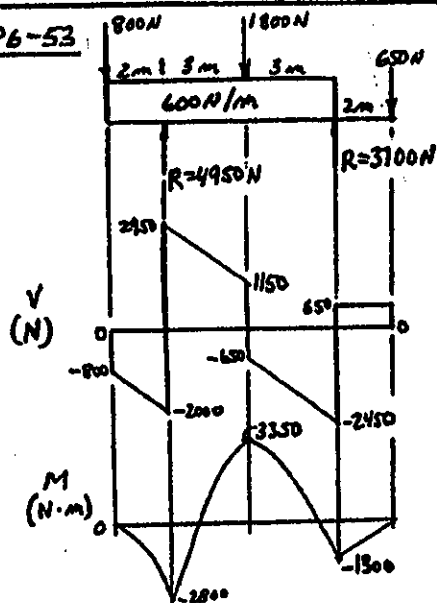
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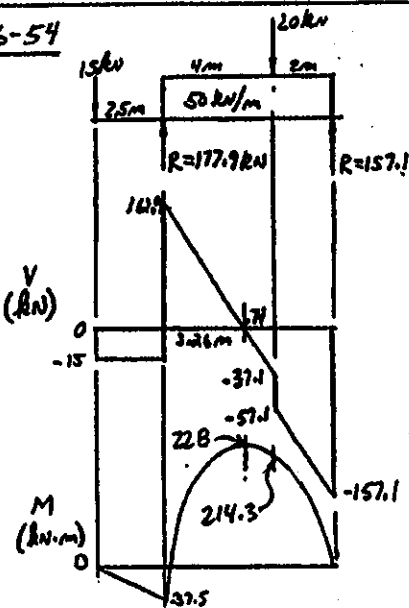
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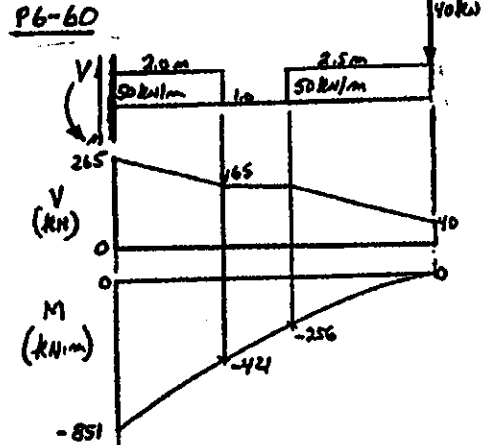
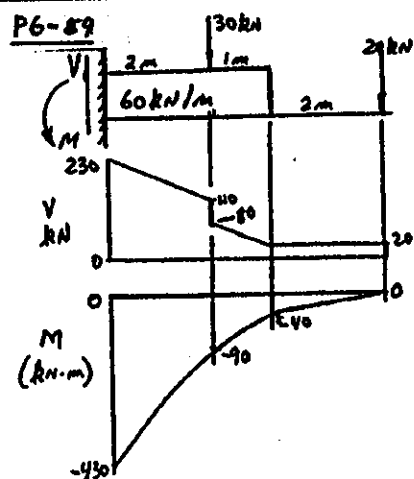
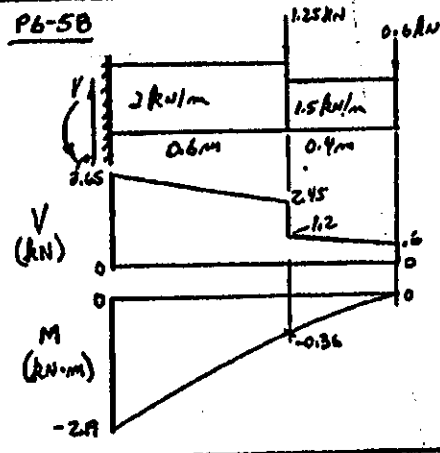
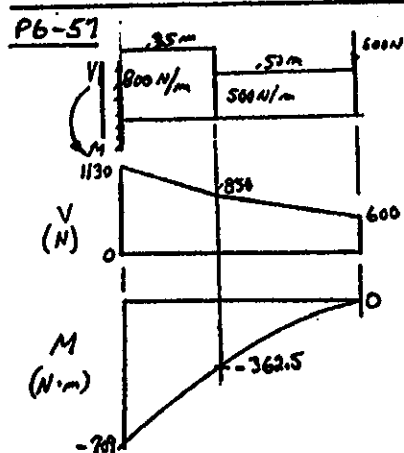
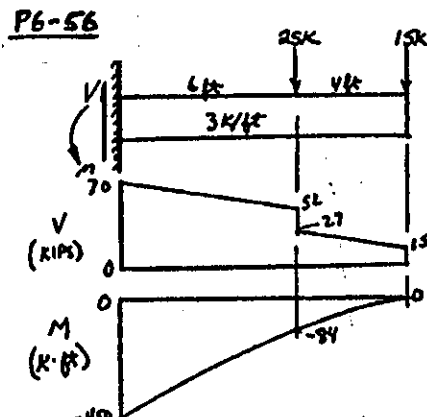
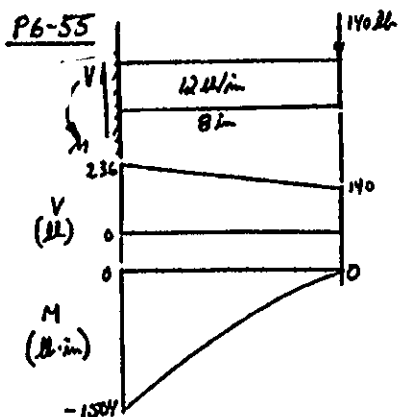


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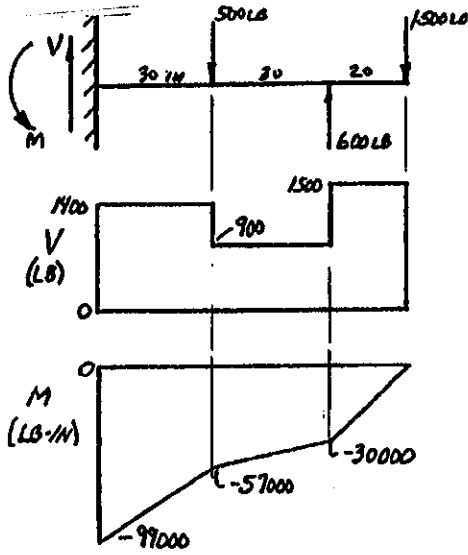
P6-54



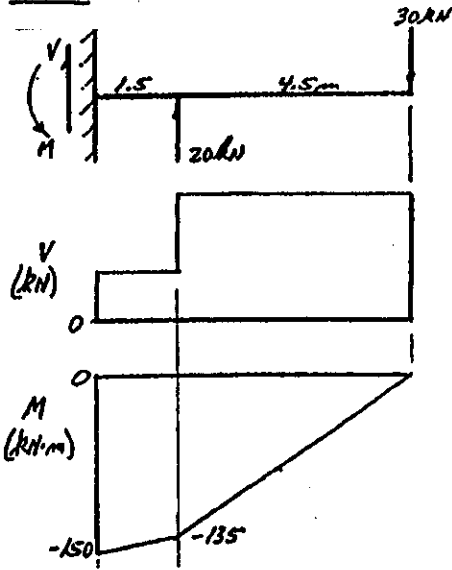




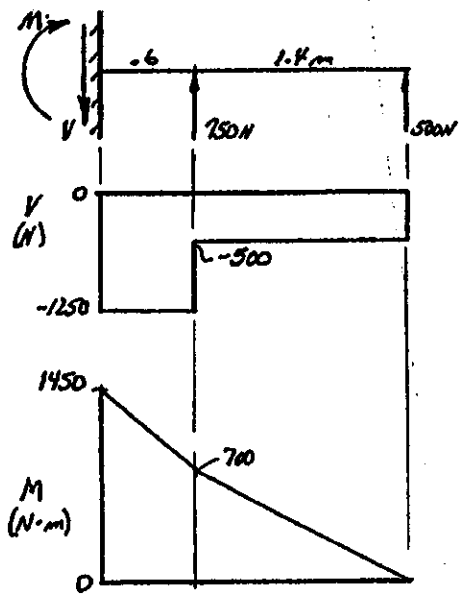
P6-61



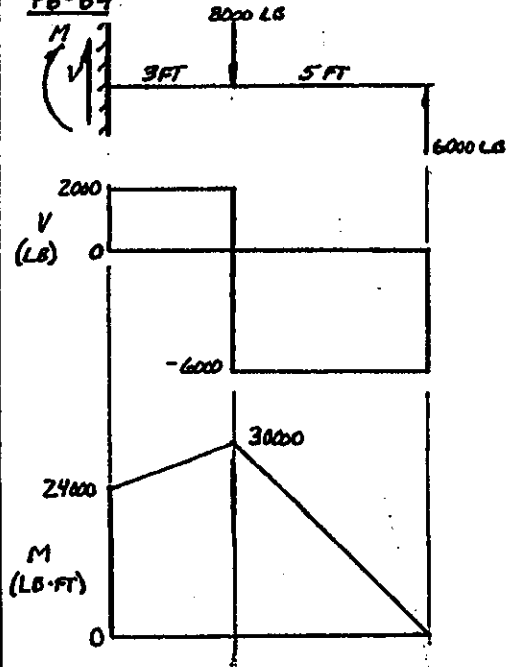
P6-62



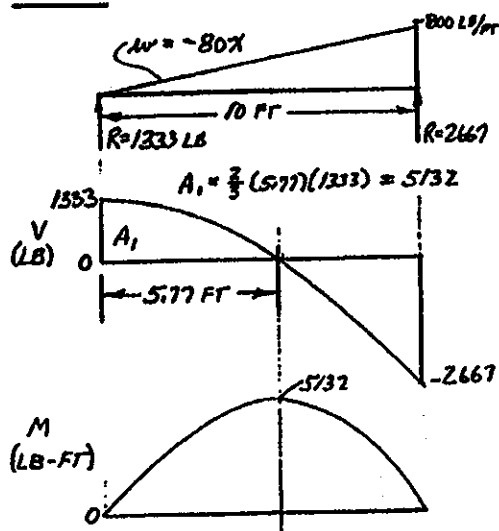
P6-63



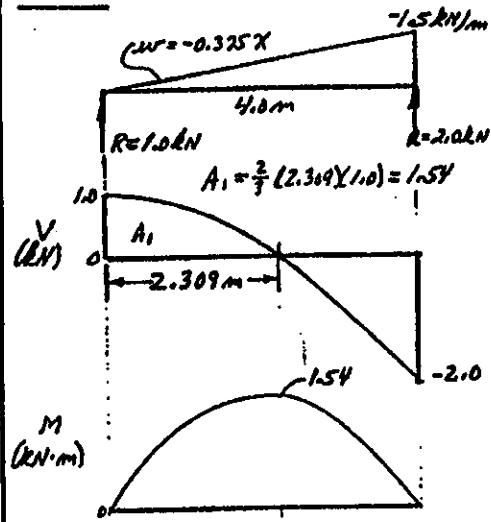
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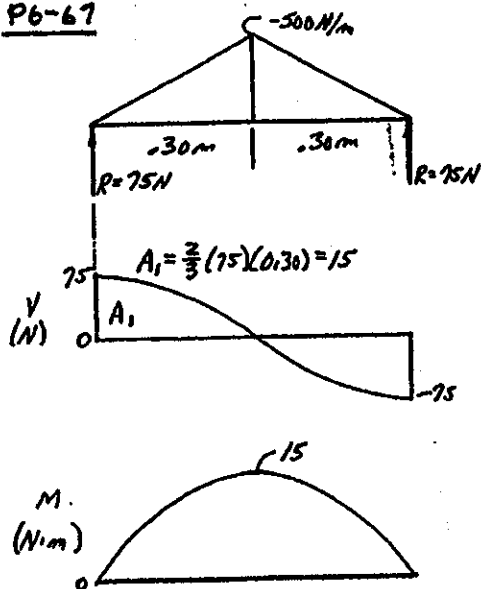
P6-65



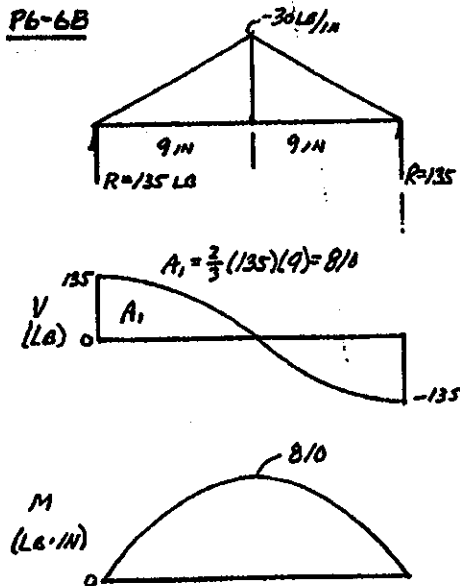
P6-66



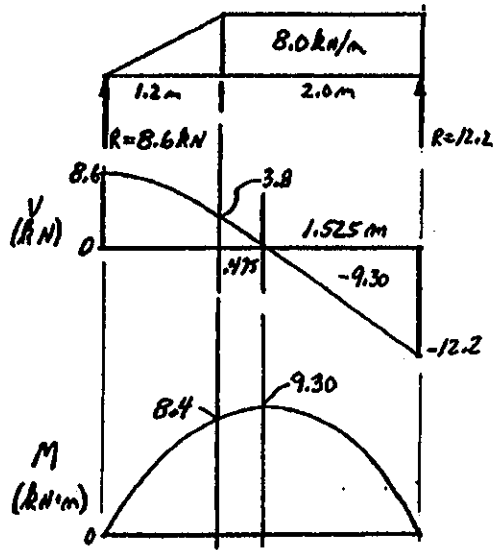
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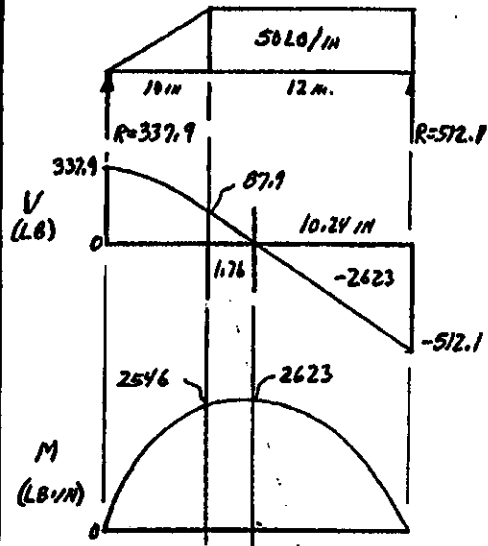
P6-68



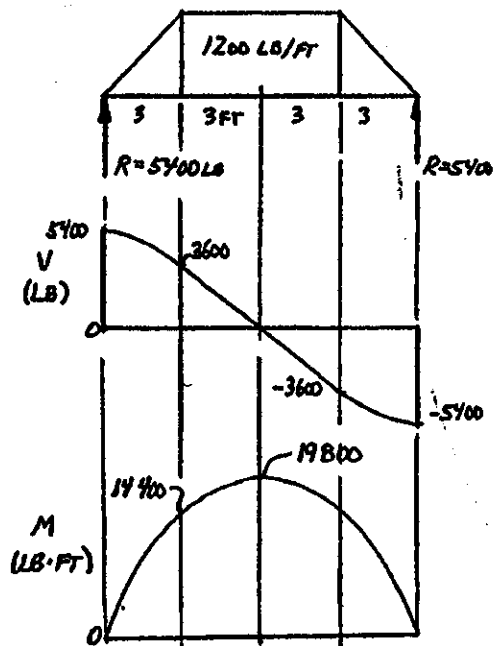
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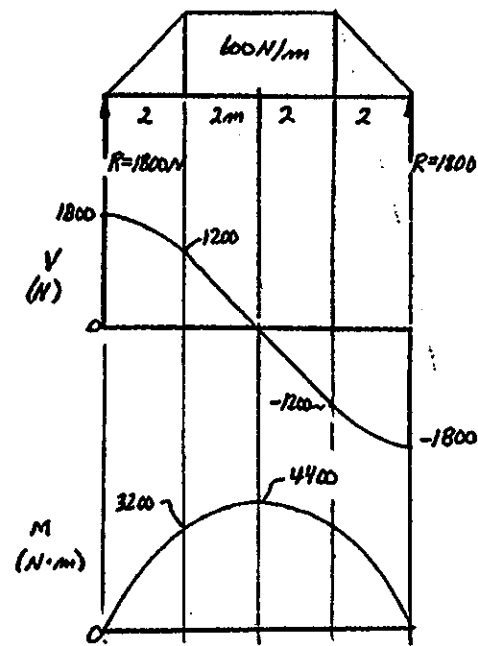
P6-70



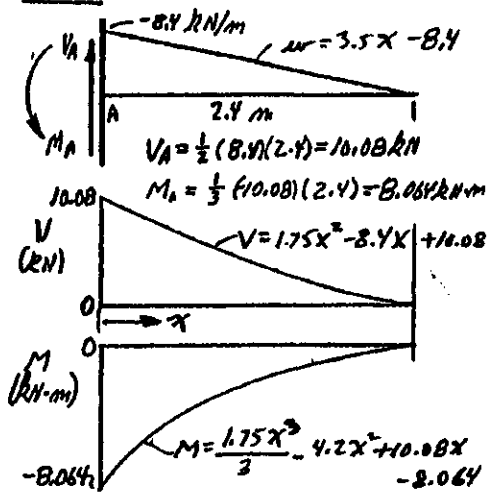
P6-71



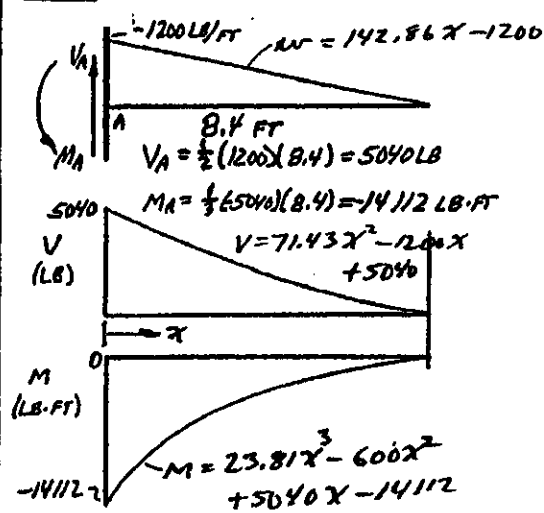
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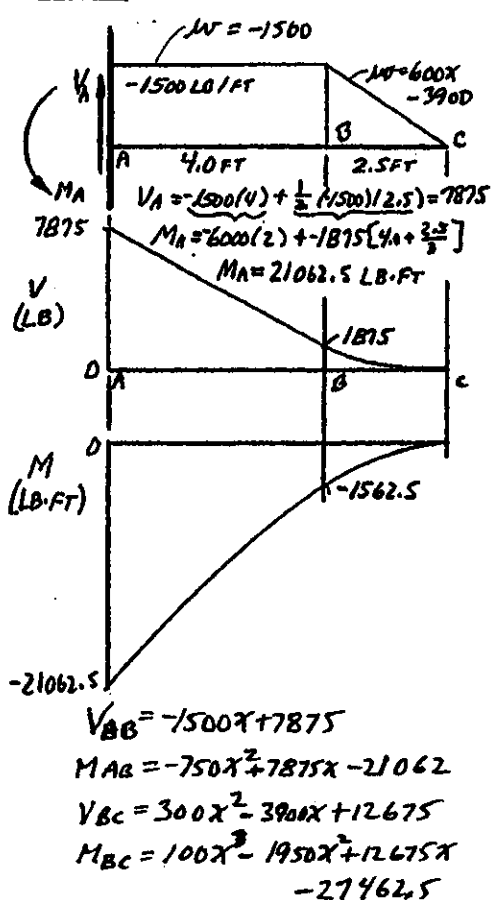
P6-73



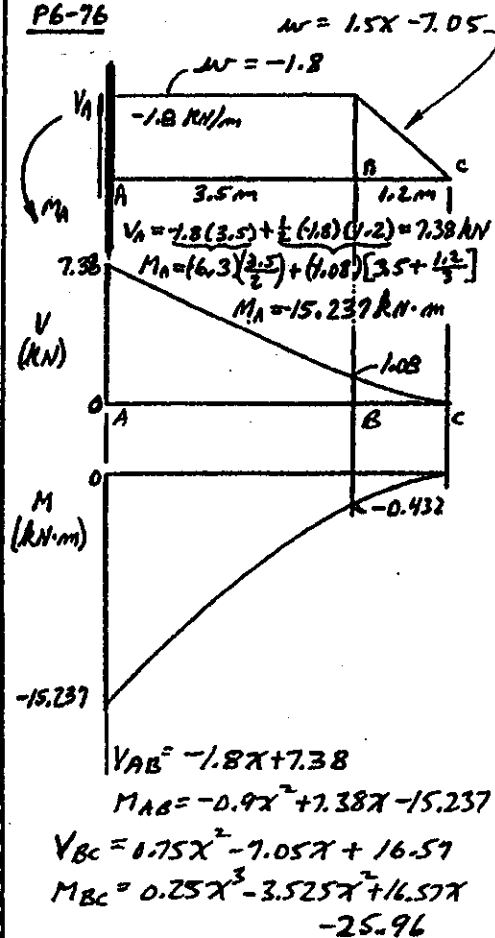
P6-74



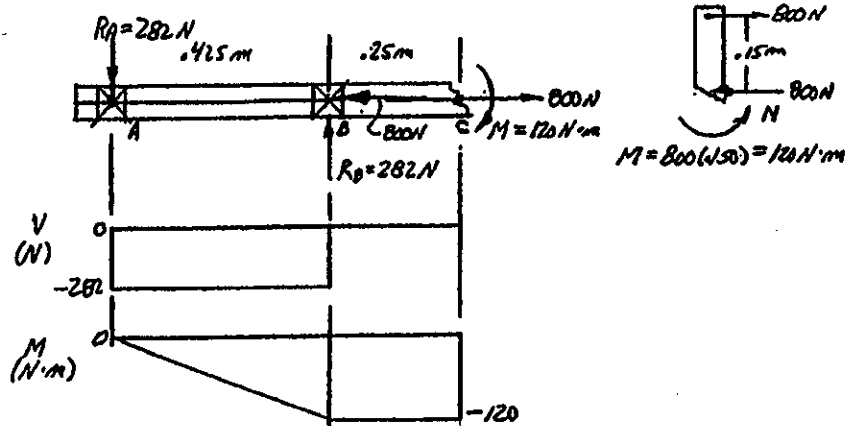
P6-75



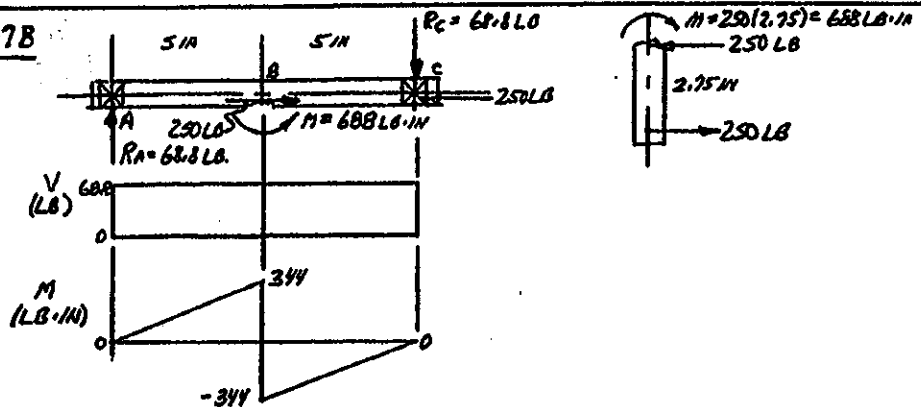
P6-76



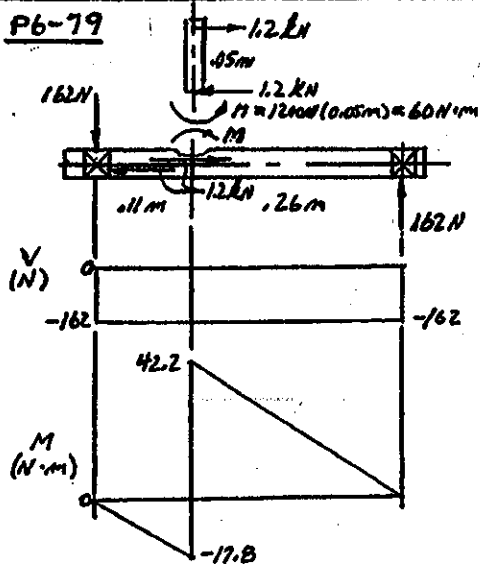
P6-77



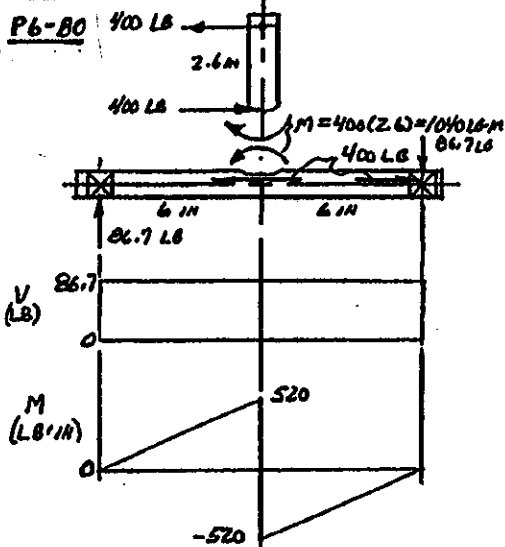
P6-78



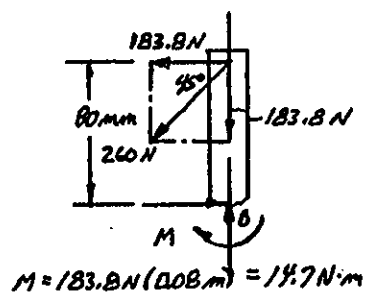
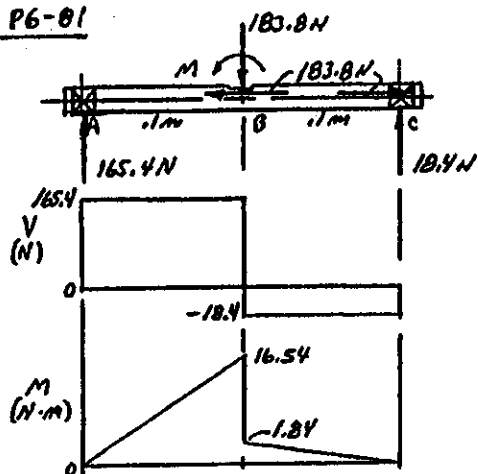
P6-79



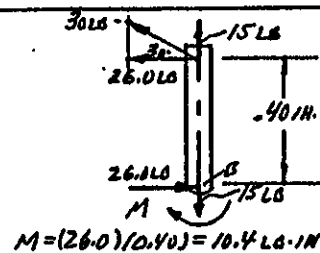
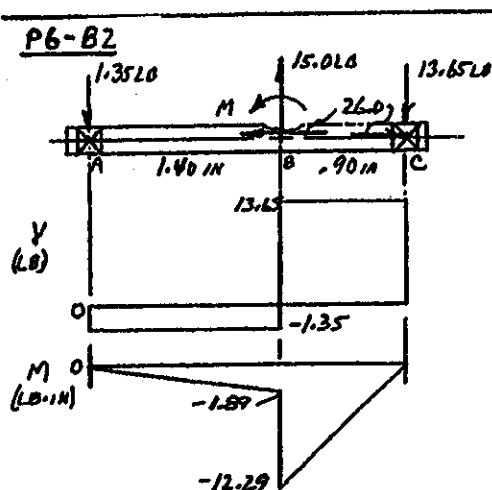
P6-80



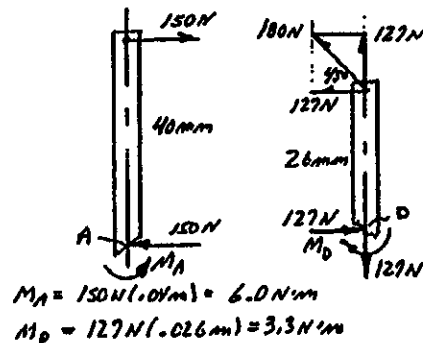
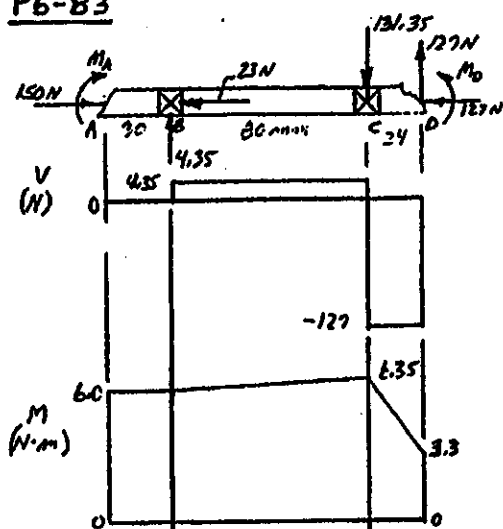
P6-81



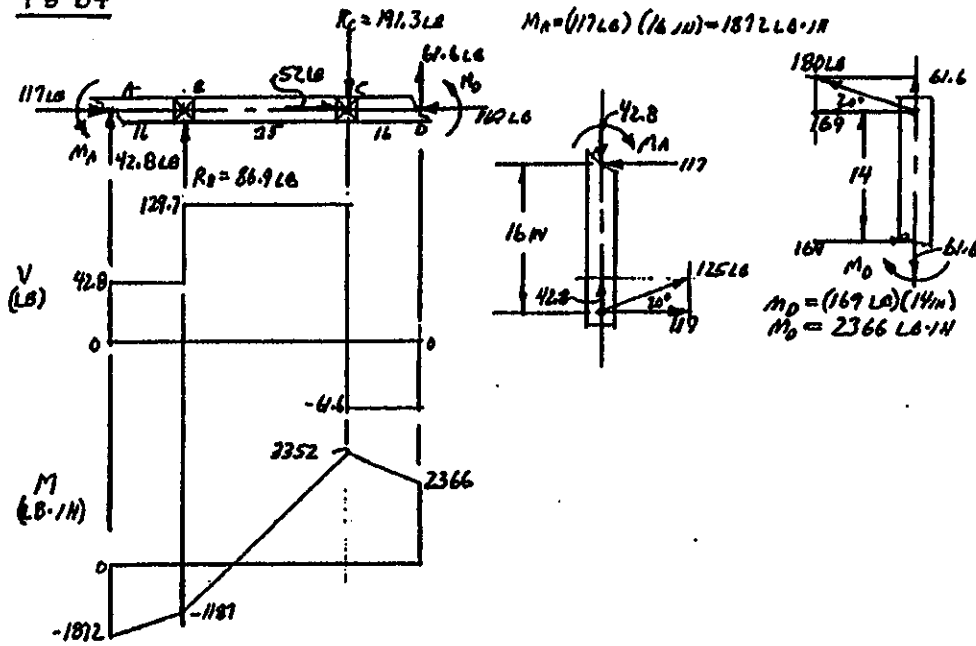
P6-82



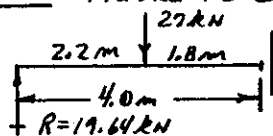
P6-83



P6-B4



P6-85 FIGURE P6-8.

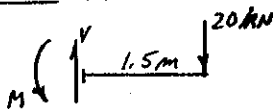


$$V = 19.64 - 27.0 = -7.36 \text{ kN}$$

$$M = (19.64 \text{ kN})(4.0 \text{ m}) - (27.0 \text{ kN})(1.8 \text{ m})$$

$$M = 27.96 \text{ kN}\cdot\text{m}$$

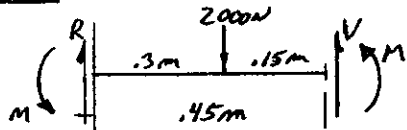
P6-86 FIGURE P6-15. USE PART OF BEAM TO RIGHT OF CUT SECTION.



$$V = 20 \text{ kN}$$

$$M = (-20 \text{ kN})(1.50 \text{ m}) = -30 \text{ kN}\cdot\text{m}$$

P6-87 FIGURE P6-22.  $R = 3000 \text{ N}$ ;  $M = -1220 \text{ N}\cdot\text{m}$

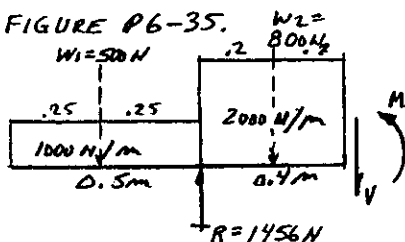


$$V = 3000 \text{ N} - 2000 \text{ N} = 1000 \text{ N}$$

$$M = -1220 \text{ N}\cdot\text{m} + 3000 \text{ N}(0.45 \text{ m}) - 2000 \text{ N}(0.15 \text{ m})$$

$$M = -170 \text{ N}\cdot\text{m}$$

P6-88 FIGURE P6-35.

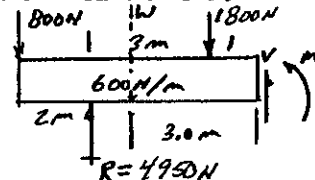


$$V = 1456 \text{ N} - 500 \text{ N} - 800 \text{ N} = 56 \text{ N}$$

$$M = (1456 \text{ N})(0.4 \text{ m}) - (500 \text{ N})(0.65 \text{ m}) - (800 \text{ N})(0.2 \text{ m})$$

$$M = 97.4 \text{ N}\cdot\text{m}$$

P6-89 FIGURE P6-53.



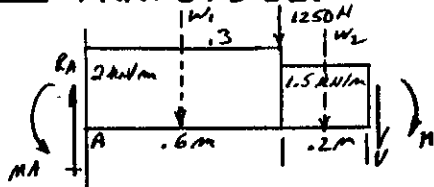
$$W = (600 \text{ N/m})(6.0 \text{ m}) = 3600 \text{ N}$$

$$V = 4950 - 800 - 3600 - 1800 = -1250 \text{ N}$$

$$M = (4950)(4) - 800(6) - 1800(1) - 3600(3)$$

$$M = 2400 \text{ N}\cdot\text{m}$$

P6-90 FIGURE P6-58.



$$W_1 = 2 \text{ kN/m}(0.6 \text{ m}) = 1.2 \text{ kN}$$

$$W_2 = 1.5 \text{ kN/m}(0.2 \text{ m}) = 0.30 \text{ kN}$$

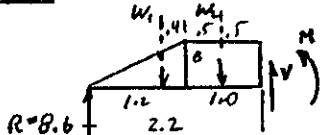
$$R_A = 3.65 \text{ kN}, M_A = -2.19 \text{ kN}\cdot\text{m}$$

$$V = 3.65 - 1.2 - 1.25 - 0.30 = 0.90 \text{ kN}$$

$$M = -2.19 + 3.65(0.8) - 1.2(0.5) - 0.3(0.1) - 1.25(0.2)$$

$$M = -0.15 \text{ kN}\cdot\text{m}$$

P6-91 FIGURE P6-69.

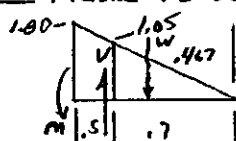


$$W_1 = (6.5)(8 \text{ kN/m})(1.2 \text{ m}) = 4.8 \text{ kN}; W_2 = 8(1) = 8 \text{ kN}$$

$$V = 8.6 - 4.8 - 8.0 = -4.2 \text{ kN}$$

$$M = 8.6(2.2) - 4.8(1.4) - 8.0(0.5) = 0.2 \text{ kN}\cdot\text{m}$$

P6-92 FIGURE P6-76. USE RIGHT PART FOR FBD.



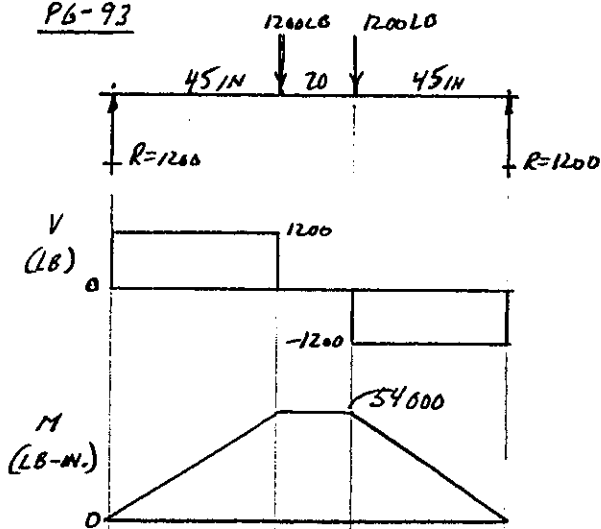
$$W = (0.5)(1.05 \text{ kN/m})(0.7 \text{ m}) = 0.377 \text{ kN}$$

$$\text{AT CUT: } V = W = 0.377 \text{ kN}$$

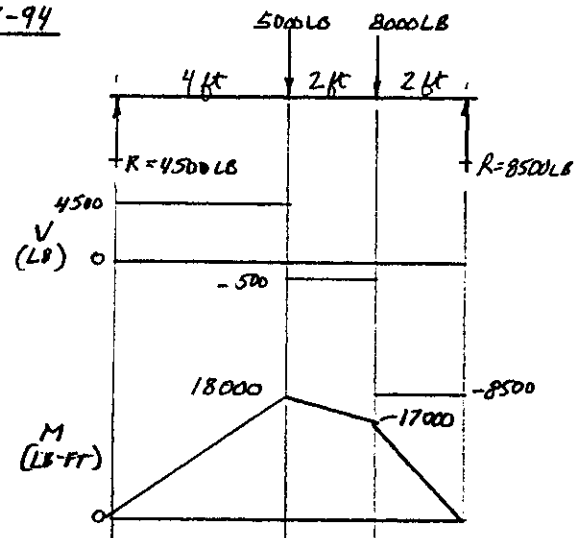
$$M = -W(0.7 - 0.467) = (0.377 \text{ kN})(0.233) = 0.0858 \text{ kN}\cdot\text{m}$$



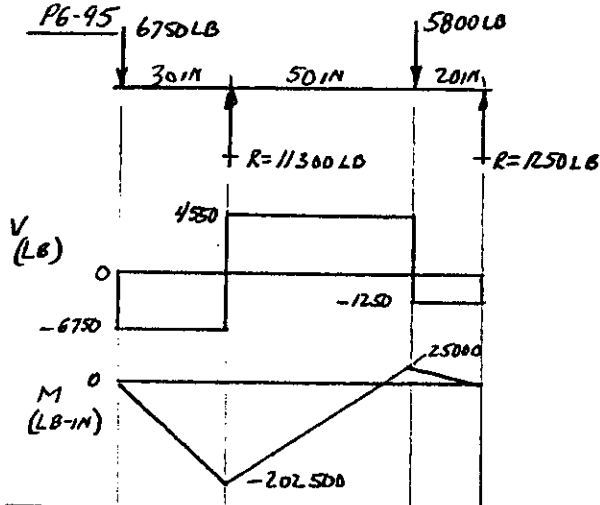
P6-93



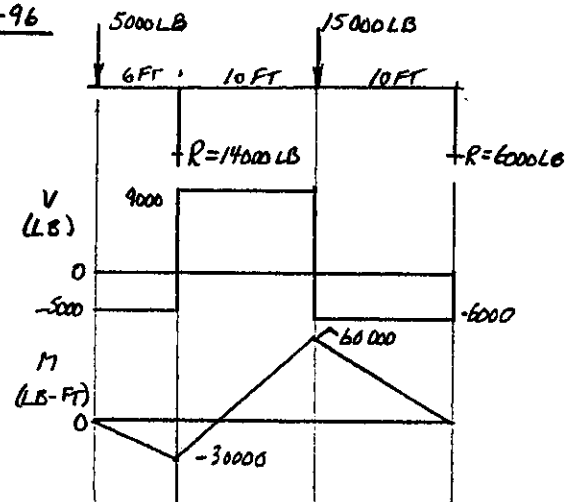
P6-94



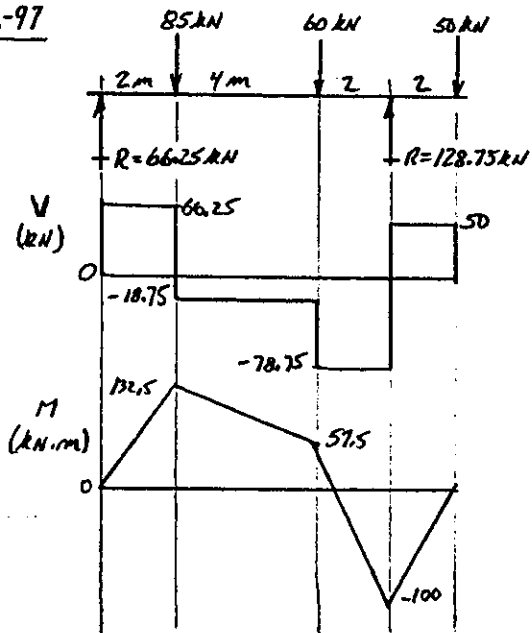
P6-95



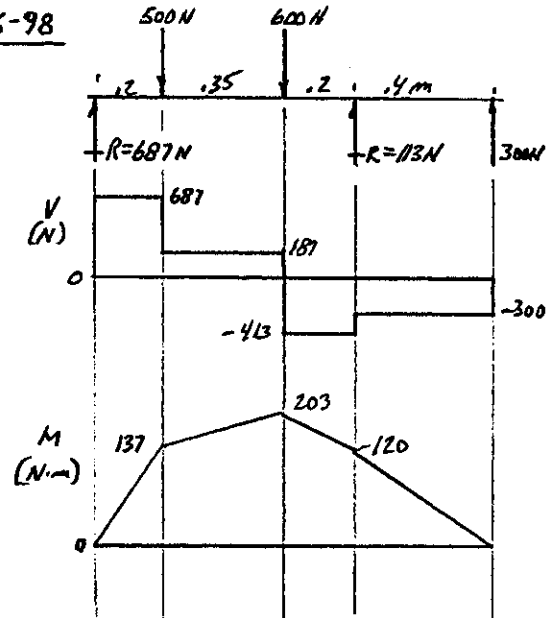
P6-96



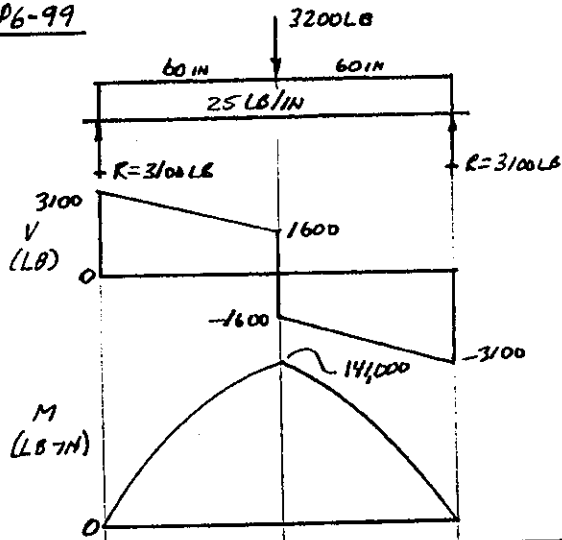
P6-97



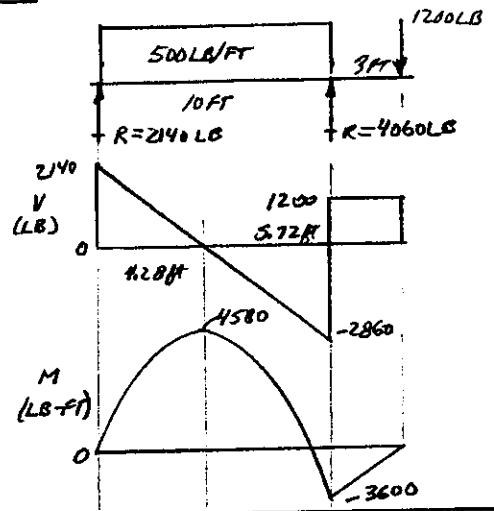
P6-98



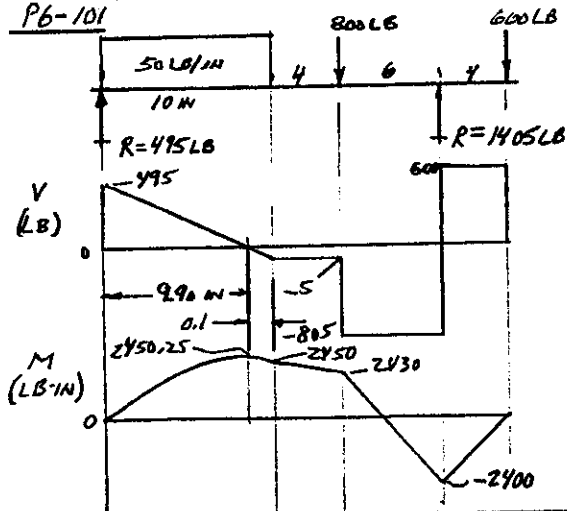
P6-99



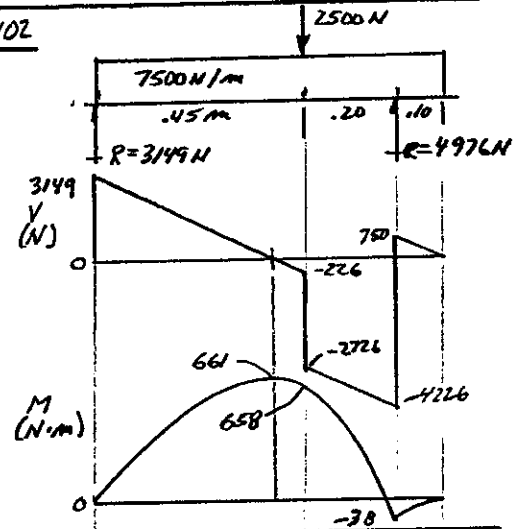
P6-100



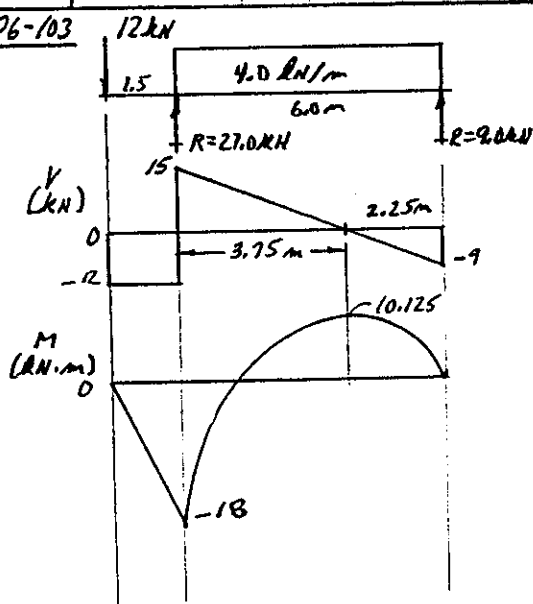
P6-101



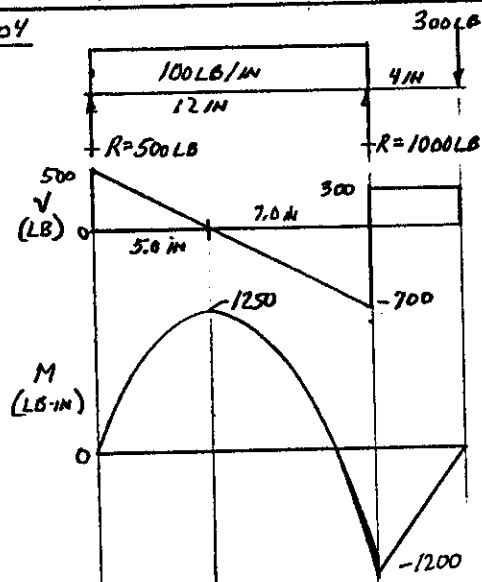
P6-102



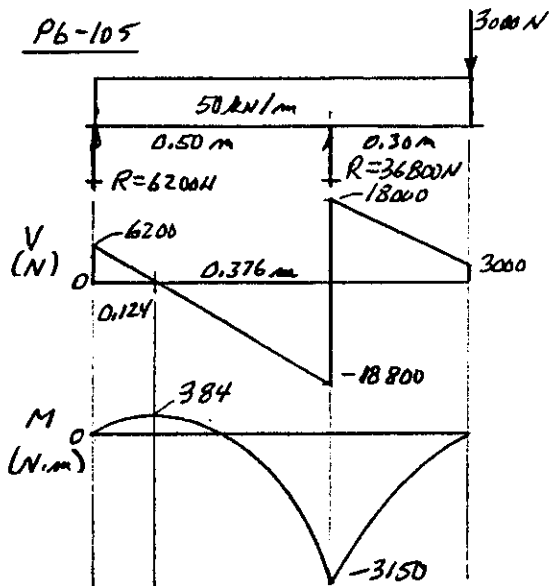
P6-103



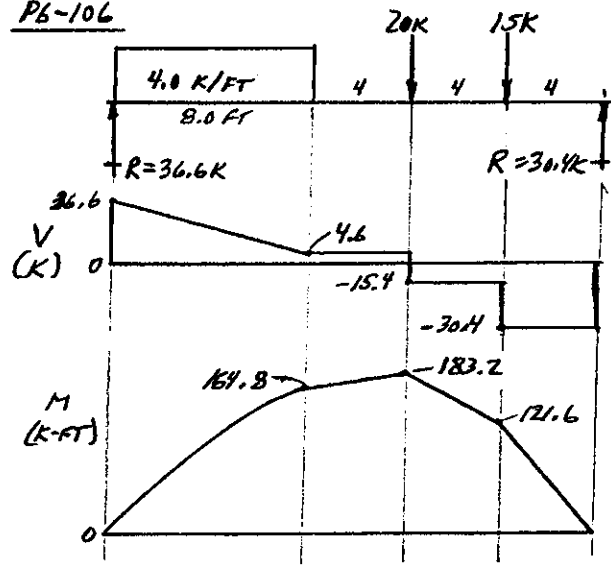
P6-104



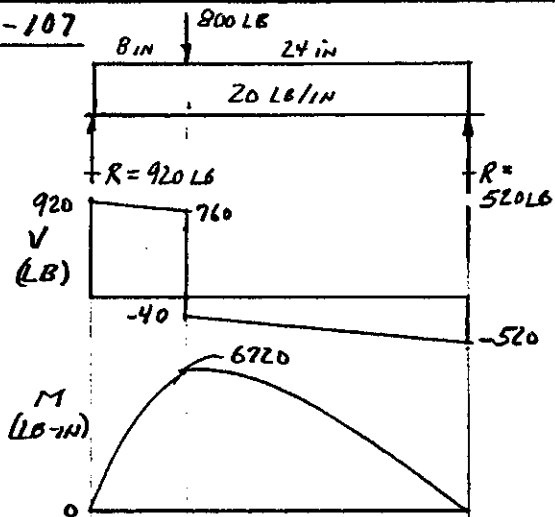
P6-105



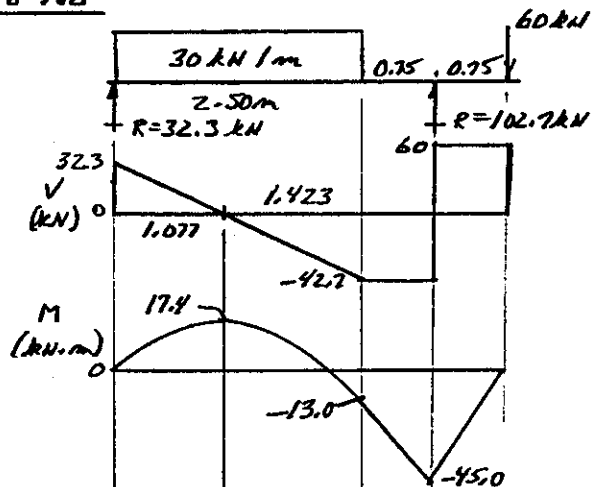
P6-106



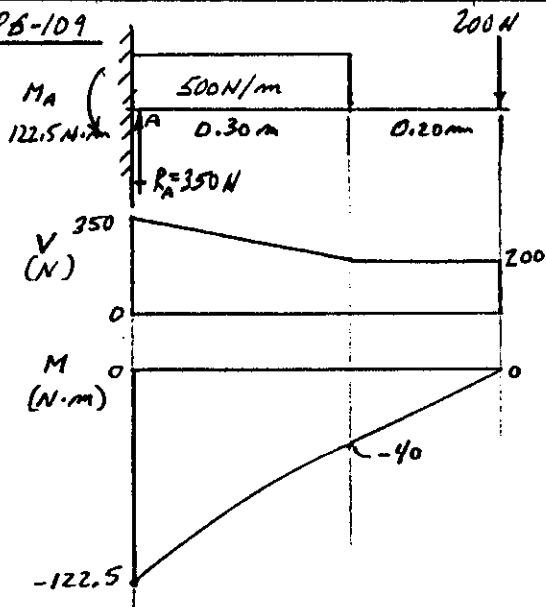
P6-107



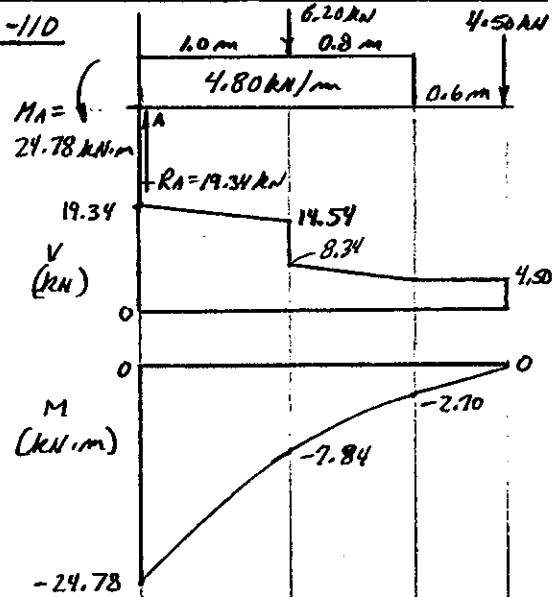
P6-108



P6-109



P6-110



## **CHAPTER 7      Centroids and Moments of Inertia of Areas**

**Notes concerning the format of solutions for Chapter 7 problems:**

- Problem solutions for the moments of inertia of the shapes shown in Figures P7-1 through P7-46 are shown in the tabular format recommended in Section 7-6.
- Calculations were completed using a spreadsheet.
- The requested result includes the vertical  $Y$  distance to the centroidal axis from the reference axis and the moment of inertia  $I$  of the composite shape relative to the horizontal centroidal axis.
- In most problems, the reference axis for computing the location of the horizontal centroidal axis was taken as the base of the section. Exceptions are noted on the top or bottom lines of the solution. For example, in Figure P7-17, the reference axis is at the axis of symmetry at the mid-height of the shape, found by inspection.
- The left-most column of the solution gives a brief description of the part of the composite shape being analyzed.
- For some shapes, internal parts removed from the outer shape are shown to be negative.
- For composite shapes having parts that are commercially available structural shapes, pipes, or tubes, or wood beams, reference should be made to the Appendix tables for pertinent data.

FIGURE P7-1		Units: Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Vertical	0.5000	1.0000	0.5000	0.16670	0.3365	0.0566	0.2233
2-Horizontal	0.3125	0.1250	0.0391	0.00163	0.5384	0.0906	0.0922
Total area =	0.8125	Sum Ay=	0.5391			Total I =	0.3156 in <sup>4</sup>
Y=	0.6635	in					

FIGURE P7-2		Units: Inches	NOTE: 6x8 rectangle with 5x6 rectangle removed				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Total 6x8	48.00	4.00	192.00	256.00	0.00	0.00	256.00
2-Void 5x6	-30.00	4.00	-120.00	-90.00	0.00	0.00	-90.00
Total area =	18.00	Sum Ay=	72.00			Total I =	166.00 in <sup>4</sup>
Y=	4.00	in					

NOTE: Reference axis is base of the shape

FIGURE P7-3		Units: Inches	NOTE: 6x8 rectangle with 4x6 rectangle removed				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Total 6x8	48.00	4.00	192.00	256.00	0.00	0.00	256.00
2-Void 4x6	-24.00	4.00	-96.00	-72.00	0.00	0.00	-72.00
Total area =	24.00	Sum Ay=	96.00			Total I =	184.00 in <sup>4</sup>
Y=	4.00	in					

NOTE: Reference axis is base of the shape

FIGURE P7-4		Units: mm	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Vertical	5000	100	5.000E+05	1.667E+07	52.50	1.38E+07	3.04E+07
2-Horizontal	4375	213	9.297E+05	2.279E+05	60.00	1.58E+07	1.60E+07
Total area =	9375	Sum Ay=	1.430E+06			Total I =	4.64E+07 mm <sup>4</sup>
Y=	152.50	mm					

FIGURE P7-5		Units: mm	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Vertical	250	30.0	7.500E+03	5.208E+04	5.00	6.25E+03	5.83E+04
2-Horiz-bot	100	2.5	2.500E+02	2.080E+02	32.50	1.06E+05	1.06E+05
3-Horiz-top	200	57.5	1.150E+04	4.170E+02	22.50	1.01E+05	1.02E+05
Total area =	550	Sum Ay=	1.925E+04			Total I =	2.66E+05 mm <sup>4</sup>
Y=	35.00	mm					

FIGURE P7-6	Units: mm	NOTE: Both vertical rectangles (10x30) combined					
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Ver-20X30	600	15	9.000E+03	4.500E+04	2.50	3.75E+03	4.88E+04
2-Hor-20X10	200	5	1.000E+03	1.667E+03	7.50	1.12E+04	1.29E+04
Total area =	800	Sum Ay=	1.000E+04			Total I =	6.17E+04 mm <sup>4</sup>
Y=	12.50 mm	NOTE: Reference axis is base of the shape					

FIGURE P7-7	Units: mm	NOTE: Entire vertical stem; 2 horiz. flanges each 5x15					
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Ver-5X40	200	20.0	4.000E+03	2.667E+04	0.00	0E+00	2.67E+04
2-Horiz-bot	75	2.5	1.875E+02	1.562E+02	17.50	2.30E+04	2.31E+04
3-Horiz-top	75	37.5	2.812E+03	1.562E+02	17.50	2.30E+04	2.31E+04
Total area =	350	Sum Ay=	7.000E+03			Total I =	7.29E+04 mm <sup>4</sup>
Y=	20.00 mm	NOTE: Reference axis is base of the shape					

FIGURE P7-8	Units: mm	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Ver-5X40	200	20.0	4.000E+03	2.667E+04	0.00	0E+00	2.67E+04
2-Ver-5x40	200	20.0	4.000E+03	2.667E+04	0.00	0E+00	2.67E+04
3-Hor-30x5	150	20.0	3.000E+03	3.125E+02	0.00	0E+00	3.12E+02
Total area =	550	Sum Ay=	1.100E+04			Total I =	5.36E+04 mm <sup>4</sup>
Y=	20.00 mm						

FIGURE P7-9	Units: mm	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Ver-5X30	150	20.0	3.000E+03	1.125E+04	0.00	0E+00	1.12E+04
2-Horiz-bot	200	2.5	5.000E+02	4.167E+02	17.50	6.12E+04	6.17E+04
3-Horiz-top	200	37.5	7.500E+03	4.167E+02	17.50	6.12E+04	6.17E+04
Total area =	550	Sum Ay=	1.100E+04			Total I =	1.35E+05 mm <sup>4</sup>
Y=	20.00 mm						

FIGURE P7-10	Units: mm	NOTE: Reference axis is base of the shape					
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Ver-5X50	250	30.0	7.500E+03	5.208E+04	0.00	0E+00	5.21E+04
2-Horiz-bot	140	2.5	3.500E+02	2.917E+02	27.50	1.06E+05	1.06E+05
3-Horiz-top	140	57.5	8.050E+03	2.917E+02	27.50	1.06E+05	1.06E+05
Total area =	530	Sum Ay=	1.590E+04			Total I =	2.64E+05 mm <sup>4</sup>
Y=	30.00 mm						

FIGURE P7-11      Units: mm      NOTE: Both vert. combined 10x45; horiz. flanges combined 5x30

Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Ver-10X45	450	22.5	1.012E+04	7.594E+04	0.69	2.14E+02	7.62E+04
2-Hor-5x30	150	2.5	3.750E+02	3.125E+02	19.31	5.59E+04	5.62E+04
3-Hor-5x25	125	42.5	5.312E+03	2.604E+02	20.69	5.35E+04	5.38E+04
Total area =	725	Sum Ay=	1.581E+04			Total I =	1.86E+05 mm <sup>4</sup>
Y=	21.81 mm						NOTE: Reference axis is base of the shape

FIGURE P7-12      Units: mm      NOTE: All verticals massed together

Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Ver-16X16	256	8	2.048E+03	5.461E+03	4.39	4.92E+03	1.04E+04
2-Hor-4X50	200	18	3.600E+03	2.667E+02	5.61	6.30E+03	6.57E+03
Total area =	456	Sum Ay=	5.648E+03			Total I =	1.70E+04 mm <sup>4</sup>
Y=	12.39 mm						NOTE: Reference axis is base of the shape

FIGURE P7-13      Units: mm      NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Hor-5x10	50	2.5	1.250E+02	1.045E+02	20.83	2.17E+04	2.18E+04
2-Ver-5X55	275	27.5	7.562E+03	6.932E+04	4.17	4.77E+03	7.41E+04
3-Hor-5x20	100	27.5	2.750E+03	2.083E+02	4.17	1.74E+03	1.94E+03
4-Ver-5x30	150	15.0	2.250E+03	1.125E+04	8.33	1.04E+04	2.17E+04
5-Hor-5x5	25	52.5	1.312E+03	5.208E+01	29.17	2.13E+04	2.13E+04
Total area =	600	Sum Ay=	1.400E+04			Total I =	1.41E+05 mm <sup>4</sup>
Y=	23.33 mm						

FIGURE P7-14      Units: Inches      NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Bot plate	0.5200	0.1000	0.0520	0.00173	0.4330	0.0975	0.0992
2-Bot flanges	0.1200	0.2500	0.0300	0.0001	0.2830	0.0096	0.0097
3-2 Vert webs	0.3000	0.9500	0.2850	0.05625	0.4169	0.0522	0.1084
4-Horiz-top	0.12	1.65	0.1980	0.0001	1.1170	0.1497	0.1498
Total area =	1.06	Sum Ay=	0.5650			Total I =	0.3672 in <sup>4</sup>
Y=	0.5330 in						

FIGURE P7-15      Units: Inches      NOTE: Reference axis is base of the shape

Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Bot flange	0.1000	0.0500	0.0050	0.00008	1.0176	0.1036	0.1036
2-2 Verticals	0.4800	1.2000	0.5760	0.23040	0.1323	0.0084	0.2388
3-Mid-Horiz.	0.1	1.4500	0.1450	0.0000833	0.3823	0.0146	0.0147
Total area =	0.68	Sum Ay=	0.7260			Total I =	0.3572 in <sup>4</sup>
Y=	1.0676 in						

FIGURE P7-16		Units: Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Rectangle	1.1250	0.7500	0.8438	0.21094	0.1491	0.0250	0.2360
2-Semicircle	0.2209	1.6590	0.3665	0.0022148	0.7598	0.1275	0.1297
Total area =	1.34589	Sum Ay=	1.2102			Total I =	0.3657 in <sup>4</sup>
Y=	0.8992	in					

FIGURE P7-17		Units: mm	NOTE: Reference axis taken at y=125				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Rectangle	11400	0.0	0E+00	3.430E+07	0.00	0E+00	3.43E+07
2-Semic-bot	1414	-107.7	-1.52E+05	9.072E+04	107.72	1.64E+07	1.65E+07
3-Semic-top	1414	107.7	1.52E+05	9.072E+04	107.72	1.64E+07	1.65E+07
Total area =	14227	Sum Ay=	0E+00			Total I =	6.73E+07 mm <sup>4</sup>
Y=	0.00	mm					

FIGURE P7-18		Units: mm	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Rectangle	1200	20.0	2.400E+04	1.60E+05	7.27	6.35E+04	2.23E+05
2-Rect rem.	-400	20.0	-8.00E+03	-1.33E+04	7.27	-2.1E+04	-3.4E+04
3-Triangle	300	46.7	1.40E+04	6.67E+03	19.39	1.13E+05	1.20E+05
Total area =	1100	Sum Ay=	3.000E+04			Total I =	3.08E+05 mm <sup>4</sup>
Y=	27.27	mm					

FIGURE P7-19		Units: Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Hor-.5x1.4	0.700	0.250	0.1750	0.0146	0.681	0.3242	0.3388
2-Ver-.6x2.5	1.500	1.250	1.8750	0.7813	0.319	0.1531	0.9343
3-2 Tr-.7x1.5	0.910	0.933	0.8493	0.0854	0.003	0.0000	0.0854
4-Tri-rem	-0.460	0.807	-0.3711	-0.0216	0.124	-0.0071	-0.0287
5-Hole-rem	-0.049	2.200	-0.1080	-0.0002	1.269	-0.0791	-0.0793
Total area =	3	Sum Ay=	2.4202			Total I =	1.2506 in <sup>4</sup>
Y=	0.9305	in					

FIGURE P7-20		Units: Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-2 Vert rect	1.2000	1.0000	1.2000	0.4000	0.2798	0.0940	0.4940
2-2 Triangles	0.5100	1.1333	0.5780	0.0819	0.1465	0.0109	0.0928
3-Top-.3x2.4	0.7200	1.8500	1.3320	0.0054	0.5701	0.2341	0.2395
Total area =	2.4300	Sum Ay=	3.1100			Total I =	0.8263 in <sup>4</sup>
Y=	1.2798	in					



FIGURE P7-21		Units:	Inches	NOTE: Reference axis is base of the shape			
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Vert rect	8.2500	4.2500	35.0625	20.7969	0	0.0000	20.7969
2-Bot flange	5.2500	0.7500	3.9375	0.9844	3.5	64.3125	65.2969
3-Top flange	5.2500	7.7500	40.6875	0.9844	3.5	64.3125	65.2969
Total area =	18.7500	Sum Ay=	79.6875			Total I =	151.3906 in <sup>4</sup>
Y=	4.2500	in					

FIGURE P7-22		Units:	Inches	NOTE: 7.25x7 rectangle with 4.25x5.5 rectangle removed			
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Tot 7.25x7	50.75	3.50	177.63	207.23	0.00	0.00	207.23
2-4.25x5.5 re	-23.38	3.50	-81.81	-58.92	0.00	0.00	-58.92
Total area =	27.38	Sum Ay=	95.81			Total I =	148.30 in <sup>4</sup>
Y=	3.50	in					

NOTE: Reference axis is base of the shape

FIGURE P7-23		Units:	Inches	NOTE: 24x4.5 rectangle with 21x3.5 rectangle removed			
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Tot 24x4.5	108.00	2.25	243.00	182.25	0.00	0.00	182.25
2-21x3.5 rem	-73.50	2.25	-165.38	-75.03	0.00	0.00	-75.03
Total area =	34.50	Sum Ay=	77.63			Total I =	107.22 in <sup>4</sup>
Y=	2.25	in					

NOTE: Reference axis is base of the shape

FIGURE P7-24		Units:	Inches	NOTE: Reference axis is base of the shape			
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-2 Verticals	33.75	5.63	189.84	355.96	2.13	152.40	508.36
2-Top flange	16.88	12.00	202.50	3.16	4.25	304.80	307.97
Total area =	50.63	Sum Ay=	392.34			Total I =	816.33 in <sup>4</sup>
Y=	7.75	in					

FIGURE P7-25		Units:	Inches	NOTE: Depth of W14x43 is 13.66; Ref. axis at centroid; Y=7.33			
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-W14x43	12.60	0.00	0.00	428.00	0.00	0.00	428.00
2-Bot plate	7.25	-7.08	-51.33	0.15	7.08	363.42	363.57
3-Top plate	7.25	7.08	51.33	0.15104	7.08	363.42	363.57
Total area =	19.85	Sum Ay=	0.00			Total I =	1155.13 in <sup>4</sup>
Y=	0.00	in					

FIGURE P7-26	Units:	Inches	NOTE: Web for C is .387 thick; Y-Y axis down .674 from top				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-S12x50	14.70	6.00	88.20	305.00	1.90	53.31	358.31
2-C12x25	7.35	11.71	86.09	4.47	3.81	106.62	111.09
Total area =	22.05	Sum Ay=	174.29			Total I =	469.40 in <sup>4</sup>
Y=	7.90 in						
			NOTE: For C12x25; y = 12.0+.387-.674 = 11.713				
			NOTE: Reference axis is base of the shape				

FIGURE P7-27	Units:	Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-I12x14.292	12.15	6.00	72.92	317.33	1.40	23.73	341.06
2-Top plate	3.50	12.25	42.88	0.07	4.85	82.41	82.49
Total area =	15.65	Sum Ay=	115.79			Total I =	423.55 in <sup>4</sup>
Y=	7.40 in						

FIGURE P7-28	Units:	Inches	NOTE: Depth of C12 is 12.0; Ref. axis at centroid; y=6.50 from bot				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Two C12	14.07	0.00	0.00	319.520	0.00	0.00	319.52
2-Bot plate	5.00	-6.25	-31.25	0.104	6.25	195.31	195.42
3-Top plate	5.00	6.25	31.25	0.104	6.25	195.31	195.42
Total area =	19.07	Sum Ay=	0.00			Total I =	710.35 in <sup>4</sup>
Y=	0.00 in						

FIGURE P7-29	Units:	Inches	NOTE: Overall depth=7.0; Ref axis=centroid, 3.50 in from bottom				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Vert. plate	3.00	0.000	0.0000	9.0000	0.000	0.000	9.000
2-2 Bot angles	2.72	-2.364	-6.4301	0.9580	2.364	15.201	16.159
3-2 Top angles	2.72	2.364	6.4301	0.9580	2.364	15.201	16.159
4-Top plate	2.25	3.250	7.3125	0.0469	3.250	23.766	23.813
5-Bot plate	2.25	-3.250	-7.3125	0.0469	3.250	23.766	23.813
Total area =	12.94	Sum Ay=	0.0000			Total I =	88.942 in <sup>4</sup>
Y=	0.00 in						

FIGURE P7-30	Units:	Inches	NOTE: Overall depth=6.0; Ref axis=centroid, 3.0 in from bottom				
Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-2 v. plates	3.000	0.000	0.000	9.000	0.000	0.000	9.000
2-Bot channel	1.760	-2.545	-4.479	0.305	2.545	11.400	11.705
3-Top plate	1.760	2.545	4.479	0.305	2.545	11.400	11.705
Total area =	4.760	Sum Ay=	0.000			Total I =	32.409 in <sup>4</sup>
Y=	0.000 in						
			NOTE: For C3x6; y = 3.0-0.455 = 2.545; See App. A-6; x=0.455				

FIGURE P7-31      Units: Inches      NOTE: Ref axis=centroid, 3.0 in from center of either pipe

Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-vert. plate	2.050	0.000	0.000	2.870	0.000	0.000	2.870
2-Bot pipe	1.070	-3.000	-3.210	0.391	3.000	9.630	10.021
3-Top pipe	1.070	3.000	3.210	0.391	3.000	9.630	10.021
Total area =	3.120	Sum Ay=	0.000			Total I =	22.912 in <sup>4</sup>
Y=	0.000 in						

NOTE: Length of plate = 6.0 in - pipe dia = 6.0 - 1.90 = 4.10 in

FIGURE P7-32      Units: Inches      NOTE: Reference axis=centroid=12 in from CL of pipes

Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-2 bot pipes	4.46	-12.00	-53.52	6.04	12.00	642.24	648.28
2-2 Top pipes	4.46	12.00	53.52	6.04	12.00	642.24	648.28
Total area =	8.92	Sum Ay=	0.00			Total I =	1296.56 in <sup>4</sup>
Y=	0.00 in						

FIGURE P7-33      Units: Inches      NOTE: Reference axis is base of the shape

Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-Base plate	2.5000	0.1250	0.3125	0.0130	2.5980	16.8734	16.8864
2-2 angles	3.3800	1.4900	5.0362	5.5400	1.2330	5.1382	10.6782
3-Top plate	6.0000	4.5000	27.0000	0.1250	1.7770	18.9473	19.0723
Total area =	11.8800	Sum Ay=	32.3487			Total I =	46.6370 in <sup>4</sup>
Y=	2.7230 in						

FIGURE P7-34      Units: Inches      NOTE: Reference axis=centroid=3.00 in from bottom

Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-channel	3.83	0.00	0.00	17.40	0.00	0.00	17.40
2-channel	3.83	0.00	0.00	17.40	0.00	0.00	17.40
Total area =	7.66	Sum Ay=	0.00			Total I =	34.80 in <sup>4</sup>
Y=	0.00 in						

FIGURE P7-35      Units: Inches      NOTE: Reference axis is base of the shape

Part	Area	y	Ay	lc	d	Ad <sup>2</sup>	lc+Ad <sup>2</sup>
1-2 angles	1.876	0.592	1.111	0.696	2.020	7.658	8.354
2-Bot channel	2.640	0.478	1.262	0.632	2.134	12.027	12.659
3-2 Ver webs	4.500	3.000	13.500	13.500	0.388	0.676	14.176
4-top channel	2.640	5.522	14.578	0.632	2.910	22.349	22.981
Total area =	11.656	Sum Ay=	30.451			Total I =	58.171 in <sup>4</sup>
Y=	2.612 in						

FIGURE P7-36							
Units:		Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-W6x15	4.430	2.995	13.268	29.100	0.638	1.801	30.901
2-2 angles	1.876	0.852	1.598	0.696	1.505	4.252	4.948
Total area =	6.306	Sum Ay=	14.866			Total I =	35.848 in <sup>4</sup>
Y=	2.357	in					

FIGURE P7-37							
Units:		Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Bot tube	2.590	2.000	5.180	4.690	1.500	5.828	10.518
2-Top tube	2.590	5.000	12.950	1.540	1.500	5.828	7.368
Total area =	5.180	Sum Ay=	18.130			Total I =	17.885 in <sup>4</sup>
Y=	3.500	in					

FIGURE P7-38							
Units:		Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-6x6x1/2	10.400	3.000	31.200	50.500	0.050	0.026	50.526
2-4x2x1/4	2.590	1.500	3.885	1.540	1.450	5.447	6.987
3-3x2x1/4	2.090	4.500	9.405	1.150	1.550	5.020	6.170
Total area =	15.080	Sum Ay=	44.490			Total I =	63.683 in <sup>4</sup>
Y=	2.950	in					

FIGURE P7-39							
Units:		Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-bot channel	1.590	0.457	0.727	0.319	2.898	13.358	13.677
2-6x2x1/4	3.590	3.184	11.431	13.800	0.171	0.106	13.906
3-Top channel	1.590	6.641	10.559	0.319	3.286	17.164	17.483
Total area =	6.770	Sum Ay=	22.716			Total I =	45.065 in <sup>4</sup>
Y=	3.355	in					

FIGURE P7-40							
Units:		Inches	NOTE: Reference axis is base of the shape				
Part	Area	y	Ay	Ic	d	Ad <sup>2</sup>	Ic+Ad <sup>2</sup>
1-Bot channel	1.881	0.730	1.373	0.980	0.321	0.193	1.173
2-I-beam	1.726	1.400	2.416	0.680	0.349	0.211	0.891
Total area =	3.607	Sum Ay=	3.790			Total I =	2.064 in <sup>4</sup>
Y=	1.051	in					

FIGURE P7-41

		Units: Inches		NOTE: Reference axis is base of the shape			
Part	Area, $A_i$	$y_i$	$A_i y_i$	$I_c$	$d_i$	$A_i d_i^2$	$I_c + A d^2$
1-W12x30	8.790	6.170	54.234	238.000	2.029	36.169	274.169
2-C6x13	3.830	12.854	49.231	1.050	4.655	83.010	84.060
Total area =	12.620 in <sup>2</sup>	Sum $A y =$	103.465 in <sup>3</sup>			Total $I_c =$	358.229 in <sup>4</sup>
		$Y =$	8.199 in				

FIGURE P7-42

		Units: Inches		NOTE: Reference axis is centroid of W-beam shape			
Part	Area, $A_i$	$y_i$	$A_i y_i$	$I_c$	$d_i$	$A_i d_i^2$	$I_c + A d^2$
1-W4x13	3.830	0.000	0.000	11.300	0.000	0.000	11.300
2-4x2x1/4	2.590	3.080	7.977	1.540	3.080	24.570	26.110
3-4x2x1/4	2.590	-3.080	-7.977	1.540	3.080	24.570	26.110
Total area =	9.010 in <sup>2</sup>	Sum $A y =$	0.000 in <sup>3</sup>			Total $I_c =$	63.520 in <sup>4</sup>
		$Y =$	0 in				

FIGURE P7-43

		Units: Inches		NOTE: Reference axis is bottom of the shape			
Part	Area, $A_i$	$y_i$	$A_i y_i$	$I_c$	$d_i$	$A_i d_i^2$	$I_c + A d^2$
1-W12x30	8.790	6.170	54.234	238.000	2.058	37.229	275.229
2-L4x3x1/4	1.690	13.580	22.950	2.770	5.352	48.408	51.178
3-L4x3x1/4	1.690	13.580	22.950	2.770	5.352	48.408	51.178
Total area =	12.170 in <sup>2</sup>	Sum $A y =$	100.135 in <sup>3</sup>			Total $I_c =$	377.585 in <sup>4</sup>
		$Y =$	8.228 in				

FIGURE P7-44

		Units: Inches		NOTE: Reference axis is centroid of 6x2 tube			
Part	Area, $A_i$	$y_i$	$A_i y_i$	$I_c$	$d_i$	$A_i d_i^2$	$I_c + A d^2$
1-6x2x1/4	3.590	0.000	0.000	13.800	0.000	0.000	13.800
2-1/2x2 PI	1.000	3.250	3.250	0.010	3.250	10.563	10.573
3-1/2x2 PI	1.000	-3.250	-3.250	0.010	3.250	10.563	10.573
Total area =	5.590 in <sup>2</sup>	Sum $A y =$	0.000 in <sup>3</sup>			Total $I_c =$	34.946 in <sup>4</sup>
		$Y =$	0 in				

FIGURE P7-45

		Units: Inches		NOTE: Reference axis is bottom of the shape			
Part	Area, $A_i$	$y_i$	$A_i y_i$	$I_c$	$d_i$	$A_i d_i^2$	$I_c + A d^2$
1-C8x4.147	3.526	4.000	14.104	37.400	1.023	3.692	41.092
2-C8x4.147	3.526	4.000	14.104	37.400	1.023	3.692	41.092
3-C8x4.147	3.526	7.070	24.929	3.250	2.047	14.770	18.020
Total area =	10.578 in <sup>2</sup>	Sum $A y =$	53.137 in <sup>3</sup>			Total $I_c =$	100.205 in <sup>4</sup>
		$Y =$	5.023 in				

**FIGURE P7-46**

Units: Inches

NOTE: Reference axis is bottom of the shape

Part	Area, $A_i$	$y_i$	$A_i y_i$	$I_c$	$d_i$	$A_i d_i^2$	$I_c + A d^2$
1-1/2x18 PI	9.000	0.250	2.250	0.188	1.862	31.194	31.382
2-3/4x10 PI	7.500	5.500	41.250	62.500	3.388	86.103	148.603
3-L8x4x1/2	5.750	1.359	7.814	6.740	0.753	3.258	9.998
4-L8x4x1/2	5.750	1.359	7.814	6.740	0.753	3.258	9.998
Total area =	28.000 in <sup>2</sup>	Sum $A y =$	59.129 in <sup>3</sup>			Total $I_c =$	199.981 in <sup>4</sup>
		$Y =$	2.112 in				

**FIGURE P7-47**

Units: mm

NOTE: Reference axis is bottom of the shape

Part	Area, $A_i$	$y_i$	$A_i y_i$	$I_c$	$d_i$	$A_i d_i^2$	$I_c + A d^2$
1-Flanges(2)	72.000	1.500	108.0	54.0	13.1	12434.2	12488.2
2-Vert. webs(2)	150.000	12.500	1875.0	7812.5	2.1	687.9	8500.4
3-Semicircle(+)	157.080	29.240	4593.0	1120.0	14.6	33476.6	34596.6
4-Semicircle(-)	-76.970	27.968	-2152.7	-268.9	13.3	-13669.7	-13938.6
Total area =	302.110 mm <sup>2</sup>	Sum $A y =$	4423.3 mm <sup>3</sup>			Total $I_c =$	41646.6 mm <sup>4</sup>
		$Y =$	14.641 mm				

**FIGURE P7-48**

Units: Inches

NOTE: Reference axis is base of the shape

Part	Area, $A_i$	$y_i$	$A_i y_i$	$I_c$	$d_i$	$A_i d_i^2$	$I_c + A d^2$
1-Rect. (2)	1.250	0.625	0.781	0.163	0.391	0.191	0.353
2-Semicircle	0.884	1.568	1.385	0.035	0.552	0.270	0.305
Total area =	2.134 in <sup>2</sup>	Sum $A y =$	2.167 in <sup>3</sup>			Total $I_c =$	0.659 in <sup>4</sup>
		$Y =$	1.016 in				

# **SOLUTIONS FOR PROBLEMS 7-49 THROUGH 7-66**

Each problem requires the computation of the radius of gyration  $r_x = (I_x/A)^{1/2}$  with respect to the horizontal centroidal axis. Data for  $I$  and  $A$  are taken from the solution for the given figure number.

Prob. No.	Fig. No.	$I_x$	$A$	$r_x$
7-49	P7-2	166.0 in <sup>4</sup>	18.00 in <sup>2</sup>	3.04 in
7-50	P7-3	184.0 in <sup>4</sup>	24.00 in <sup>2</sup>	2.77 in
7-51	P7-4	4.64E+07 mm <sup>4</sup>	9375 mm <sup>2</sup>	70.35 mm
7-52	P7-5	2.66E+05 mm <sup>4</sup>	550 mm <sup>2</sup>	21.99 mm
7-53	P7-6	6.17E+04 mm <sup>4</sup>	800 mm <sup>2</sup>	8.78 mm
7-54	P7-8	5.38E+04 mm <sup>4</sup>	550 mm <sup>2</sup>	9.87 mm
7-55	P7-9	1.35E+05 mm <sup>4</sup>	550 mm <sup>2</sup>	15.87 mm
7-56	P7-11	1.86E+05 mm <sup>4</sup>	725 mm <sup>2</sup>	16.02 mm
7-57	P7-12	1.70E+04 mm <sup>4</sup>	456 mm <sup>2</sup>	6.11 mm
7-58	P7-14	0.3672 in <sup>4</sup>	1.08 in <sup>2</sup>	0.59 in
7-59	P7-15	0.3572 in <sup>4</sup>	0.68 in <sup>2</sup>	0.72 in
7-60	P7-16	0.3657 in <sup>4</sup>	1.35 in <sup>2</sup>	0.52 in
7-61	P7-17	6.73E+07 mm <sup>4</sup>	14227 mm <sup>2</sup>	68.78 mm
7-62	P7-21	151.4 in <sup>4</sup>	18.75 in <sup>2</sup>	2.84 in
7-63	P7-22	148.3 in <sup>4</sup>	27.38 in <sup>2</sup>	2.33 in
7-64	P7-23	107.2 in <sup>4</sup>	34.50 in <sup>2</sup>	1.76 in
7-65	P7-24	816.3 in <sup>4</sup>	50.63 in <sup>2</sup>	4.02 in
7-66	P7-25	1155.1 in <sup>4</sup>	19.85 in <sup>2</sup>	7.63 in

# SOLUTIONS FOR PROBLEMS 7-67 THROUGH 7-81

Each problem requires the computation of the radius of gyration  $r_y = (I_y/A)^{1/2}$  with respect to the vertical centroidal axis.

Data for  $A$  are taken from the solution for the given figure number.

All sections have a vertical axis of symmetry and they can be broken into parts that all have their centroidal axis on the axis of symmetry.

Therefore the total  $I$  is the algebraic sum of the  $I$  for all parts.

Prob. No.	Fig. No.	$I_1$	$I_2$	$I_3$	Total $I_y$	$A$	$r_y$
7-67	P7-2	18.00 in <sup>4</sup>	18.00 in <sup>4</sup>	0.50 in <sup>4</sup>	36.50 in <sup>4</sup>	18.00 in <sup>2</sup>	1.424 in
7-68	P7-3*	144.00 in <sup>4</sup>	-32.00 in <sup>4</sup>	0.00 in <sup>4</sup>	112.00 in <sup>4</sup>	24.00 in <sup>2</sup>	2.160 in
7-69	P7-4	2.60E+05 mm <sup>4</sup>	1.12E+07 mm <sup>4</sup>	0.00 mm <sup>4</sup>	1.14E+07 mm <sup>4</sup>	9375 mm <sup>2</sup>	34.91 mm
7-70	P7-5	3333.33 mm <sup>4</sup>	520.83 mm <sup>4</sup>	26867 mm <sup>4</sup>	3.05E+04 mm <sup>4</sup>	550 mm <sup>2</sup>	7.449 mm
7-71	P7-16	0.0527 in <sup>4</sup>	0.0078 in <sup>4</sup>	0.00 in <sup>4</sup>	0.0605 in <sup>4</sup>	1.35 in <sup>2</sup>	0.212 in
7-72	P7-17	3.42E+06 mm <sup>4</sup>	3.18E+05 mm <sup>4</sup>	3E+05 mm <sup>4</sup>	4.08E+06 mm <sup>4</sup>	14227 mm <sup>2</sup>	16.885 mm
7-73	P7-21	5.36 in <sup>4</sup>	1.55 in <sup>4</sup>	5.36 in <sup>4</sup>	12.27 in <sup>4</sup>	18.75 in <sup>2</sup>	0.809 in
7-74	P7-22*	222.30 in <sup>4</sup>	-35.18 in <sup>4</sup>	0.00 in <sup>4</sup>	187.11 in <sup>4</sup>	27.38 in <sup>2</sup>	2.614 in
7-75	P7-23*	5184.00 in <sup>4</sup>	-2701.13 in <sup>4</sup>	0.00 in <sup>4</sup>	2482.88 in <sup>4</sup>	34.50 in <sup>2</sup>	8.483 in
7-76	P7-24	25.31 in <sup>4</sup>	177.98 in <sup>4</sup>	0.00 in <sup>4</sup>	203.29 in <sup>4</sup>	50.63 in <sup>2</sup>	2.004 in
7-77	P7-25	45.20 in <sup>4</sup>	127.03 in <sup>4</sup>	127.03 in <sup>4</sup>	299.25 in <sup>4</sup>	19.85 in <sup>2</sup>	3.883 in
7-78	P7-26	15.70 in <sup>4</sup>	144.00 in <sup>4</sup>	0.00 in <sup>4</sup>	159.70 in <sup>4</sup>	22.05 in <sup>2</sup>	2.691 in
7-79	P7-27	35.48 in <sup>4</sup>	14.29 in <sup>4</sup>	0.00 in <sup>4</sup>	49.77 in <sup>4</sup>	15.65 in <sup>2</sup>	1.783 in
7-80	P7-42	3.86 in <sup>4</sup>	4.69 in <sup>4</sup>	4.69 in <sup>4</sup>	13.24 in <sup>4</sup>	9.01 in <sup>2</sup>	1.212 in
7-81	P7-44	2.31 in <sup>4</sup>	0.33 in <sup>4</sup>	0.33 in <sup>4</sup>	2.98 in <sup>4</sup>	5.59 in <sup>2</sup>	0.730 in

\*  $I_1$  is for the large outside rectangle.  $I_2$  is for the internal rectangle and is negative.



## CHAPTER 8 Stress Due to Bending

### ANALYSIS OF BENDING STRESSES

8-1  $\sigma = M c / I = (425 \text{ N}\cdot\text{m}) (15 \text{ mm}) / 67500 \text{ mm}^4 \times \frac{10^3 \text{ mm}}{\text{m}} = 94.4 \text{ MPa}$   
 $I = (30)^4 / 12 = 67500 \text{ mm}^4$

8-2  $I = \pi D^4 / 64 = \pi (20)^4 / 64 = 7854 \text{ mm}^4$   
 $\sigma = \frac{M c}{I} = \frac{(120 \text{ N}\cdot\text{m}) (10 \text{ mm})}{7854 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = 152.8 \text{ MPa}$

8-3 (a)  $I = b h^3 / 12 = 0.75 (1.5)^3 / 12 = 0.211 \text{ in}^4$   
 $\sigma = \frac{M c}{I} = \frac{(5800 \text{ lb}\cdot\text{in}) (0.75 \text{ in})}{0.211 \text{ in}^4} = 20620 \text{ psi}$   
 (b)  $I = 1.5 (0.75)^3 / 12 = 0.0527 \text{ in}^4$   
 $\sigma = \frac{(5800 \text{ lb}\cdot\text{in}) (0.375 \text{ in})}{0.0527 \text{ in}^4} = 41240 \text{ psi}$

8-4  $I = 1.5 (7.25)^3 / 12 = 47.63 \text{ in}^4$ ;  $C = 7.25 / 2 = 3.625 \text{ in}$   
 $\sigma = \frac{M c}{I} = \frac{(15500 \text{ lb}\cdot\text{in}) (3.625 \text{ in})}{47.63 \text{ in}^4} = 1180 \text{ psi}$

8-5  $M = 30 \text{ k}\cdot\text{ft}$ ;  $S = 17.1 \text{ in}^3$   
 $\sigma = \frac{M}{S} = \frac{30000 \text{ lb}\cdot\text{ft}}{17.1 \text{ in}^3} \times \frac{12 \text{ in}}{\text{ft}} = 21050 \text{ psi}$

8-6  $M = 60 \text{ k}\cdot\text{ft}$ ;  $S = 38.2 \text{ in}^3$   
 $\sigma = \frac{M}{S} = \frac{60000 \text{ lb}\cdot\text{ft}}{38.2 \text{ in}^3} \times \frac{12 \text{ in}}{\text{ft}} = 18850 \text{ psi}$

8-7 ALUM. C 4x2.33;  $I = 1.02 \text{ in}^4$ ;  $C_c = 0.78 \text{ in}$ ;  $C_b = 1.47 \text{ in}$   
 BEAM IS IN NEGATIVE BENDING.  
 TENSILE - TOP  
 $\sigma = \frac{M C_c}{I} = \frac{(9000 \text{ lb}\cdot\text{in}) (0.78 \text{ in})}{1.02 \text{ in}^4} = 6882 \text{ psi}$   
 COMP. - BOTTOM  
 $\sigma = \frac{-M C_b}{I} = \frac{-(9000 \text{ lb}\cdot\text{in}) (1.47 \text{ in})}{1.02 \text{ in}^4} = -12970 \text{ psi}$

8-8  $\sigma = \frac{M}{S} = \frac{4550 \text{ lb}\cdot\text{in}}{0.326 \text{ in}^3} = 13960 \text{ psi}$

8-9

$$M = 71.5 \text{ kN}\cdot\text{m} = 71500 \text{ N}\cdot\text{m} \times 8.851 \text{ lb}\cdot\text{in}/\text{N}\cdot\text{m} = 6.33 \times 10^5 \text{ lb}\cdot\text{in}$$

$$I = 710.4 \text{ in}^4 ; C = 7 = 6.50 \text{ in}$$

$$\sigma = \frac{Mc}{I} = \frac{6.33 \times 10^5 \text{ lb}\cdot\text{in} (6.50 \text{ in})}{710.4 \text{ in}^4} = 5794 \text{ psi} \quad (39.9 \text{ MPa})$$

8-10

$$S = 22.67 \text{ in}^3 \times 1.639 \times 10^4 \text{ mm}^3/\text{in}^3 = 3.716 \times 10^5 \text{ mm}^3$$

$$\sigma = \frac{M}{S} = \frac{(43.2 \text{ kN}\cdot\text{m})}{3.716 \times 10^5 \text{ mm}^3} \times \frac{10^3 \text{ N}}{\text{kN}} \times \frac{10^3 \text{ mm}}{\text{m}} = 116 \text{ MPa}$$

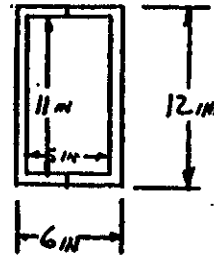
$$(16800 \text{ psi})$$

8-11

$$I = \frac{6(12)^3}{12} - \frac{5(11)^3}{12} = 309.4 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{(6000 \text{ lb}\cdot\text{ft}) (6 \text{ in}) \times 12 \text{ in}}{309.4 \text{ in}^4 \text{ ft}}$$

$$\sigma = 13963 \text{ psi}$$



DESIGN OF BEAMS

8-12  $\sigma = Mc/I = M/S ; S = \pi D^3/32 ; D = \sqrt[3]{32S/\pi}$

$$\text{REQ'D } S = \frac{M}{S_u} = \frac{240 \text{ N}\cdot\text{m}}{125 \text{ N/mm}^2} \times \frac{10^3 \text{ mm}}{\text{m}} = 1920 \text{ mm}^3$$

$$D = \sqrt[3]{32(1920 \text{ mm}^3)/\pi} = 26.9 \text{ mm}$$

8-13

$$S = bh^2/6 = b(36)^2/6 = 96^3/6 = 1.56^3$$

$$b = \sqrt[3]{S/1.5} = \sqrt[3]{2636 \text{ mm}^3/1.5} = 12.1 \text{ mm}$$

$$\text{REQ'D } S = \frac{M}{S_u} = \frac{145 \text{ N}\cdot\text{m}}{55 \text{ N/mm}^2} \times \frac{10^3 \text{ mm}}{\text{m}} = 2636 \text{ mm}^3$$

$$b = 12.1 \text{ mm} ; h = 36 = 36.3 \text{ mm}$$

8-14

$$C_b = \bar{y} = 152.5 \text{ mm} ; C_t = 225 - 152.5 = 72.5 \text{ mm}$$

$$\sigma_{\text{bot}} = \frac{Mc_b}{I} = \frac{(28000 \text{ N}\cdot\text{m})(152.5 \text{ mm})}{46.4 \times 10^6 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = 92.0 \text{ MPa}$$

FOR AISC 1020 HR,  $S_y = 331 \text{ MPa}$

$$\sigma_a = S_y/2 = 331 \text{ MPa}/2 = 165.5 \text{ MPa} ; \text{SINCE } < S_y, \text{ OK}$$

8-15

$$I = 2.66 \times 10^5 \text{ mm}^4 ; C_b = \bar{y} = 35.0 \text{ mm}$$

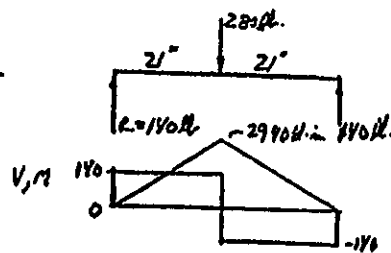
$$\sigma = \frac{Mc_b}{I} = \frac{(675 \text{ N}\cdot\text{m})(35.0 \text{ mm})}{2.66 \times 10^5 \text{ mm}^4} \times \frac{10^3 \text{ mm}}{\text{m}} = 36.2 \text{ MPa}$$

$$\text{LET } \sigma = \sigma_a = S_u/8$$

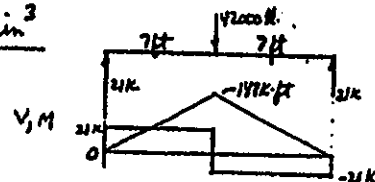
$$\text{REQ'D } S_u = 8S = 8(36.2 \text{ MPa}) = 290 \text{ MPa}$$

COULD USE 6061-T6,  $S_u = 310 \text{ MPa}$ ; 17% ELONGATION

B-16 REQ'D  $S = \frac{M}{\sigma_d} = \frac{2940 \text{ lb} \cdot \text{in}}{10000 \text{ lb/in}^2} = 0.294 \text{ in}^3$   
 USE  $1\frac{1}{2}$  IN SCH. 40 PIPE,  $= 0.3262 \text{ in}^3$



B-17  $S = \frac{M}{\sigma_d} = \frac{147000 \text{ lb} \cdot \text{ft}}{20000 \text{ lb/in}^2 \times \frac{12 \text{ in}}{\text{ft}}} = 88.2 \text{ in}^3$   
 USE A W20X66;  $S = 119 \text{ in}^3$

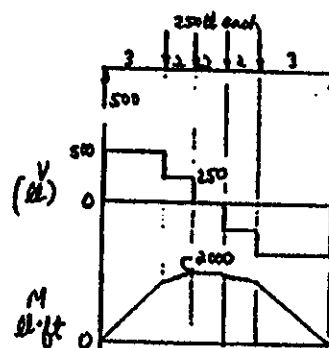


B-18  $I = 107.2 \text{ in}^4$ ;  $C = 4.50 \text{ in}/2 = 2.25 \text{ in}$

$\sigma = \frac{M c}{I} = \frac{(2000 \text{ lb} \cdot \text{ft})(2.25 \text{ in})(12 \text{ in/ft})}{(107.2 \text{ in}^4)}$

$\sigma = 504 \text{ psi}$

FROM TABLE A/B, MINIMUM ALLOWABLE  
 BENDING STRESS = 625 psi - OK



B-19  $I = 30(3)^3/12 - 2[14(2)^3/12] = 48.83 \text{ in}^4$ ;  $C = 1.50 \text{ in}$

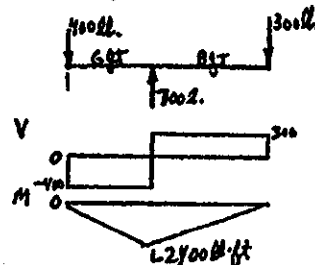
$\sigma = \frac{M c}{I} = \frac{2400 \text{ lb} \cdot \text{ft}(1.50 \text{ in})(12 \text{ in/ft})}{48.83 \text{ in}^4}$

$\sigma = 885 \text{ psi}$

FOR IMPACT;

REQUIRED  $S_u = 12\sigma = 12(885) = 10620 \text{ psi}$

FOR 6061-T4  $S_u = 35000 \text{ psi}$  - OK



B-20  $I = 1.86 \times 10^5 \text{ mm}^4$ ;  $C_b = \bar{y} = 21.8 \text{ mm}$ ;  $C_x = 45 - 21.8 = 23.2 \text{ mm}$

MAX STRESS AT TOP

$\sigma = \frac{M C_x}{I} = \frac{(318 \text{ N} \cdot \text{m})(23.2 \text{ mm}) \times 10^3 \text{ mm}}{1.86 \times 10^5 \text{ mm}^4} = 39.7 \text{ MPa (comp.)}$

REQ'D  $S_y = 2\sigma = 2(39.7 \text{ MPa}) = 79.4 \text{ MPa}$  OK FOR 6061-T4

$S_y = 145 \text{ MPa}$

B-21  $I = 1.7 \times 10^4 \text{ mm}^4$ ;  $C_b = \bar{y} = 12.39 \text{ mm}$

$\sigma = \frac{M C}{I} = \frac{(195 \text{ N} \cdot \text{m})(12.39 \text{ mm}) \times 10^3 \text{ mm}}{1.7 \times 10^4 \text{ mm}^4} = 142 \text{ MPa}$

REQ'D  $S_y = 2\sigma = 284 \text{ MPa}$  - 2014-T4 HAS  $S_y = 290 \text{ MPa}$

B-22

$$REQ'D S = \frac{M}{\sigma_d} = \frac{11250 \text{ N}\cdot\text{m}}{80 \text{ N/mm}^2} \times \frac{\text{mm}^3}{\text{m}} = 0.141 \times 10^6 \text{ mm}^3$$

(a) ROUND:  $S = \pi D^3 / 32$

$$D = \sqrt[3]{32(0.141 \times 10^6) / \pi} = 112.8 \text{ mm}$$

$$A = \frac{\pi D^2}{4} = 9999 \text{ mm}^2$$

(b) SQUARE:  $S = b^3 / 6$

$$b = \sqrt[3]{6(0.141 \times 10^6)} = 94.6 \text{ mm}$$

$$A = b^2 = 8945 \text{ mm}^2$$

(c) RECT.;  $h = 4b$ ;  $S = \frac{bh^2}{6}$ ;  $\frac{b(4b)^2}{6} = \frac{16b^3}{6} = 8b^3/3$

$$b = \sqrt[3]{3S/8} = \sqrt[3]{3(0.141 \times 10^6)/8} = 37.5 \text{ mm}$$

$$h = 4b = 150 \text{ mm}$$

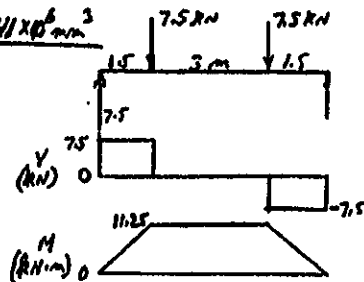
$$A = bh = 5625 \text{ mm}^2$$

(d) S-BEAM;  $S = 0.141 \times 10^6 \text{ mm}^3 \times \frac{6.102 \times 10^{-5} \text{ in}^3}{\text{mm}^3} = 8.60 \text{ in}^3$

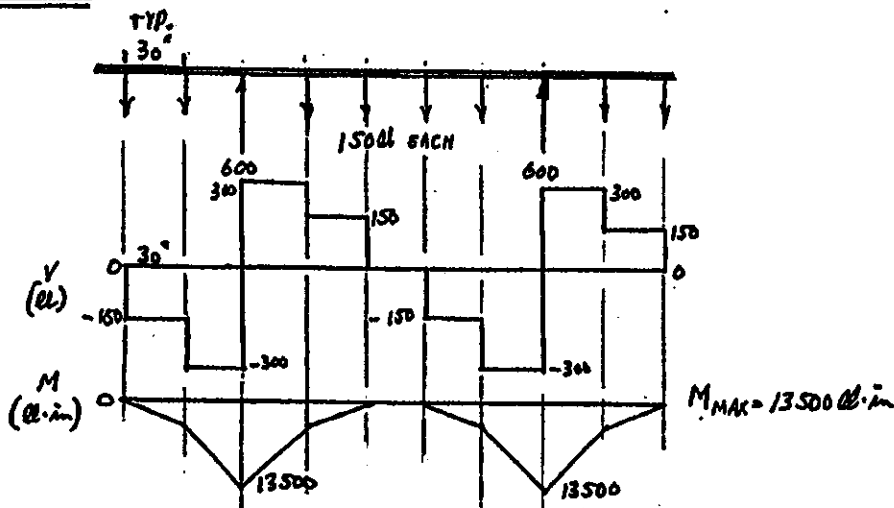
$$S_{8 \times 18.4} = 14.1 \text{ in}^3$$

$$A = 5.41 \text{ in}^2 \times \frac{645.16 \text{ mm}^2}{\text{in}^2} = 3490 \text{ mm}^2$$

LIGHTEST



B-23



$$REQ'D S = \frac{M}{\sigma_d} = \frac{13500 \text{ lb}\cdot\text{ft}}{10000 \text{ lb/in}^2} = 1.35 \text{ in}^3$$

$$USE \underline{3 \text{ IN. SCH 40 PIPE}}, S = 1.724 \text{ in}^3$$

8-24  $\sigma_a = S_y/4 = 39,000 \text{ psi}/4 = 9750 \text{ psi}$   
 $M = (4800 \text{ lb})(14 \text{ in}) = 67200 \text{ lb}\cdot\text{in}$   
 $\text{REQ'D. } S = M/\sigma_a = (67200 \text{ lb}\cdot\text{in})/(9750 \text{ lb/in}^2) = 6.89 \text{ in}^3$   
 USE EITHER  $6 \times 4 \times 1/4$  OR  $8 \times 2 \times 1/4$ : EACH WEIGHS 15.6 LB/FT.

8-25 LET  $\sigma_a = S_y/4 = 40,000 \text{ psi}/4 = 10,000 \text{ psi}$   
 $\text{REQ'D. } S = M/\sigma_a = 67200/10000 = 6.72 \text{ in}^3 \Rightarrow \underline{6 \text{ I} \times 4.030}$

8-26  $\sigma_a = S_y/4 = 36,000 \text{ psi}/4 = 9000 \text{ psi}$   
 $\text{REQ'D. } S = M/\sigma_a = 67200/9000 = 7.47 \text{ in}^3 \Rightarrow \underline{W 8 \times 10}$

8-27  $\text{REQ'D } S = 7.47 \text{ in}^3$  FROM 8-26: NO SUITABLE CHANNEL

8-28  $S_y = 36 \text{ ksi}$  - THEN  $\text{REQ'D. } S = 7.47 \text{ in}^3$  (FROM 8-26): 6 IN SCH 40 PIPE

8-29 DESIGN PROBLEM - MULTIPLE SOLUTIONS POSSIBLE

8-30 FROM FIG P1-15;  $I = 0.3572 \text{ in}^4$ ;  $C_b = 1.068 \text{ in}$ ;  $C_t = 1.332 \text{ in}$

$S_{\min} = \frac{I}{C_t} = \frac{0.3572 \text{ in}^4}{1.332 \text{ in}} = 0.268 \text{ in}^3$

$\sigma = \frac{M}{S} = \frac{117 \text{ lb}\cdot\text{ft} \cdot \frac{12 \text{ in}}{\text{ft}}}{0.268 \text{ in}^3} = 5239 \text{ psi}$

$\text{REQ'D } \sigma_a = 4\sigma = 20,960 \text{ psi}$

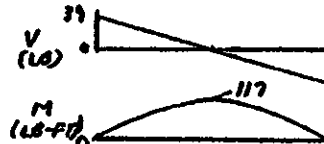
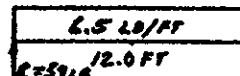
SEVERAL POSSIBLE CHOICES: APP. A-19

EXAMPLE: NYLON; POLYESTER

HIGH MODULUS OF ELASTICITY

GOOD ELECTRICAL PROPERTIES (TABLE 2-5)

MUST CHECK EXTRUDABILITY.



8-31

$M_{\max} = 24 \text{ kN}\cdot\text{m}$  (SEE PROB. P6-34)

$S = \frac{M}{\sigma_a} = \frac{24,000 \text{ N}\cdot\text{m}}{150 \text{ N/mm}^2} \times \frac{10^3 \text{ mm}^3}{\text{m}} = 160,000 \text{ mm}^3 \times \frac{6.02 \times 10^{-5} \text{ m}^3}{\text{mm}^3}$

$S = 9.76 \text{ in}^3$ ; USE W 10 x 12,  $S = 10.9 \text{ in}^3$

8-32

FROM PROB. P6-35,  $M_{\max} = 125 \text{ N}\cdot\text{m}$

$\sigma_a = S_u/8 = 648 \text{ MPa}/8 = 81 \text{ MPa}$

$S = \frac{M}{\sigma_a} = \frac{125 \text{ N}\cdot\text{m}}{81 \text{ N/mm}^2} \times \frac{10^3 \text{ mm}^3}{\text{m}} = 1543 \text{ mm}^3 = \pi D^3/32$

$D = \sqrt[3]{32S/\pi} = \underline{25.1 \text{ mm}}$

**8-33** Specify the lightest wide-flange beam. ASTM A36 structural steel.  $s_y = 36$  ksi.

$$\text{Design stress: } \sigma_d = 0.66 s_y = (0.66)(36\,000 \text{ psi}) = 23\,760 \text{ psi}$$

$$\text{From Fig. P6-3: } M_{\max} = (45.7 \text{ K}\cdot\text{ft})(1000 \text{ lb/K})(12 \text{ in/ft}) = 548\,400 \text{ lb}\cdot\text{in}$$

$$\text{Required section modulus: } S = M/\sigma_d = (548\,400 \text{ lb}\cdot\text{in})/(23\,760 \text{ in}^2) = 23.1 \text{ in}^3$$

Specify: W14x26 steel beam.

**8-34** Specify the lightest wide-flange beam. ASTM A36 structural steel.  $s_y = 248$  MPa.

$$\text{Design stress: } \sigma_d = 0.66 s_y = (0.66)(248 \text{ MPa}) = 164 \text{ MPa}$$

Because beam data are given in U.S. Cust. units, bending moments converted to lb in.

$$\text{Use } \sigma_d = 0.66 s_y = (0.66)(36\,000 \text{ psi}) = 23\,760 \text{ psi}$$

$$\text{From Fig. P6-7: } M_{\max} = (71.5 \text{ kN}\cdot\text{m})(8\,851 \text{ lb}\cdot\text{in/kN}\cdot\text{m}) = 6.33 \times 10^5 \text{ lb}\cdot\text{in}$$

$$\text{Required section modulus: } S = M/\sigma_d = (6.33 \times 10^5 \text{ lb}\cdot\text{in})/(23\,760 \text{ in}^2) = 26.6 \text{ in}^3$$

Specify: W14x26 steel beam.

Problems 8-33 to 8-42 are similar to 8-33 and 8-34 with the same design stress. Maximum bending moment varies with the beam loading shown in the indicated figure from Chapter 6.

Problems 8-43 to 8-52 use the same set of beam loadings as 8-33 to 8-42 but the objective is to specify the lightest American Standard S-beam. The required section modulus,  $S$ , is the same.

Problems 8-53 to 8-62 use the same set of beam loadings as 8-33 to 8-42 but the material is ASTM A572 Grade 60 with  $s_y = 60$  ksi. Then,

$$\sigma_d = 0.66 s_y = (0.66)(60\,000 \text{ psi}) = 39\,600 \text{ psi} \quad (273 \text{ MPa})$$

The objective is to specify the lightest wide flange beam.

The results of these three sets of problems are summarized below.

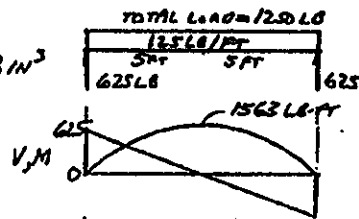
Prob. No.	Fig. No.	M max	Reqd. S (in <sup>3</sup> )	Lightest W-beam	Prob. No.	Lightest S-beam	Prob. No.	Reqd. S (in <sup>3</sup> )	Lightest W-beam
8-33	P6-3	45.7 K-ft	23.1	W14x26	8-43	S10x25.4	8-53	13.9	W12x16
8-34	P6-7	71.5 N-m	26.6	W14x26	8-44	S10x35	8-54	16.0	W12x16
8-35	P6-8	43.2 kN-m	16.1	W12x16	8-45	S8x23	8-55	9.66	W10x12
8-36	P6-11	60.0 K-ft	30.3	W14x26	8-46	S12x35	8-56	18.2	W14x26
8-37	P6-16	170 kN-m	63.3	W18x40	8-47	S15x50	8-57	38.0	W12x30
8-38	P6-36	10.0 K-ft	5.05	W8x10	8-48	S6x12.5	8-58	3.03	W8x10
8-39	P6-40	40.0 K-ft	20.2	W14x26	8-49	S10x25.4	8-59	12.1	W10x15
8-40	P6-52	148 K-ft	74.7	W18x55	8-50	S20x66	8-60	44.8	W18x40
8-41	P6-63	1450 N-m	0.54	W8x10	8-51	S3x5.7	8-61	0.32	W8x10
8-42	P6-64	30.0 K-ft	15.15	W12x16	8-52	S8x23	8-62	9.09	W10x12

8-63

$\sigma_d = 1000 \text{ psi}$

REQ'D.  $S = \frac{M}{\sigma_d} = \frac{(1563 \text{ LB} \cdot \text{FT})(12 \text{ IN/FT})}{850 \text{ LB/IN}^2} = 18.8 \text{ IN}^3$

USE 2x10 WOOD BEAM



8-64

$\sigma_d = 1150 \text{ psi}$

PART	A	$y$	$Ay$	$I$	$d$	$Ad^2$	$I + Ad^2$
1	5.25	1.75	9.188	5.36	1.907	19.085	24.445
2	16.87	4.25	71.698	3.164	.593	5.939	9.103

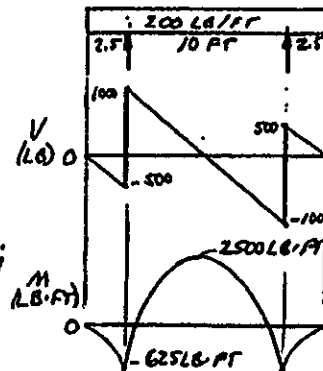
$\Sigma A = 22.12$   $\Sigma Ay = 80.886$

$I_T = 33.55 \text{ IN}^4$

$\bar{Y} = 3.66 \text{ IN} = C$

$\sigma = \frac{M C}{I} = \frac{(2500 \text{ LB} \cdot \text{FT})(12 \text{ IN/FT})(3.66 \text{ IN})}{33.55 \text{ IN}^4} = 3273 \text{ psi}$

UNSAFE



8-65

PROCEDURE SAME AS 8-64;  $\sigma_d = 1150 \text{ psi}$

WITH 2x8 VERT. MEMBER,  $\sigma = 1064 \text{ psi}$

WITH 2x8 VERT. MEMBER:

$\bar{Y} = C = 6.286 \text{ IN}$ ;  $I = 177.3 \text{ IN}^4$

$\sigma = \frac{(2500)(12)(6.286)}{177.3} = 1064 \text{ psi}$  OK

8-66

PROCEDURE SAME AS 8-64 WITH DOUBLE-WIDTH WEB.

WITH 2-2x6 VERTICAL MEMBERS:  $\bar{Y} = C = 4.519 \text{ IN}$ ;  $I = 146.9 \text{ IN}^4$

$\sigma = \frac{(2500)(12)(4.519)}{146.9} = 923 \text{ psi}$  OK

8-67

DESIGN PROBLEM - MULTIPLE SOLUTIONS POSSIBLE

8-68

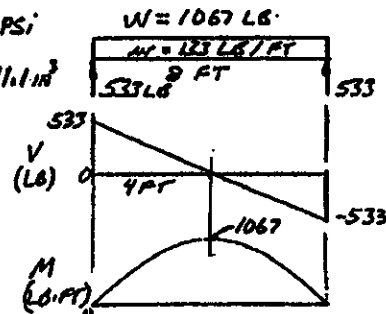
IN THE MIDDLE OF THE DECK, EACH FOOT OF JOIST LENGTH CARRIES A 16 IN WIDE PART OF THE DECK LOAD.

$w = \frac{100 \text{ LB}}{\text{FT}^2} \times \frac{(16 \text{ IN})(12 \text{ IN})}{144 \text{ IN}^2} \times \frac{1 \text{ FT}^2}{1 \text{ FT}} = 133 \text{ LB/FT}$

SPECIFY NO.2 HEMLOCK:  $\sigma_d = 1150 \text{ psi}$

REQ'D.  $S = \frac{M}{\sigma_d} = \frac{(1067 \text{ LB} \cdot \text{FT})(12 \text{ IN/FT})}{1150 \text{ LB/IN}^2} = 11.1 \text{ IN}^3$

USE 2x8;  $S = 13.14 \text{ IN}^3$



8-69

JOISTS ARE 12 FT LONG: BEAMS

AT ENDS:  $w = 133 \text{ LB/FT}$

$V_{\text{MAX}} = 800 \text{ LB}$  AT SUPPORTS:

$M_{\text{MAX}} = 2400 \text{ LB} \cdot \text{FT}$  (28800 LB-IN)

REQ'D  $S = \frac{M}{\sigma_d} = \frac{28800 \text{ LB} \cdot \text{IN}}{1150 \text{ LB/IN}^2} = 25.0 \text{ IN}^3$  -USE 2x12;  $S = 31.6 \text{ IN}^3$

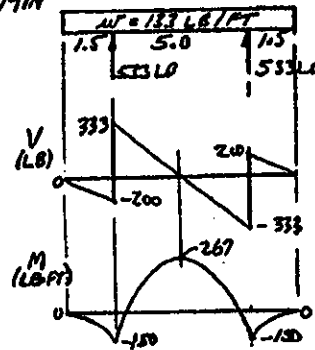
B-70

$$REQ'D S = \frac{M}{\sigma_b} = \frac{(267 \text{ LB}\cdot\text{FT})(12 \text{ IN/FT})}{1150 \text{ LB/IN}^2} = 2.79 \text{ IN}^3$$

USE 2X4 JOISTS:  $S = 3.06 \text{ IN}^3$

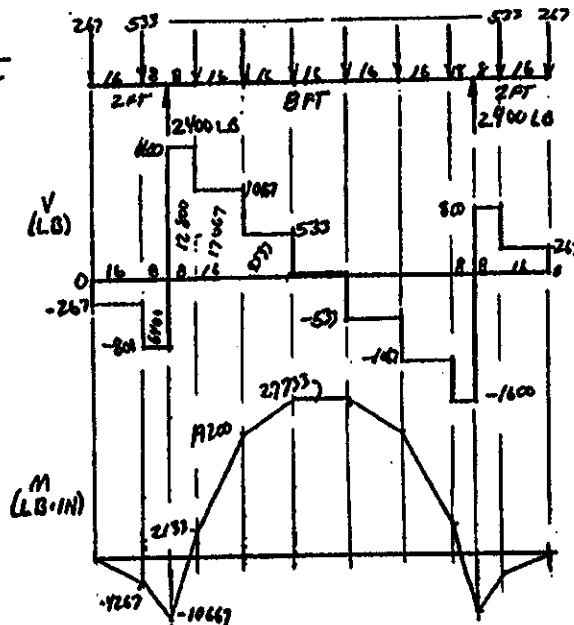
B-71

JOISTS ARE 12 FT LONG, BEAMS 1.5 FT FROM EACH END:  $w = 133 \text{ LB/FT}$   
 $V_{MAX} = 600 \text{ LB}$  AT SUPPORTS:  
 $M_{MAX} = 1200 \text{ LB}\cdot\text{FT}$  (18400 LB·IN)  
 $REQ'D S = \frac{M}{\sigma_b} = \frac{(18400 \text{ LB}\cdot\text{IN})}{1150 \text{ LB/IN}^2} = 16.0 \text{ IN}^3$   
 USE 2X8:  $S = 13.1 \text{ IN}^3$



B-72

$M_{MAX} = 27733 \text{ LB}\cdot\text{IN}$   
 $REQ'D S = \frac{M}{\sigma_b} = \frac{27733 \text{ LB}\cdot\text{IN}}{1150 \text{ LB/IN}^2}$   
 $S = 24.1 \text{ IN}^3$   
 COULD USE:  
 $2 \times 12: S = 31.6 \text{ IN}^3$   
 $4 \times 8: S = 30.7 \text{ IN}^3$



### DESIGN PROBLEM

B-73

TRY 2X6 DECK BOARDS WITH 3.0 FT SPAN

HEMLOCK:  $\sigma_b = 1150 \text{ LB/IN}^2$

$$S = bh^2/6 = (6.5)(1.5)^2/6 = 2.06 \text{ IN}^3$$

$$W = \frac{60 \text{ LB}}{\text{FT}^2} \times \frac{(6.5)(36) \text{ IN}^2}{144 \text{ IN}^2} \times \frac{1 \text{ FT}^2}{144 \text{ IN}^2} = 82.5 \text{ LB TOTAL}$$

$$w = 82.5 \text{ LB}/36 \text{ IN} = 2.29 \text{ LB/IN}$$

$$\sigma = \frac{M}{S} = \frac{371 \text{ LB}\cdot\text{IN}}{2.06 \text{ IN}^3} = 180 \text{ PSI OK}$$

MUST ALSO CHECK DEFLECTION AND SHEAR

BEAMS: TOTAL LOAD =  $(60 \text{ LB/FT}^2)(3 \text{ FT}^2)$   
 $w_T = 180 \text{ LB}$

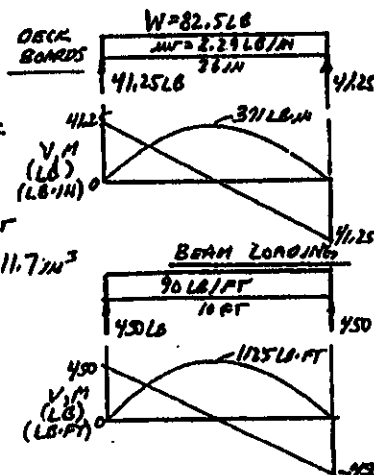
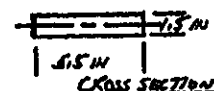
EACH BEAM CARRIES  $90 \text{ LB/FT} = 90 \text{ LB/FT}$

$$REQ'D S = \frac{M}{\sigma_b} = \frac{(1125 \text{ LB}\cdot\text{FT})(12 \text{ IN/FT})}{1150 \text{ LB/IN}^2} = 11.7 \text{ IN}^3$$

$$USE 4X6 BEAM: S = 17.65 \text{ IN}^3$$

NOTE: MANY OTHER DESIGNS ARE POSSIBLE AND PRACTICAL.

COULD USE 2X8:  $S = 13.14 \text{ IN}^3$





B-74  $\sigma_b = 1150 \text{ psi}$  FROM PROB. B-73.

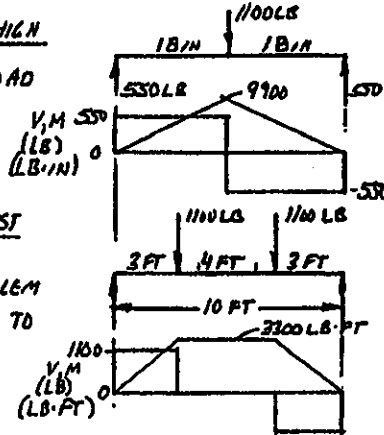
ASSUME DECK BOARDS CARRY  $\frac{1}{2}$  OF TOTAL LOAD WHEN HORSE STEPS ON IT WITH ONE HOOF.

$$\sigma = \frac{A}{S} = \frac{9900 \text{ LB} \cdot \text{IN}}{2.06 \text{ IN}^3} = 4805 \text{ PSI TOO HIGH}$$

ASSUME SIDE BEAMS CARRY FULL LOAD IF HORSE WALKS TOWARD ONE SIDE. LOAD SPLIT BETWEEN FRONT AND BACK; ASSUME 4 FT SPREAD.

$$\sigma = \frac{M}{S} = \frac{(3300 \text{ LB} \cdot \text{FT})(12 \text{ IN/FT})}{17.65 \text{ IN}^3} = 2244 \text{ PSI TOO HIGH}$$

CONCLUSION: BRIDGE DESIGNED IN PROBLEM B-41 WOULD NOT BE SAFE FOR HORSE TO CROSS.



B-75

WOOD BEAM: ASSUME NO. 2 SOUTHERN

PINE -  $\sigma_b = 1000 \text{ PSI}$

$$\text{REQ'D } S = \frac{M}{\sigma_b} = \frac{(15750 \text{ LB} \cdot \text{FT})(12 \text{ IN/FT})}{1000 \text{ LB/IN}^2} = 189 \text{ IN}^3$$

USE 10x12 BEAM:  $S = 209 \text{ IN}^3$

STEEL W-BEAM: ASTM A36 STEEL,  $S_y = 36 \text{ ksi}$

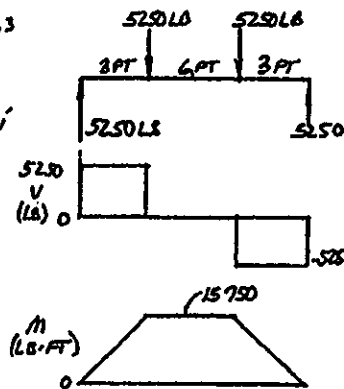
$$\sigma_b = 0.66 S_y = 0.66 (36000 \text{ PSI}) = 23760 \text{ PSI}$$

$$\text{REQ'D } S = \frac{M}{\sigma_b} = \frac{(15750 \text{ LB} \cdot \text{FT})(12 \text{ IN/FT})}{23760 \text{ LB/IN}^2}$$

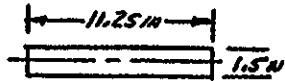
$$S = 7.95 \text{ IN}^3$$

USE: W6x15;  $S = 9.72 \text{ IN}^3$

OR W10x12;  $S = 10.9 \text{ IN}^3$



B-76



$$\text{FOR } 2 \times 12: S = \frac{bh^2}{2} = \frac{(11.25)(1.5)^2}{2}$$

$$S = 4.22 \text{ IN}^3$$

$$\sigma = \frac{M}{S} = \frac{(1870 \text{ LB} \cdot \text{FT})(12 \text{ IN/FT})}{4.22 \text{ IN}^3}$$

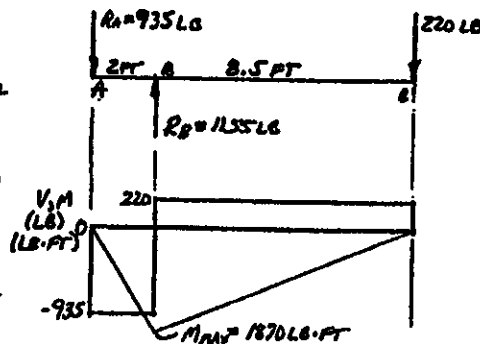
$$\sigma = 5319 \text{ PSI}$$

FOR NO. 2 S. PINE:  $\sigma_b = 1000 \text{ PSI}$

UNSAFE

POSSIBLE REDSIGN APPROACHES:

SHORTER PLANK; STRONGER WOOD; THICKER PLANK; BUILT-UP BEAM.



8-77

$$W = m \cdot g = 135 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 1324 \text{ N}$$

$$W/2 = 662 \text{ N ON EACH ROPE}$$

$$\text{FOR NO. 3 HEMLOCK: } \sigma_u = 4.3 \text{ MPa}$$

$$\text{AT A: } S = \pi D^3/32 = \pi (180)^3/32$$

$$S = 5.73 \times 10^5 \text{ mm}^3$$

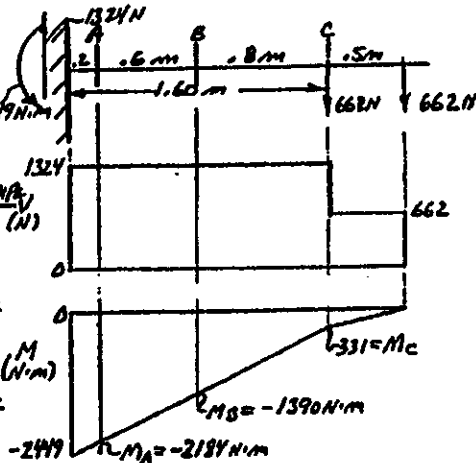
$$\sigma = \frac{M}{S} = \frac{2184 \text{ N}\cdot\text{m}}{5.73 \times 10^5 \text{ mm}^3} \cdot \frac{10^3 \text{ mm}}{\text{m}} = 3.81 \text{ MPa} < \sigma_u \text{ (N)}$$

$$\text{AT B: } S = \pi (140)^3/32 = 2.69 \times 10^5 \text{ mm}^3$$

$$\sigma = \frac{M}{S} = \frac{1390 \times 10^3 \text{ N}\cdot\text{mm}}{2.69 \times 10^5 \text{ mm}^3} = 5.16 \text{ MPa}$$

$$\text{AT C: } S = \pi (90)^3/32 = 7.16 \times 10^4 \text{ mm}^3$$

$$\sigma = \frac{M}{S} = \frac{(32 \times 10^3 \text{ N}\cdot\text{mm})}{7.16 \times 10^4 \text{ mm}^3} = 4.62 \text{ MPa}$$



8-78

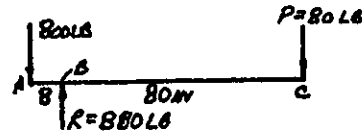
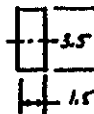
$$M_{\text{MAX}} = 6400 \text{ L}\cdot\text{IN AT B}$$

ASSUME 2x4 PLACED AS:

$$S = 3.06 \text{ IN}^3$$

$$\sigma = \frac{M}{S} = \frac{6400 \text{ L}\cdot\text{IN}}{3.06 \text{ IN}^3} = 2092 \text{ PSI}$$

$$\sigma_u = 1000 \text{ PSI FOR NO. 2 SO. PINE - UNSAFE}$$



8-79

$$\text{FROM P7-6: } I = 6.167 \times 10^4 \text{ mm}^4$$

$$C = 17.5 \text{ mm: } \sigma = \frac{M}{S}$$

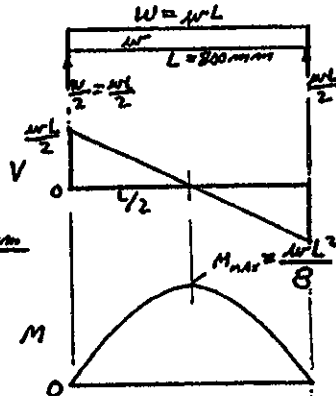
$$\text{FOR NYLON 6/6: } S_u = 24 \text{ MPa}$$

$$\text{THEN } \sigma_B = S_u/2 = 12.0 \text{ MPa}$$

$$M_{\text{MAX}} = \frac{\sigma_B \cdot I}{C} = \frac{12.0 \text{ MPa} \cdot 6.167 \times 10^4 \text{ mm}^4}{17.5 \text{ mm}}$$

$$M_{\text{MAX}} = 4.25 \times 10^5 \text{ N}\cdot\text{mm} = w L^2/8$$

$$w_{\text{MAX}} = \frac{8 M_{\text{MAX}}}{L^2} = \frac{(8 \times 4.25 \times 10^5 \text{ N}\cdot\text{mm})}{(800 \text{ mm})^2} = 5.31 \text{ N/mm}$$



8-80

$$\text{FROM P7-9: } I = 1.35 \times 10^5 \text{ mm}^4, C = 20.0 \text{ mm}$$

$$M_{\text{MAX}} = (2.25 \text{ kN} \times 0.20 \text{ m}) = 450 \text{ N}\cdot\text{m}$$

$$\sigma = \frac{M}{I} = \frac{(450 \text{ N}\cdot\text{m})(20.0 \text{ mm})}{1.35 \times 10^5 \text{ mm}^4} \cdot \frac{10^3 \text{ mm}}{\text{m}} = 66.7 \text{ MPa}$$

$$\text{REQ'D. FLEXURAL STRENGTH} = 3\sigma = 3(66.7) = 200 \text{ MPa}$$

COULD USE: NYLON 6/6, POLYESTER PET, POLYIMIDE, EPOXY

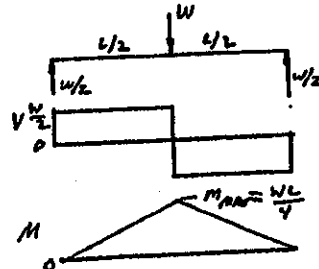
8-81

$$\text{FROM P7-5: } I = 2.66 \times 10^5 \text{ mm}^4, C = 35 \text{ mm}$$

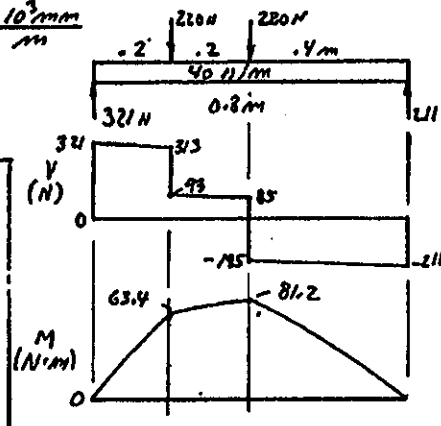
$$\sigma_B = \frac{S_u}{2} = \frac{97 \text{ MPa}}{2} = 48.5 \text{ MPa} = \frac{M_{\text{MAX}} C}{I}$$

$$M_{\text{MAX}} = \frac{\sigma_B I}{C} = \frac{48.5 \text{ MPa} \cdot 2.66 \times 10^5 \text{ mm}^4}{35 \text{ mm}} = 3.69 \times 10^6 \text{ N}\cdot\text{mm}$$

$$w_{\text{MAX}} = \frac{4 M_{\text{MAX}}}{L} = \frac{4(3.69 \times 10^6 \text{ N}\cdot\text{mm})}{1250 \text{ mm}} = 1180 \text{ N}$$



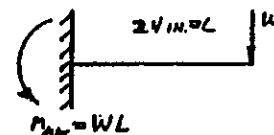
B-82 FROM P7-12:  $I = 1.7 \times 10^4 \text{ mm}^4$ ,  $C = 12.39 \text{ mm}$   
 $\sigma = \frac{M C}{I} = \frac{(81.2 \text{ N}\cdot\text{m})(12.39 \text{ mm})}{1.7 \times 10^4 \text{ mm}^4} = \frac{10^3 \text{ mm}}{\text{m}}$   
 $\sigma = 59.3 \text{ N/mm}^2 = 59.3 \text{ MPa}$   
 $N = \frac{S_c}{\sigma} = \frac{131 \text{ MPa}}{59.3 \text{ MPa}} = 2.21$



B-83 FROM P7-20:  $I = 0.8263 \text{ in}^4$ ,  $C = 1.28 \text{ in}$   
 $\sigma_b = \frac{S_c}{3} = \frac{17000 \text{ psi}}{3} = 5667 \text{ psi} = \frac{M_{\max} C}{I}$   
 $M_{\max} = \frac{\sigma_b I}{C} = \frac{5667 \text{ psi} \cdot 0.8263 \text{ in}^4}{1.28 \text{ in}} = 3658 \text{ lb}\cdot\text{in} = WL/4 \text{ (See 8-81)}$   
 $W_{\max} = \frac{4 M_{\max}}{L} = \frac{4(3658 \text{ lb}\cdot\text{in})}{14.0 \text{ in}} = 1045 \text{ lb}$

B-84 FROM P7-15:  $I = 0.3572 \text{ in}^4$ ,  $C = 1.332 \text{ in}$ ,  $L = 8 \text{ ft} \times 12 \text{ in/ft} = 96 \text{ in}$   
 $\sigma_b = S_y/2 = 21000 \text{ psi}/2 = 10500 \text{ psi} = \frac{M_{\max} C}{I}$   
 $M_{\max} = \frac{\sigma_b I}{C} = \frac{10500 \text{ psi} \cdot 0.3572 \text{ in}^4}{1.332 \text{ in}} = 2816 \text{ lb}\cdot\text{in} = \frac{WL^2}{8} \text{ (See 8-79)}$   
 $W_{\max} = \frac{8 M_{\max}}{L^2} = \frac{8(2816 \text{ lb}\cdot\text{in})}{(96 \text{ in})^2} = 2.44 \frac{\text{lb}}{\text{in}} \times \frac{12 \text{ in}}{\text{ft}} = 29.3 \text{ lb/ft}$

B-85 FROM P7-14:  $I = 0.3672 \text{ in}^4$ ,  $C = 1.167 \text{ in}$ ,  $\sigma_b = S_y/8 = \frac{62000 \text{ psi}}{8} = 7750 \text{ psi}$   
 $M_{\max} = \frac{\sigma_b I}{C} = \frac{7750 \text{ psi} \cdot 0.3672 \text{ in}^4}{1.167 \text{ in}} = 2439 \text{ lb}\cdot\text{in}$   
 $W_{\max} = \frac{M}{L} = \frac{2439 \text{ lb}\cdot\text{in}}{24 \text{ in}} = 102 \text{ lb}$



B-86 HAT SECTION FROM P7-14 WITHOUT LOWER PLATE

PART	A	$x_y$	$A x_y$	T	d	$A d^2$	$I + A d^2$	
1	.12	.05	.006					BOTTOM FLANGE (24.6 x .1)
2	.80	.75	.225					2 WEBS 2 x 1.5 x .1
3	.12	1.45	.174					TOP FLANGE (1.2 x .1)

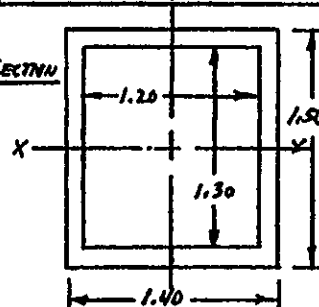
$\Sigma A = .54$ ,  $\Sigma A x_y = .405$

$\bar{y} = .405/.54 = 0.75 \text{ in}$  SYMMETRICAL  $\rightarrow$  EQUIVALENT SECTION

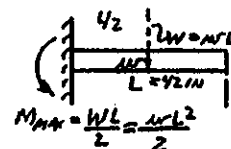
$I_x = \frac{1.40(1.50)^3}{12} - \frac{1.20(1.30)^3}{12} = 0.174 \text{ in}^4$

$M_{\max} = \frac{\sigma_b I}{C} = \frac{7750 \text{ psi} \cdot 0.174 \text{ in}^4}{0.75 \text{ in}} = 1798 \text{ lb}\cdot\text{in}$

$W_{\max} = \frac{M}{L} = \frac{1798 \text{ lb}\cdot\text{in}}{24 \text{ in}} = 75 \text{ lb}$



B-87 FROM P7-19:  $I = 1.2506 \text{ in}^4$ ,  $C = 1.570 \text{ in}$   
 $\sigma_b = S_y/6 = 45 \text{ ksi}/6 = 7.5 \text{ ksi} = 7500 \text{ psi}$   
 $M_{\max} = \frac{\sigma_b I}{C} = \frac{7500 \text{ psi} \cdot 1.2506 \text{ in}^4}{1.570 \text{ in}} = 5971 \text{ lb}\cdot\text{in}$   
 $W_{\max} = \frac{2 M_{\max}}{L^2} = \frac{2(5971 \text{ lb}\cdot\text{in})}{(42 \text{ in})^2} = 6.77 \text{ lb/in}$



B-88 FROM P7-20:  $I = 0.8263 \text{ in}^4$ ,  $C = 1.28 \text{ in}$ ,  $\sigma_b = \frac{S_y}{6} = \frac{33 \text{ ksi}}{6} = 5.50 \text{ ksi}$   
 $M_{\max} = \frac{\sigma_b I}{C} = \frac{5500 \text{ psi} \cdot 0.8263 \text{ in}^4}{1.28 \text{ in}} = 3551 \text{ lb}\cdot\text{in}$   
 $W = \frac{2 M_{\max}}{L^2} = \frac{2(3551 \text{ lb}\cdot\text{in})}{(42 \text{ in})^2} = 4.03 \text{ lb/in}$

BEAMS WITH STRESS CONCENTRATIONS  
AND VARYING CROSS SECTIONS

B-89

$M_A = \text{ALLOWABLE MOMENT AT JOINT}$

$\sigma = K_t M_x / S \quad (S = 2.39 \text{ in}^3 \text{ FOR } 3 \frac{1}{2} \text{ in PIPE})$

$$M_x = \frac{\sigma S}{K_t} = \frac{(20000 \text{ LB/IN}^2)(2.39 \text{ IN}^3)}{1.78} = 26900$$

$$r/d = \frac{0.25}{4.00} = 0.063 \quad \left. \begin{array}{l} \\ \end{array} \right\} K_t = 1.78$$

$$D/d = 4.50/4.00 = 1.125$$

$$M_A = 26900 \text{ LB/IN} (1 \text{ FT/12 IN}) = 2242 \text{ LB-FT}$$

$$\text{LET } M_x = -4900 + 700X = -2242$$

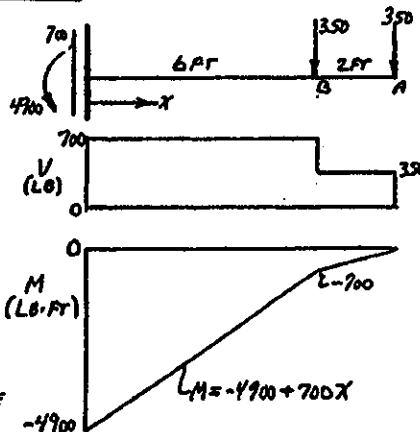
$$X = 2658/700 = 3.80 \text{ FT AT JOINT}$$

AT D AT WALL:  $S = 3.215 \text{ in}^3$  FOR 4 IN PIPE

$$\sigma = \frac{K_t M}{S} = \frac{(1.0)(4900 \text{ LB-FT})}{3.215 \text{ in}^3} \times \frac{12 \text{ IN}}{\text{FT}}$$

$$\sigma = 18300 \text{ PSI IF NO SIGNIFICANT } K_t \text{ EXISTS AT WALL.}$$

OK



B-90

$M = 0 \text{ AT A, E}$

POINT B IS CRITICAL

$$\sigma = \frac{M K_t}{S}$$

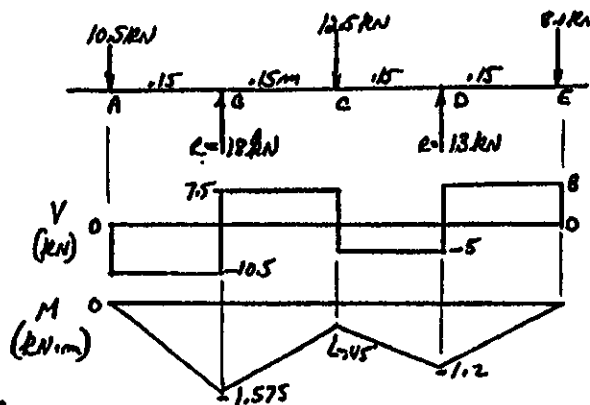
$$S = \frac{\pi (45)^3}{32} = 8946 \text{ mm}^3$$

FOR FILLET

$$\frac{r}{d} = \frac{2}{45} = 0.044 \quad \left. \begin{array}{l} \\ \end{array} \right\} K_t = 2.17$$

$$\frac{D}{d} = \frac{55}{45} = 1.22$$

$$\sigma = \frac{M K_t}{S} = \frac{1.575 \times 10^3 \text{ N}\cdot\text{m} (2.17) \times 10^3 \text{ mm}^3}{8946 \text{ mm}^3} = 382 \text{ MPa}$$



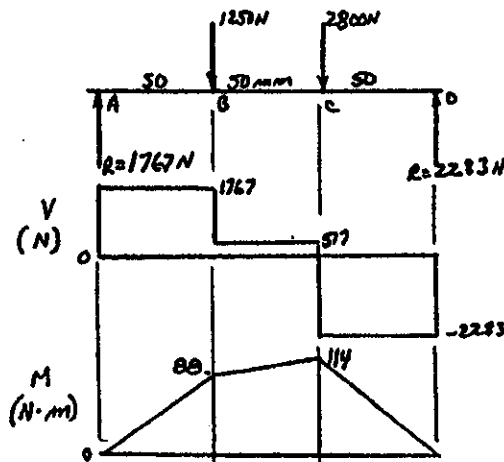
B-91

AT C

$$S = \frac{\pi (18)^3}{32} = 572.6 \text{ mm}^3$$

$$\sigma = \frac{M K_t}{S} = \frac{(114 \text{ N}\cdot\text{m})(2.0) \times 10^3 \text{ mm}^3}{572.6 \text{ mm}^3}$$

$$\sigma = 398 \text{ MPa}$$



8-92

FOR  $D_1$ :  $S = \pi D_1^3 / 32 = \pi (1.65)^3 / 32 = 0.03187 \text{ in}^3$

FOR  $D_2$ :  $S = \pi (1.00)^3 / 32 = 0.0982 \text{ in}^3$

FOR  $D_3$ :  $S = \pi (0.94)^3 / 32 = 0.09154 \text{ in}^3$

AT C:  $K_t = 2.0$  (KEYSEAM)

$\sigma = \frac{K_t \cdot M_c}{S} = \frac{2.0 (1535)}{0.0982} = 31270 \text{ psi}$

AT D:  $d/w = 0.94/1.00 = 0.94$ ;  $d/w = 1.00/0.94 = 1.06$

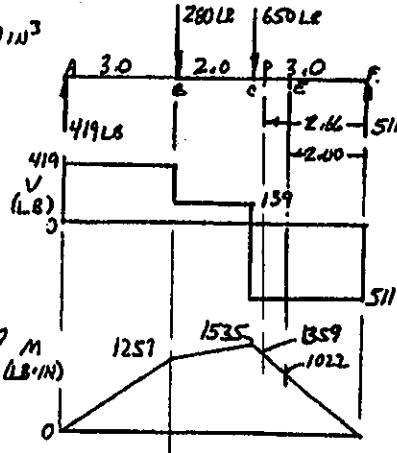
$K_t = 2.55$ ;  $M_D = 1359 \text{ LB-IN}$  (GROOVE)

$\sigma = \frac{(2.55)(1359)}{0.09154} = 42500 \text{ psi}$

AT E:  $d/w = 1.06/1.06 = 1.0$ ;  $d/w = 1.00/0.94 = 1.06$

$K_t = 1.78$ ;  $M_E = 1022 \text{ LB-IN}$  (STEP)

$\sigma = \frac{1.78 (1022)}{0.03087} = 58930 \text{ psi}$  HIGHEST



8-93

AT FULCRUM C:

$S = b h^3 / 6 = .75 (2.0)^3 / 6 = 0.50 \text{ in}^3$

$\sigma = \frac{M}{S} = \frac{4000 \text{ LB-IN}}{0.50 \text{ in}^3} = 8000 \text{ psi}$

AT HOLE B<sub>1</sub>:  $D_H = 0.75 = d$

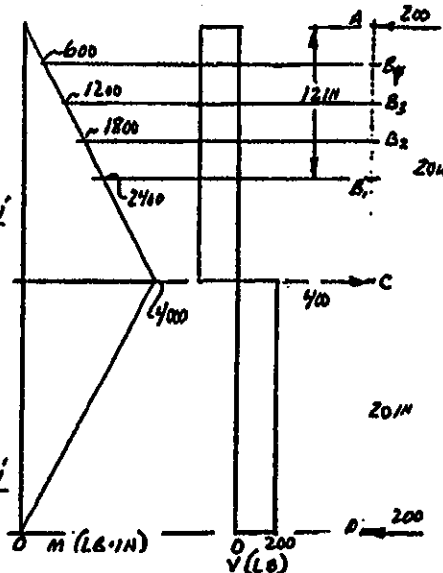
$d/w = .75/2.0 = 0.375$ ;  $K_t = 1.0$

$\sigma = \frac{K_t \cdot M_{B1}}{(w^3 - d^3)/6} = \frac{(1.0)(6)(2400)(2.0)}{(2.0^3 - .75^3)(.75)} = 5060 \text{ psi}$

AT B<sub>2</sub>:  $M = 1700$ ;  $\sigma = 3800 \text{ psi}$

AT B<sub>3</sub>:  $M = 1200$ ;  $\sigma = 2634 \text{ psi}$

AT B<sub>4</sub>:  $M = 600$ ;  $\sigma = 1267 \text{ psi}$



8-94

AT C:  $\sigma = 8000 \text{ psi}$  (FROM 8-93)

AT B<sub>1</sub>:  $d/w = 1.38/2.0 = 0.69$ ;  $K_t = 1.46$

$\sigma = \frac{K_t \cdot M_{B1}}{(w^3 - d^3)/6} = \frac{1.46(6)(2400)(2.0)}{(2.0^3 - 1.38^3)(.75)} = 10000 \text{ psi}$

AT B<sub>2</sub>:  $\sigma = 7506 \text{ psi}$ ; AT B<sub>3</sub>:  $\sigma = 5004 \text{ psi}$

AT B<sub>4</sub>:  $\sigma = 2500 \text{ psi}$

8-95

$d/w = 1.25/2.00 = 0.625$ ;  $K_t = 1.27$  AT HOLE

$\sigma_c = \frac{K_t \cdot M_{B1}}{(w^3 - d^3)/6} = \frac{1.27(6)(2400)}{(2.0^3 - 1.25^3)(.75)} \cdot M_{B1} = 3.36 M_{B1}$

AT FULCRUM C:

$S_c = 0.50 \text{ in}^3$  (SEE 8-93)

$\sigma_c = M_c / S_c = 2.0 M_c$

PIVOT	CD	$M_c$	$\sigma_c$	$R_A$	AB <sub>1</sub>	$M_{B1}$	$\sigma_{B1}$
a) END HOLE	20 IN.	4000 LB-IN.	8000 psi	200 LB	12 IN.	2400 LB-IN.	8064 psi
b) HOLE B <sub>4</sub>	17 IN.	3400	6800	170	9	1530	5141
c) HOLE B <sub>3</sub>	14 IN.	2800	5600	140	6	840	2822
d) HOLE B <sub>2</sub>	11 IN.	2200	4400	110	3	330	1109
e) HOLE B <sub>1</sub>	8 IN.	1600	3200	80	0	0	0

8-96

$M = F (52 + 25/2) = 2500 \text{ N} (64.5 \text{ mm}) = 161250 \text{ N-mm}$

AT A-A:  $S = b h^3 / 6 = 16 (25)^3 / 6 = 1667 \text{ mm}^3$

$\sigma = M / S = 161250 \text{ N-mm} / 1667 \text{ mm}^3 = 96.8 \text{ MPa}$

8-97

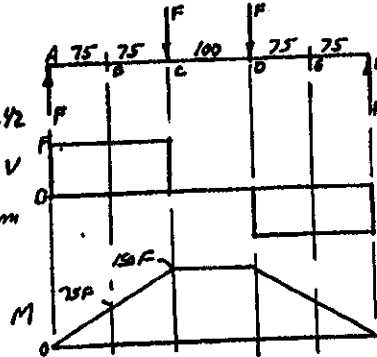
AT B-B:  $d/w = 12/25 = 0.48$ ;  $K_t = 1.0$

$\sigma = \frac{K_t \cdot M_{B1}}{(w^3 - d^3)/6} = \frac{1.0(6)(161250)(25)}{(25^3 - 12^3)(16)} = 108.8 \text{ MPa}$

8-98 AT B-B:  $d/w = 15/25 = 0.6 \rightarrow K_t = 1.21$   
 $\sigma = \frac{(1.21)(6)(161250)(2.5)}{(25^3 - 15^3)(16)} = 149.3 \text{ MPa}$

8-99 LET  $\sigma = \sigma_B = S_u/8$ ; REQ'D.  $S_u = 8\sigma = 8(149.3) = 1195 \text{ MPa}$   
 POSSIBLE STEEL: AISI 4140 OQT 900,  $S_u = 1289 \text{ MPa}$ ; 15% ELONG.

8-100 AT C:  $S = bh^2/6 = (12)(60)^2/6 = 7200 \text{ mm}^3$   
 AT B:  $S = 12(40)^2/6 = 3200 \text{ mm}^3$   
 $h/h_0 = 10/40 = 0.25$ ;  $H/h = 60/40 = 1.50 \rightarrow K_t = 1.42$   
 $\sigma_B = S_u/8 = 669/8 = 83.6 \text{ MPa} = \frac{K_t M}{S}$   
 $M_{MAX} = \frac{\sigma_B S}{K_t} = \frac{83.6 \text{ N/mm}^2 (3200 \text{ mm}^3)}{1.42} = 188450 \text{ N}\cdot\text{mm}$   
 $M_B = 75F$ ;  $F = \frac{M}{75} = \frac{188450 \text{ N}\cdot\text{mm}}{75 \text{ mm}} = 2513 \text{ N}$   
 AT C:  $M_{MAX} = \frac{(83.6)(7200)}{1.0} = 602100 \text{ N}\cdot\text{mm}$   
 $M_C = 150F$ ;  $F = \frac{M}{150} = \frac{602100}{150} = 4014 \text{ N}$



8-101 AT B:  $h/h_0 = 2.0/40 = 0.05$ ;  $H/h = 1.50$ ;  $K_t = 2.2$   
 $M_{MAX} = \frac{\sigma_B S}{K_t} = \frac{83.6 \text{ N/mm}^2 (3200 \text{ mm}^3)}{2.2} = 121636 = 75F$   
 $F = M_{MAX}/75 = 1622 \text{ N}$

8-102 LET DISTANCE FROM A TO B BE  $a$ . THEN  $M_B = Fa$ ; LET  $\sigma_B = \sigma_C$   
 $\sigma_B = \frac{K_t M_B}{S_B} = \frac{1.42(Fa)}{3200} = \frac{F \cdot a}{2254}$  (DATA FROM PROB 8-100)  
 $\sigma_C = \frac{K_t M_C}{S_C} = \frac{(1.0)(150F)}{7200} = \frac{F}{48}$   
 $\frac{Fa}{2254} = \frac{F}{48}$ ;  $a = \frac{2254}{48} = 47 \text{ mm}$

8-103  $\frac{K_t M_B}{S_B} = \frac{K_t M_C}{S_C}$ ;  $K_t B = \frac{M_C}{M_B} \cdot \frac{S_B}{S_C} \cdot K_t C = \frac{150F}{75F} \cdot \frac{3200}{7200} \cdot 1.0 = 0.889$   
 IMPOSSIBLE

8-104 AT B:  $S = 3200 \text{ mm}^3$ ;  $h/h_0 = 0.25$ ;  $H/h = 75/40 = 1.88$ ;  $K_t = 1.42$  SAME AS PROB 8-100  
 NO CHANGE IN LIMITING VALUE OF  $F = 2513 \text{ N}$

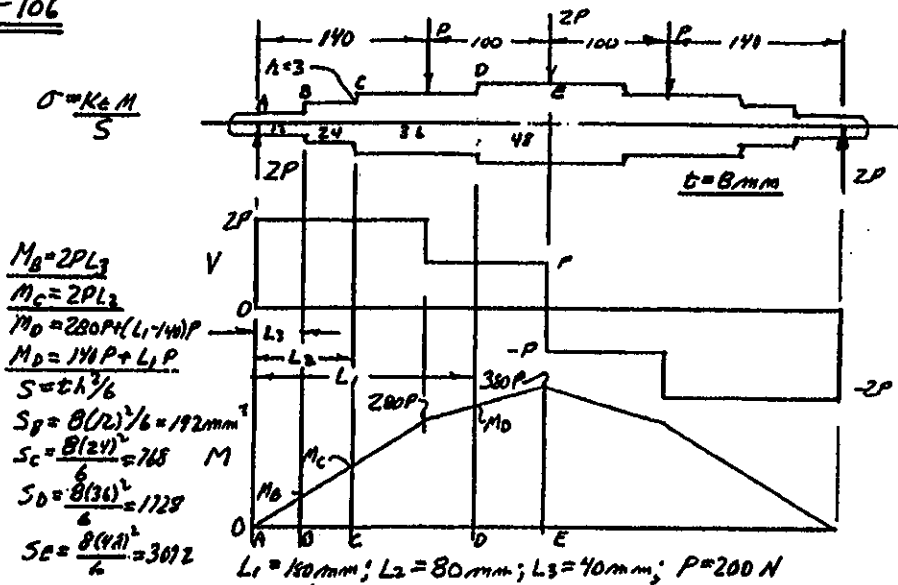
8-105 BECAUSE MAXIMUM STRESS OCCURS AT B, A HOLE CAN BE DRILLED AT C.

$\sigma = \sigma_B = 83.6 \text{ MPa} = \frac{K_t M}{(w^3 - d^3)K_t}$   
 $F = 2513 \text{ N}$  MAXIMUM FROM PROB 8-100. THEN  $M_C = 150F = 3.77 \times 10^5 \text{ N}\cdot\text{mm}$   
 $w = 60$ ,  $t = 12$ ; BUT IF  $d > 0.5w$ ,  $K_t > 1.0$  (APP. A21-4)

SOLVE FOR  $d$ :  
 $d = \left[ w^3 - \frac{K_t M}{\sigma_B t} \right]^{1/3} = \left[ 60^3 - \frac{(1.0)(3.77 \times 10^5)}{83.6(12)} \right]^{1/3}$   
 $d = [2.16 \times 10^5 - (1.35 \times 10^5)K_t]^{1/3}$

BY ITERATION: IF  $K_t = 1.0$ ,  $d = 43.3$ ; BUT  $d/w = 0.72$  AND  $K_t = 1.42$   
 $K_t = 1.30$ ,  $d = 34.3$ ;  $d/w = 0.57$ ,  $K_t = 1.14$   
 $K_t = 1.20$ ,  $d = 37.8$ ;  $d/w = 0.63$ ,  $K_t = 1.24$   
 $K_t = 1.22$ ,  $d = 37.2 \text{ mm}$ ;  $d/w = 0.62$ ,  $K_t = 1.22$  OK  
 MAXIMUM HOLE SIZE

B-106



POINT	M (N-mm)	S (mm <sup>3</sup> )	r/h	M/A	K <sub>t</sub>	$\sigma$ (MPa)
B	16000	192	.25	2.0	1.42	118.3 MAXIMUM
C	32000	768	.125	1.50	1.68	70.0
D	64000	1728	.083	1.33	1.82	67.4
E	76000	3072	-	-	1.0	24.7

B-107

LET  $\sigma = \sigma_d = S_u / B$

READ  $S_u = B\sigma = 8(118.3 \text{ MPa}) = 946 \text{ MPa}$  & AISI 1141 OR 900,  $S_u = 1017 \text{ MPa}$

ONE POSSIBLE CHOICE

B-108

DATA FROM B-106 EXCEPT:

AT B:  $r/h = 1.5/2 = 0.125 \rightarrow K_t = 1.75; \sigma_B = 145.8 \text{ MPa MAXIMUM}$

AT C:  $r/h = 1.5/24 = 0.063 \rightarrow K_t = 2.00; \sigma_C = 83.3 \text{ MPa}$

AT D:  $r/h = 1.5/36 = 0.042 \rightarrow K_t = 2.27; \sigma_D = 84.1 \text{ MPa}$

B-109

$P = 400 \text{ N}$ ,  $K_t$  AND  $S$  FROM PROB 8-106

$\sigma_B = S_u / B = 1170 \text{ MPa} / B = 146 \text{ MPa} = \frac{K_t M}{S}; M_{MAX} = \frac{\sigma_B S}{K_t}$

AT B:  $M_{B,MAX} = \frac{146 \text{ N} (192 \text{ mm}^3)}{\text{mm}^2 \cdot 1.42} = 19740 \text{ N-mm} = 2PL_3$

$L_3 = \frac{19740 \text{ N-mm}}{2(400 \text{ N})} = 24.7 \text{ mm}$

AT C:  $M_{C,MAX} = \frac{(146)(768)}{1.68} = 66742 \text{ N-mm} = 2PL_2$

$L_2 = \frac{66742}{2(400)} = 83.4 \text{ mm}$

AT D:  $M_{D,MAX} = \frac{(146)(1728)}{1.82} = 138620 \text{ N-mm} = P(L_4 + L_1)$

$L_1 = \frac{M_D}{P} - L_4 = \frac{138620}{400} - 140 = 206.5 \text{ mm}$

B-110

$$\sigma = S_u / B = 1014 \text{ MPa} / B = 126.8 \text{ MPa} = \frac{K_u M}{S} \text{ ; } K_u \text{ AND } S \text{ FROM B-106}$$

$$\text{AT B: } M_{\max} = \frac{\sigma S_u}{K_u} = \frac{126.8 \text{ N} \cdot \frac{192 \text{ mm}^3}{1.42}}{\text{mm}^2} = 17138 \text{ N} \cdot \text{mm} = 2PL_3$$

$$P_{\max} = \frac{M}{2L_3} = \frac{17138 \text{ N} \cdot \text{mm}}{2(40 \text{ mm})} = 214 \text{ N} \text{ GOVERNING VALUE}$$

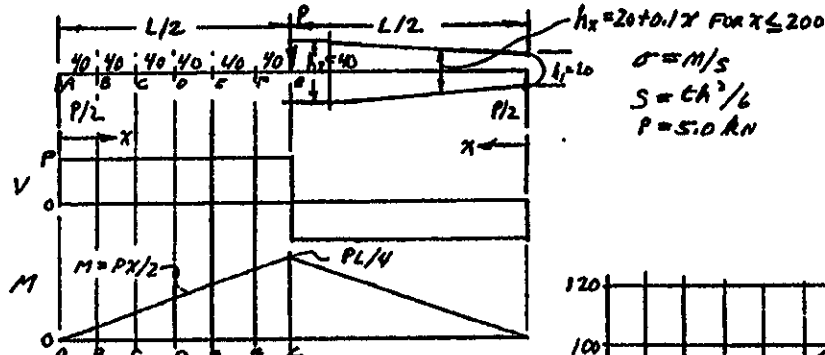
$$\text{AT C: } M_{\max} = \frac{(126.8)(1968)}{1.68} = 57943 \text{ N} \cdot \text{mm} = 2PL_2$$

$$P_{\max} = \frac{M}{2L_2} = \frac{57943}{2(80)} = 362 \text{ N}$$

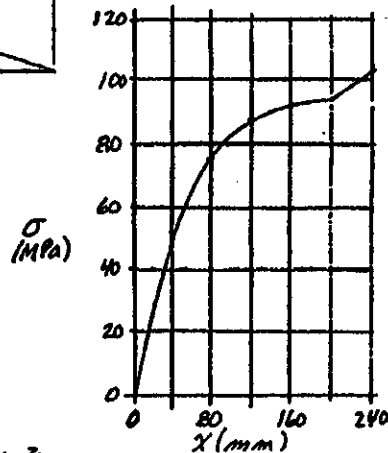
$$\text{AT D: } M_{\max} = \frac{(126.8)(1728)}{1.82} = 120342 \text{ N} \cdot \text{mm} = P(L_1 + 140)$$

$$P_{\max} = \frac{M}{L_1 + 140} = \frac{120342}{180 + 140} = 376 \text{ N}$$

B-111



x (m)	M (kN·m)	h (mm)	S (mm <sup>3</sup> )	σ (MPa)
0	0	20	1333	0
0.040	0.20P = 0.20	24	1920	52.1
0.080	0.40P = 0.20	28	2613	76.5
0.120	0.60P = 0.30	32	3413	87.9
0.160	0.80P = 0.40	36	4320	92.6
0.200	1.00P = 0.50	40	5333	93.8
0.240	1.20P = 0.60	40	5333	112.5



B-112

$$\sigma_u = \frac{K_u M}{S} = 131 \text{ MPa} / S = 32.75 \text{ MPa} = \frac{M}{S}$$

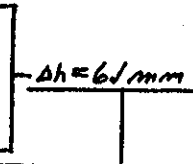
$$M = Px/2; h_x = 20 + 0.2x \text{ for } x \leq 200; S = \frac{th^3}{6}$$

x	h	S	M	$\sigma = \frac{M}{S}$	P for $\sigma = 32.75 \text{ MPa}$
0	20	1333	0	0	-
40	28	2613	20P	P/130.7	4280
80	36	4320	40P	P/108	3537
120	44	6453	60P	P/107.6	3524 - GOVERNING VALUE WITHIN TABLE
160	52	9013	80P	P/112.7	3691
200	60	12000	100P	P/120	3930
240	60	12000	120P	P/100	3275 MAXIMUM PERMISSIBLE



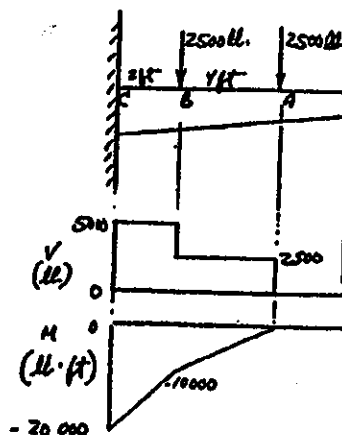
B-113

$\sigma_a = S_u/8 = 793 \text{ MPa}/8 = 99.1 \text{ MPa}; P = 1.20 \text{ kN} = 1200 \text{ N}$   
 BASED ON RESULTS OF PROBS. 111 AND 112,  $h_1$  IS CRITICAL AT MIDDLE  
 $M = PL/4 = (1200 \text{ N})(480 \text{ mm})/4 = 144000 \text{ N}\cdot\text{mm}$   
 $\text{REQD } S = \frac{M}{\sigma} = \frac{144000 \text{ N}\cdot\text{mm}}{99.1 \text{ N/mm}^2} = 1453 \text{ mm}^3 = t h^2/6$   
 $h_1 = \sqrt{\frac{6S}{t}} = \sqrt{\frac{6(1453 \text{ mm}^3)}{20 \text{ mm}}} = 20.9 \text{ mm}$   
 AT  $X=120 \text{ mm}$ ;  $M = 60P = 60(1200 \text{ N}) = 72000 \text{ N}\cdot\text{mm}$   
 $\text{REQD } S = \frac{M}{\sigma} = \frac{72000}{99.1} = 727 \text{ mm}^3$   
 $h_{120} = \sqrt{\frac{6(727)}{20}} = 14.8 \text{ mm}$   
 FOR LINEAR SIDES;  $h_2 = h_{120} - 6.1 = 8.7 \text{ mm}$   
 LET  $h_2 = 8 \text{ mm}$ ;  $h_1 = 22 \text{ mm}$  FOR CONVENIENT DIMENSIONS  
 A CHECK OF STRESS AS IN B-111 SHOWS  $\sigma < \sigma_a$  FOR ALL SECTIONS.



B-114

$\sigma_a = \frac{S_y}{4} = \frac{60000}{4} = 15000 \text{ psi}$   
 AT B,  $M = 10000 \text{ lb}\cdot\text{ft}$   
 $\text{REQD } S = \frac{M}{\sigma_a} = \frac{10000 \text{ lb}\cdot\text{ft}(12)}{15000 \text{ psi}} = 8.00 \text{ in}^3$   
 $S = \frac{b h^2}{6} = \frac{(1.5) h^2}{6} = 0.25 h^2$   
 $h = \sqrt{S/0.25} = \sqrt{8.00/0.25} = 5.66 \text{ in}$   
 AT C,  $M = 20000 \text{ lb}\cdot\text{ft}$   
 $S = \frac{M}{\sigma_a} = \frac{20000(12)}{15000} = 16.00 \text{ in}^3$   
 $h = \sqrt{S/0.25} = 8.00 \text{ in}$

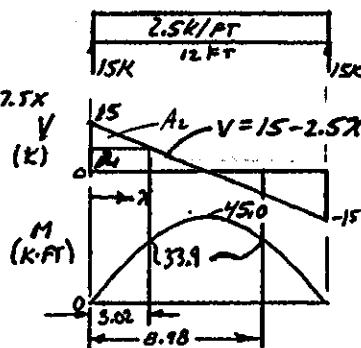


B-115

$\sigma_a = 0.66 S_v = 0.66(36000 \text{ psi}) = 23760 \text{ psi}$   
 $M_{\text{MAX}} = wL^2/8 = (2.5 \text{ k/ft})(12 \text{ ft})^2/8 = 45.0 \text{ k}\cdot\text{ft} (54000 \text{ lb}\cdot\text{in})$  (SEE B-79)  
 $\text{REQD. } S = \frac{M}{\sigma_a} = \frac{54000 \text{ lb}\cdot\text{in}}{23760 \text{ lb/in}^2} = 2.273 \text{ in}^3$  W14X26 ( $S = 25.3 \text{ in}^3$ )

B-116

FOR COMPOSITE SECTION:  $I = 103 + 2[(1.5)(25)(6.12)^2] = 168.5 \text{ in}^4$   
 $C = [1.99 + 2(0.25)]/2 = 6.245 \text{ in}$ ;  $S = I/C = 168.5/6.245 = 26.99 \text{ in}^3$  OK  
 FOR W12X16 ALONE:  $S = 17.1 \text{ in}^3$   
 $M_{\text{MAX}} = \sigma_a \cdot S = (23760 \text{ lb/in}^2)(17.1 \text{ in}^3) = 406796 \text{ lb}\cdot\text{in} \cdot \frac{1 \text{ k}}{1000 \text{ lb}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 33.9 \text{ k}\cdot\text{ft}$   
 FIND  $X$  WHERE  $M_x = 33.9 \text{ k}\cdot\text{ft}$   
 IN GENERAL:  $M_x = A_1 + A_2 = Vx + 0.5(15 - V)x$   
 $M_x = 15x + 7.5x - 0.5Vx = 0.5Vx + 22.5x$   
 $= 0.5(15 - 2.5x)x + 22.5x = 7.5x - 1.25x^2 + 22.5x$   
 $M_x = -1.25x^2 + 30x$   
 FOR  $M_x = 33.9$ ;  
 $-1.25x^2 + 30x - 33.9 = 0$   
 BY QUADRATIC EQUATION:  $x = 3.02 \text{ FT}$  OR  $8.98 \text{ FT}$  (K-FT)  
 EXTEND COVER PLATE OVER MIDDLE 6 FT  
 ALTERNATE DEVELOPMENT FOR  $M_x$ :  
 $M_x = \int V dx = \int (15 - 2.5x) dx = 15x - 1.25x^2$



8-117

COMPOSITE FIB P7-26:  $I = 469.4 \text{ in}^4$ ;  $C = 7.90 \text{ in}$

$$M_{\text{MAX}} = \frac{Q_0 I}{C} = \frac{(23760)(469.4)}{7.90 \text{ in}} = 1.41 \times 10^6 \text{ LB-IN (1 FT/12 IN)} = 117.6 \text{ K-FT} = \text{W-L}/8$$

$$W = \frac{8M}{L^2} = \frac{8(117.6 \text{ K-FT})}{(15 \text{ FT})^2} = 4.18 \text{ K/FT}$$

FOR S12 X50:  $S = 50.8 \text{ in}^3$ ;  $M_{\text{MAX}} = Q_0 S = (23760)(50.8) = 1.207 \times 10^6 \text{ LB-IN} = 100.6 \text{ K-FT}$

$$W = \frac{8M}{L^2} = \frac{8(100.6 \text{ K-FT})}{(15 \text{ FT})^2} = 3.58 \text{ K/FT}$$

### FLEXURAL CENTER

8-118

$$e = \frac{b^2 h^2 t}{4 I_x} : I_x = \frac{38(76)^3}{12} - \frac{34(68)^3}{12} = 4.992 \times 10^5 \text{ mm}^4$$

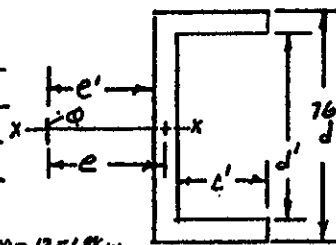
$$b = 38 - 2 = 36 \text{ mm}; h = 76 - 4 = 72 \text{ mm}; t = 4 \text{ mm}$$

$$e = \frac{(36^2)(72)(4)}{4(4.992 \times 10^5)} = 13.5 \text{ mm FROM MIDDLE OF WEB}$$

$Q$  IS AT  $e - z = 11.5 \text{ mm}$  FROM LEFT FACE OF WEB.

8-119

t	C'	d'	$I_x$	b	h	$e(\text{mm})$	$e' = e - \frac{t}{2}$
.50	37.5	75	71731	37.75	75.5	14.16	13.91
1.60	36.4	76.8	219745	37.20	74.4	13.94	13.14
3.00	35.0	70.0	389674	36.50	73.0	13.66	12.16



8-120

$$h = 2.00 - 0.12 = 1.88 \text{ in}; C = .50 + \frac{12}{2} = 0.56 \text{ in}; b = 2.00 - .12 = 1.88 \text{ in}$$

$$b/h = 1.00; c/h = 0.298; e/h = 0.46$$

$$e = 0.46 h = 0.46(1.88) = 0.865 \text{ in}$$

$$e' = e - \frac{t}{2} = .865 - .06 = 0.805 \text{ in FROM LEFT FACE}$$

$$I_x = \frac{C d^3}{12} - \frac{C' d'^3}{12}$$

8-121

t	h	C	b	c/h	b/h	e/h	e	e'
.020	1.980	.500	1.980	.258	1.00	.48	.950	.940
.063	1.937	.5315	1.937	.274	1.00	.46	.891	.860
.125	1.875	.5625	1.875	.300	1.00	.45	.844	.781

ALL DIMENSIONS  
IN INCHES

8-122

$$h = 80 - 3 = 77 \text{ mm}; C = 20 - 1.5 = 18.5 \text{ mm}; b = 50 - 3 = 47 \text{ mm}$$

$$b/h = 47/77 = 0.61; c/h = 18.5/77 = 0.24; \text{ THEN } e/h = 0.35$$

$$e = 0.35 h = 0.35(77) = 27.0 \text{ mm}$$

$$e' = e - \frac{t}{2} = 27.0 - 1.50 = 25.5 \text{ mm FROM LEFT FACE}$$

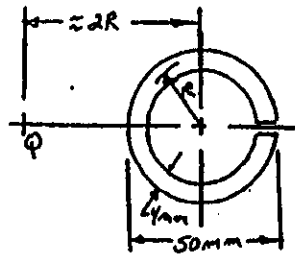
8-123

t	h	C	b	c/h	b/h	e/h	e	e'
0.50	77.5	19.75	47.5	0.248	0.623	.38	30.2	30.0
1.60	78.4	19.20	48.4	0.245	0.617	.37	29.0	28.2
3.00	77.0	18.5	47.0	0.24	0.61	0.35	27.0	25.5

8-124

$$R = \frac{D}{2} - \frac{t}{2} = 25 - 2 = 23 \text{ mm}$$

Q IS AT  $2R = 46 \text{ mm}$  FROM  
CENTER



8-125

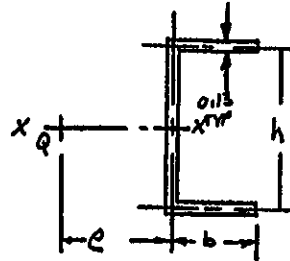
$$I_x = 0.288 \text{ in}^4 \text{ (FROM APPENDIX)}$$

OVERALL DIMENSIONS 2.00 IN. DEPTH = D  
1.00 IN. WIDTH = B

$$h = D - t = 2.00 - 0.13 = 1.87 \text{ in}$$

$$b = B - \frac{t}{2} = 1.00 - \frac{0.13}{2} = 0.935 \text{ in}$$

$$e = \frac{b^3 h^2 t}{4 I_x} = \frac{(0.935)^2 (1.87)^2 (0.13)}{4 (0.288)} \text{ in} = 0.345 \text{ in}$$



8-126

$$h = 25 + 5 = 30 \text{ mm}; C = 20 - 2.5 = 17.5 \text{ mm}; b = 45 - 5 = 40 \text{ mm}$$

$$c/h = 0.58; b/h = 1.33; \text{ THEN } e/h \approx 0.43 \text{ BY EXTRAPOLATION}$$

$$e = 0.43 h = 0.43 (30) = 12.9 \text{ mm}$$

### BEAMS MADE FROM ANISOTROPIC MATERIALS

8-127

FROM P7-15:  $I = 0.3572 \text{ in}^4$ ;  $C_b = 1.068 \text{ in}$ ;  $C_t = 1.332 \text{ in}$ ;  $L = 6 \text{ SFT} = 72 \text{ in}$

$$\text{FOR TENSION ON BOTTOM: } M_{\text{MAX}} = \frac{Q I}{C_b} = \frac{19000 \text{ LB} \cdot 0.3572 \text{ in}^4}{1.068 \text{ in}} = 6355 \text{ LB} \cdot \text{in}$$

$$\text{FOR COMP. ON TOP: } M_{\text{MAX}} = \frac{Q I}{C_t} = \frac{19000 \text{ LB} \cdot 0.3572 \text{ in}^4}{1.332 \text{ in}} = 5095 \text{ LB} \cdot \text{in}$$

FROM 8-47:  $M_{\text{MAX}} = w L^2 / 8$

$$w_{\text{MAX}} = \frac{8 M_{\text{MAX}}}{L^2} = \frac{8 (3754 \text{ LB} \cdot \text{in})}{(72 \text{ in})^2} = 4.94 \text{ LB/in}$$

8-128

$$\text{TENSION AT BOTTOM: } M_{\text{MAX}} = \frac{Q I}{C} = \frac{(19000)(0.2572)}{1.332} = 5095 \text{ LB} \cdot \text{in}$$

$$\text{COMP. ON TOP: } M_{\text{MAX}} = \frac{(21000)(0.3572)}{1.068} = 7024 \text{ LB} \cdot \text{in}$$

$$w_{\text{MAX}} = \frac{8 M}{L^2} = \frac{8 (5095)}{(72)^2} = 6.70 \text{ LB/in}$$

8-129

FROM P7-6:  $I = 6.167 \times 10^4 \text{ mm}^4$ ;  $C_b = 12.5 \text{ mm}$ ;  $C_t = 17.5 \text{ mm}$

$$\text{TENSION AT BOTTOM: } M_{\text{MAX}} = \frac{Q I}{C_b} = \frac{(100 \text{ N})(6.167 \times 10^4 \text{ mm}^4)}{12.5 \text{ mm}} = 4.93 \times 10^5 \text{ N} \cdot \text{mm}$$

$$\text{COMPRESSION AT TOP: } M_{\text{MAX}} = \frac{Q I}{C_t} = \frac{(70)(6.167 \times 10^4)}{17.5} = 2.47 \times 10^5 \text{ N} \cdot \text{mm}$$

$M_{\text{MAX}} = W L / 4$  (SEE 8-81)

LIMITING VALUE

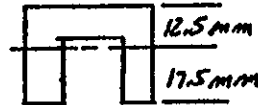
$$W_{\text{MAX}} = \frac{4 M_{\text{MAX}}}{L} = \frac{4 (2.47 \times 10^5 \text{ N} \cdot \text{mm})}{1200 \text{ mm}} = 822 \text{ N}$$

B-130FROM P7-6:  $I = 6.167 \times 10^4 \text{ mm}^4$  $C_b = 12.5 \text{ mm}$ ,  $C_t = 17.5 \text{ mm}$ COMPRESSION AT TOP:  $M_{\text{MAX}} = \frac{\sigma_c I}{C_b}$ 

$$M_{\text{MAX}} = \frac{70 \text{ N} (6.167 \times 10^4 \text{ mm}^4)}{\text{mm}^2 (12.5 \text{ mm})} = 3.45 \times 10^5 \text{ N} \cdot \text{mm} - \text{LIMITING VALUE}$$

$$\text{TENSION AT BOTTOM: } M_{\text{MAX}} = \frac{\sigma_t I}{C_t} = \frac{(100)(6.167 \times 10^4)}{17.5} = 3.52 \times 10^5 \text{ N} \cdot \text{mm}$$

$$W_{\text{MAX}} = \frac{4 M_{\text{MAX}}}{L} = \frac{4(3.45 \times 10^5 \text{ N} \cdot \text{mm})}{1200 \text{ mm}} = 1150 \text{ N}$$

B-131FROM P7-8:  $I = 5.36 \times 10^4 \text{ mm}^4$ ;  $C_b = C_t = 20 \text{ mm}$ 

$$\text{COMPR. AT TOP: } M_{\text{MAX}} = \frac{\sigma_c I}{C_b} = \frac{(70 \text{ N})(5.36 \times 10^4 \text{ mm}^4)}{\text{mm}^2 (20.0 \text{ mm})} = 1.876 \times 10^5 \text{ N} \cdot \text{mm}$$

$$\text{TENSION AT BOTTOM: } M_{\text{MAX}} = \frac{\sigma_t I}{C_t} = \frac{(100)(5.36 \times 10^4)}{20.0} = 2.68 \times 10^5 \text{ N} \cdot \text{mm}$$

$$W_{\text{MAX}} = \frac{4 M_{\text{MAX}}}{L} = \frac{4(1.876 \times 10^5 \text{ N} \cdot \text{mm})}{1200 \text{ mm}} = 625 \text{ N}$$

B-132FROM P7-9:  $I = 1.35 \times 10^5 \text{ mm}^4$ ;  $C_b = C_t = 20 \text{ mm}$ 

COMPRESSION AT TOP IS LIMITING

$$M_{\text{MAX}} = \frac{\sigma_c I}{C_b} = \frac{70 \text{ N} (1.35 \times 10^5 \text{ mm}^4)}{\text{mm}^2 (20 \text{ mm})} = 4.73 \times 10^5 \text{ N} \cdot \text{mm}$$

$$W_{\text{MAX}} = \frac{4 M_{\text{MAX}}}{L} = \frac{4(4.73 \times 10^5 \text{ N} \cdot \text{mm})}{1200 \text{ mm}} = 1575 \text{ N}$$

B-133FROM P7-4:  $I = 4.64 \times 10^7 \text{ mm}^4$ ;  $C_b = \bar{Y} = 152.5 \text{ mm}$ ;  $C_t = 72.5 \text{ mm}$ CI-ASTM A48, GR 40:  $\sigma_{bc} = \frac{S_{ut}}{4} = \frac{276 \text{ MPa}}{4} = 69 \text{ MPa}$ ;  $\sigma_{tc} = \frac{S_{ut}}{4} = \frac{276}{4} = 69 \text{ MPa}$ ;  $\sigma_{bc} = \frac{S_{ut}}{4} = \frac{276}{4} = 69 \text{ MPa}$ 

$$\sigma = \frac{M c}{I} : M_{\text{MAX}} = \frac{\sigma I}{C}$$

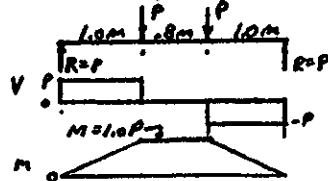
$$\text{COMPR. AT TOP: } M_{\text{MAX}} = \frac{\sigma_{bc} I}{C_b} = \frac{241 \text{ N} (4.64 \times 10^7 \text{ mm}^4)}{\text{mm}^2 (72.5 \text{ mm})}$$

$$M_{\text{MAX}} = 1.57 \times 10^8 \text{ N} \cdot \text{mm}$$

$$\text{TENS. AT BOTTOM: } M_{\text{MAX}} = \frac{\sigma_{tc} I}{C_t} = \frac{(69)(4.64 \times 10^7)}{152.5}$$

$$M_{\text{MAX}} = 2.10 \times 10^7 \text{ N} \cdot \text{mm} - \text{LIMITING VALUE} = (1.0 \text{ m})(P)$$

$$P_{\text{MAX}} = M_{\text{MAX}} / 1.0 \text{ m} = 2.10 \times 10^7 \text{ N} \cdot \text{mm} / 1000 \text{ mm} = 2.10 \times 10^4 \text{ N} = 21.0 \text{ kN}$$

B-134FROM P7-5:  $I = 2.66 \times 10^5 \text{ mm}^4$ ;  $C_b = \bar{Y} = 35.0 \text{ mm}$ ;  $C_t = 25.0 \text{ mm}$ ASTM A220, 000028:  $\sigma_{bc} = \frac{S_{ut}}{4} = \frac{657 \text{ MPa}}{4} = 164 \text{ MPa}$ ;  $\sigma_{tc} = \frac{S_{ut}}{4} = \frac{657}{4} = 164 \text{ MPa}$ 

$$\sigma = \frac{M c}{I} : M_{\text{MAX}} = \frac{\sigma I}{C}$$

$$\text{COMPR. AT TOP: } M_{\text{MAX}} = \frac{\sigma_{bc} I}{C_b} = \frac{413 \text{ N} (2.66 \times 10^5 \text{ mm}^4)}{\text{mm}^2 (25.0 \text{ mm})}$$

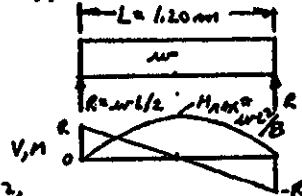
$$M_{\text{MAX}} = 4.39 \times 10^6 \text{ N} \cdot \text{mm}$$

$$\text{TENS. AT BOTTOM: } M_{\text{MAX}} = \frac{\sigma_{tc} I}{C_t} = \frac{(164)(2.66 \times 10^5)}{35.0}$$

$$M_{\text{MAX}} = 1.25 \times 10^6 \text{ N} \cdot \text{mm} - \text{LIMITING VALUE} = w L^2 / 8$$

$$w_{\text{MAX}} = \frac{8 M_{\text{MAX}}}{L^2} = \frac{8(1.25 \times 10^6 \text{ N} \cdot \text{mm})}{(1200 \text{ mm})^2} = 6.94 \text{ N} / \text{mm} = 6.94 \text{ kN} / \text{m}$$

$$\text{TOTAL LOAD} = w L = (6.94 \text{ kN} / \text{m})(1.20 \text{ m}) = 8.33 \text{ kN}$$



8-135

$I = 2.66 \times 10^5 \text{ mm}^4$ ;  $C_b = 25 \text{ mm}$ ;  $C_t = 35 \text{ mm}$  (FROM P7-5 UPPER PART)

COMPR. AT TOP:  $M_{MAX} = \frac{C_b I}{C_t} = \frac{(413)(2.66 \times 10^5)}{35} = 3.14 \times 10^6 \text{ N}\cdot\text{mm}$

TENS. AT BOTTOM:  $M_{MAX} = \frac{C_t I}{C_b} = \frac{(164)(2.66 \times 10^5)}{25} = 1.74 \times 10^6 \text{ N}\cdot\text{mm}$

$w = \frac{8M}{L^2} = \frac{8(1.74 \times 10^6) \text{ N}\cdot\text{mm}}{(1200 \text{ mm})^2} = 9.69 \text{ N/mm} = 9.69 \text{ kN/m}$

TOTAL LOAD =  $wL = (9.69 \text{ kN/m})(1.20 \text{ m}) = 11.63 \text{ kN}$

8-136

$\sigma_{dc} = S_y / f_o = 827 \text{ MPa} / f_o = 82.7 \text{ MPa}$ ;  $\sigma_{dc} = S_y / f_o = 1240 \text{ MPa} / f_o = 124 \text{ MPa}$

PART	A	$\eta$	$A\eta$	$I$	$d$	$A d^2$	$I + A d^2$	
1	18750	62.5	$1.17 \times 10^6$	$24.4 \times 10^6$	75	$105.5 \times 10^6$	$129.9 \times 10^6$	3 RIBS
2	56250	162.5	$9.14 \times 10^6$	$26.4 \times 10^6$	25	$35.16 \times 10^6$	$61.56 \times 10^6$	TOP
$\Sigma A = 75000$		$\Sigma A \eta = 10.31 \times 10^6$		$I = 191.4 \times 10^6 \text{ mm}^4$				

$\bar{Y} = 137.5 \text{ mm} = C_b$ ;  $C_t = (200 - \bar{Y}) = 62.5 \text{ mm}$

TENSION AT BOTTOM:  $M_{MAX} = \frac{C_b I}{C_t} = \frac{82.7 \text{ N} \cdot 191.4 \times 10^6 \text{ mm}^4}{62.5 \text{ mm}} = 115.1 \times 10^6 \text{ N}\cdot\text{mm}$

COMPR. AT TOP:  $M_{MAX} = \frac{C_t I}{C_b} = \frac{(124)(191.4 \times 10^6)}{162.5} = 379.8 \times 10^6 \text{ N}\cdot\text{mm}$

$M_{MAX} = 2.4 P$

$P_{MAX} = \frac{M_{MAX}}{2.4} = \frac{115.1 \times 10^6 \text{ N}\cdot\text{mm}}{2.4 \text{ m}(10^3 \text{ mm/m})} = 48.0 \text{ kN}$

8-137

INCREASE DEPTH OF RIBS TO 250 mm

THEN  $\bar{Y} = 222.5 \text{ mm} = C_b$ ;  $C_t = (325 - \bar{Y}) = 102.5 \text{ mm}$

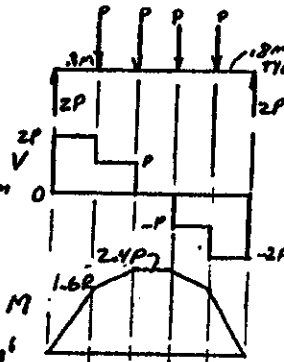
$I = 815.8 \times 10^6 \text{ mm}^4$

TENSION AT BOTTOM:  $M_{MAX} = \frac{(82.7)(815.8 \times 10^6)}{222.5}$

$M_{MAX} = 303.2 \times 10^6 \text{ N}\cdot\text{mm}$

COMPR. AT TOP:  $M_{MAX} = \frac{(124)(815.8 \times 10^6)}{102.5} = 986 \times 10^6$

$P_{MAX} = \frac{M_{MAX}}{2.4 \text{ m}} = \frac{303.2 \times 10^6 \text{ N}\cdot\text{mm}}{2400 \text{ mm}} = 126 \text{ kN}$



8-138

DESIGN PROBLEM - MULTIPLE SOLUTIONS POSSIBLE

8-139 ASTM A48 CAST IRON.  $S_u = 276 \text{ MPa}$  - BRITTLE

$$\sigma_d = S_u/6 = 276 \text{ MPa}/6 = 46.0 \text{ MPa}$$

$$\text{ACTUAL } \sigma_{\text{MAX}} = \frac{M}{S} = \frac{(2.4 \times 10^3 \text{ N})(350 \text{ mm})}{\pi(50 \text{ mm})^3/32} = 68.4 \text{ MPa} > \sigma_d$$

UNSAFE

8-140 FROM FIG. 8-15:  $M_{\text{MAX}} = 45900 \text{ LB-IN}$ .  $\sigma_d = S_u/8$  REPEATED LOAD

$$\text{AL. 6061-T6 } S_u = 45000 \text{ PSI}. \sigma_u = 45000 \text{ PSI}/8 = 5625 \text{ PSI}$$

$$\sigma = \frac{M}{S}. \text{REQ'D } S = \frac{M}{\sigma_d} = \frac{45900 \text{ LB-IN}}{5625 \text{ LB/IN}^2} = 8.16 \text{ IN}^3$$

SPECIFY 6 I x 4.692 ALUMINUM I-BEAM SHAPE.  $S = 8.50 \text{ IN}^3$

8-141 FIND MAX. STRESS. POINTS B OR D  
POSSIBLE SECTIONS

$$\text{AT B: } d = 40 \text{ mm}, M_B = 150000 \text{ N-mm}$$

$$\text{STEP: } d/D = 80/40 = 2.00 \quad \left. \begin{array}{l} K_t = 1.97 \\ r/d = 3.0/40 = 0.075 \end{array} \right\} \text{ APP A-21.9}$$

$$S_B = \frac{\pi d^3}{32} = \frac{\pi(40 \text{ mm})^3}{32} = 6283 \text{ mm}^3$$

$$\sigma_B = \frac{M_B \cdot K_t}{S_B} = \frac{(150000 \text{ N-mm})(1.97)}{6283 \text{ mm}^3} = 47.8 \text{ MPa}$$

$$\text{AT D: } d_g = 60 \text{ mm}, M_D = 300000 \text{ N-mm}$$

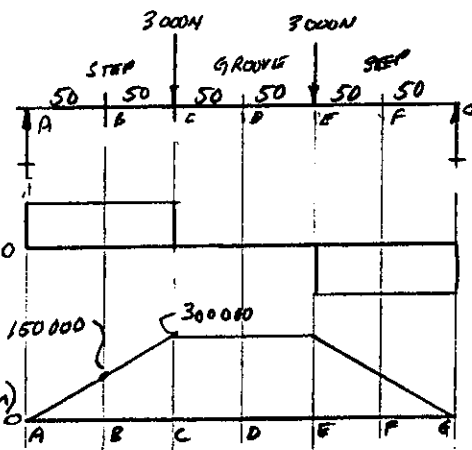
$$\text{GROOVE: } D/d_g = 80/60 = 1.33 \quad \left. \begin{array}{l} K_t = 2.03 \\ r/d_g = 6.0/60 = 0.10 \end{array} \right\} \text{ APP A-21.9}$$

$$S_D = \frac{\pi (d_g)^3}{32} = \frac{\pi(60 \text{ mm})^3}{32} = 21206 \text{ mm}^3$$

$$\sigma_D = \frac{M_D K_t}{S_D} = \frac{(300000 \text{ N-mm})(2.03)}{21206 \text{ mm}^3} = 28.7 \text{ MPa}$$

$$\sigma_{\text{MAX}} = \sigma_B = 47.8 \text{ MPa}$$

AT STEP AT B.



8-142 SHAFT  $D = 30.0 \text{ mm}$ ,  $M_{\text{MAX}} = 203 \text{ N-m}$

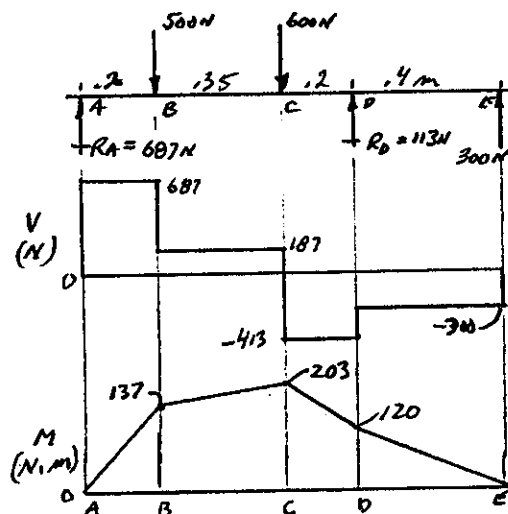
$$S = \frac{\pi D^3}{32} = \frac{\pi(30 \text{ mm})^3}{32} = 2651 \text{ mm}^3$$

$$\sigma = \frac{M}{S} = \frac{203 \text{ N-m}}{2651 \text{ mm}^3} \cdot \frac{10^3 \text{ mm}}{\text{m}} = 76.6 \text{ MPa}$$

$$\sigma_d = S_u/8 = 600 \text{ MPa}/8 = 75.0 \text{ MPa}$$

$$\text{AISI 1040 WQT 1300. } S_u = 600 \text{ MPa}$$

BECAUSE  $\sigma_{\text{MAX}} > \sigma_d$  - UNSAFE



8-143 ASTM A36,  $S_y = 36,000 \text{ psi}$ , STATIC LOAD

AISC:  $\sigma_a = 0.66 S_y = 0.66 (36,000) = 23,760 \text{ psi}$

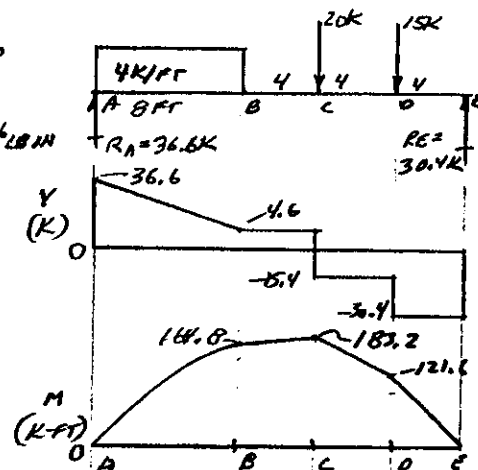
$$M_{\max} = 183.2 \text{ K-FT} \times 1000 \text{ LB/K} \times 12 \text{ IN/FT} = 2.20 \times 10^6 \text{ LB-IN}$$

$$\text{REQ'D } S = \frac{M_{\max}}{\sigma_a} = \frac{2.20 \times 10^6 \text{ LB-IN}}{23,760 \text{ LB/IN}^2}$$

$$S_{\min} = 92.5 \text{ IN}^3$$

SPECIFY  $W18 \times 55$ ;  $S = 98.3 \text{ IN}^3$

CHECK SHEAR STRESS IN WEB,  
CHECK LATERAL BRACING AND DEFLECTION



8-144 SAME AS 8-143 BUT ASTM A242

$S_y = 50,000 \text{ psi}$ ;  $\sigma_a = 0.66 S_y = 0.66 (50,000 \text{ psi}) = 33,000 \text{ psi}$

$$\text{REQ'D } S = \frac{M_{\max}}{\sigma_a} = \frac{2.20 \times 10^6 \text{ LB-IN}}{33,000 \text{ LB/IN}^2} = 66.7 \text{ IN}^3$$

SPECIFY  $W18 \times 40$ ;  $S = 68.4 \text{ IN}^3$ . LIGHTER BEAM SAVES 15 LB/FT

15 LB/FT  $\times$  20 FT = 300 LB OF STEEL.

BUT COST/LB MAY BE HIGHER. ALSO CHECK WEB SHEAR, DEFLECTION,  
AND LATERAL BRACING REQUIREMENTS FROM AISC SPECIFICATIONS.

8-145 ASTM A500 GRADE C:  $S_y = 46 \text{ ksi} = 317 \text{ MPa}$

AISC:  $\sigma_a = 0.66 S_y = 0.66 (317 \text{ MPa}) = 209 \text{ MPa}$

$M_{\max} = -12.16 \text{ KIN-M AT B}$

$$\text{REQ'D } S = \frac{M_{\max}}{\sigma_a} = \frac{12.16 \text{ KIN-M} \times 10^3 \text{ N} \times 10^3 \text{ mm}}{209 \text{ N/mm}^2 \times \text{mm}} \times \frac{\text{mm}}{\text{mm}}$$

$$S_{\min} = 5.82 \times 10^4 \text{ mm}^3$$

CONVERT TO  $\text{IN}^3$ :

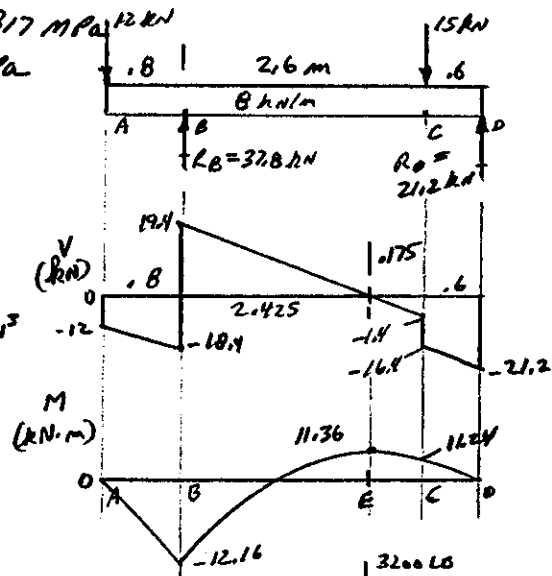
$$S_{\min} = 5.82 \times 10^4 \text{ mm}^3 \times \left( \frac{1 \text{ IN}}{25.4 \text{ mm}} \right)^3 = 3.55 \text{ IN}^3$$

SPECIFY EITHER:

$$4 \times 4 \times \frac{1}{4} ; S = 4.11 \text{ IN}^3$$

$$6 \times 2 \times \frac{1}{4} ; S = 4.60 \text{ IN}^3$$

BOTH WEIGH 12.2 LB/FT



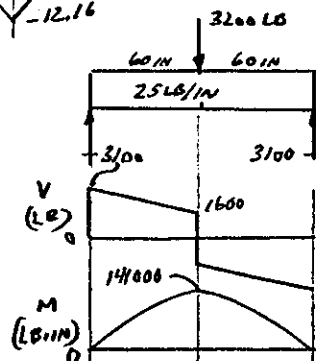
8-146 ASTM A510-C:  $S_y = 46 \text{ ksi}$

$\sigma_a = S_y/4 = 46 \text{ ksi}/4 = 11,500 \text{ psi}$

$$\text{REQ'D } S = \frac{M_{\max}}{\sigma_a} = \frac{141,000 \text{ LB-IN}}{11,500 \text{ LB/IN}^2} = 12.26 \text{ IN}^3$$

SPECIFY 8-IN SCHEDULE 40 STEEL PIPE

$$S = 16.81 \text{ IN}^3$$



8-147 ASTM A501:  $S_m = 58 \text{ ksi}$ ;  $\sigma_d = S_m/8 = 58/8 = 7.25 \text{ ksi}$

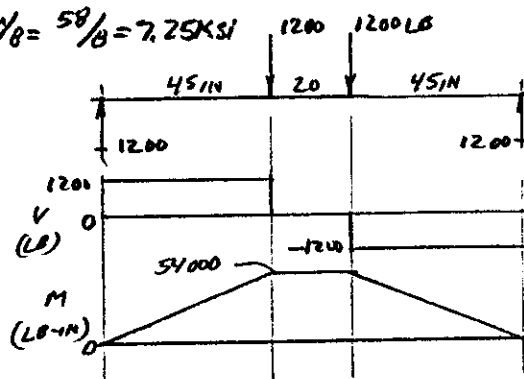
$$\text{REQ'D } S = \frac{M_{\max}}{\sigma_d} = \frac{54000 \text{ LB}\cdot\text{IN}}{7.25 \text{ LB/IN}^2}$$

$$S_{\min} = 7.45 \text{ IN}^3$$

SPECIFY  $8 \times 2 \times 1/4$  STEEL TUBE (HSS)

$$S = 7.52 \text{ IN}^3$$

$$W = 15.6 \text{ LB/FT}$$



8-148  $M_{\max} = 36F$  AT SUPPORT

ASTM A578 GRADE 60:  $S_m = 55 \text{ ksi}$

$$S_m = 170 \text{ ksi}$$

$$\sigma_{dt} = S_m/4 = 55000 \text{ PSI}/4 = 13750 \text{ PSI}$$

$$\sigma_{dc} = S_m/4 = 170000 \text{ PSI}/4 = 42500 \text{ PSI}$$

TOP IS IN TENSION:  $\sigma_t = \frac{M c_t}{I}$

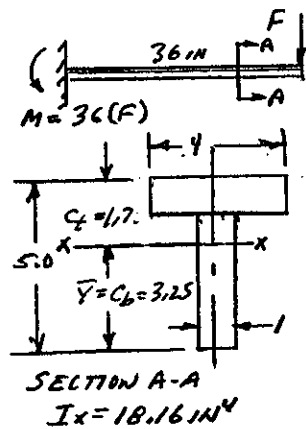
$$M_{\max} = \frac{\sigma_{dt} I}{c_t} = \frac{(13750 \text{ LB/IN}^2)(18.16 \text{ IN}^4)}{1.75 \text{ IN}}$$

$$M_{\max} = 142685 \text{ LB}\cdot\text{IN} = (36 \text{ IN})(F)$$

$$\frac{F_{\text{ALL}}}{36 \text{ IN}} = \frac{M_{\max}}{36 \text{ IN}} = \frac{142685 \text{ LB}\cdot\text{IN}}{36 \text{ IN}} = 3963 \text{ LB.}$$

BOTTOM IS IN COMPRESSION:  $\sigma_b = \frac{M c_b}{I}$  ANSWER

$$M_{\max} = \frac{\sigma_{dc} I}{c_b} = \frac{(42500)(18.16)}{3.25} = 237477 \text{ LB}\cdot\text{IN} < M_{\text{AT TOP}}$$



8-149 DESIGN PROBLEM. MANY POSSIBLE SOLUTIONS.

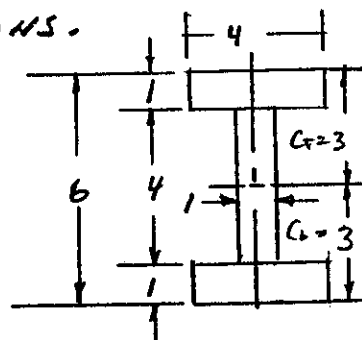
ONE DESIGN: MAKE SHAPE A FULL I-BEAM

$$\text{THEN } I = 4(6)^3/12 - 3(4)^3/12 = 56.0 \text{ IN}^4$$

$$S = \frac{I}{c} = \frac{56.0 \text{ IN}^4}{3 \text{ IN}} = 18.67 \text{ IN}^3 \text{ FOR BOTH TENSION AT TOP AND COMPRESSION AT BOTTOM}$$

$$\text{AT TOP: } \sigma = \frac{M}{S} = \frac{(6000 \text{ LB})(36 \text{ IN})}{18.67 \text{ IN}^3} = 11571 \text{ PSI} < \sigma_{dt} \text{ OK}$$

\$\sigma\_{\text{BOTTOM}}\$:



SHAPE MAY BE OPTIMIZED BY MAKING TOP FLANGE LARGER THAN BOTTOM FLANGE TO TAKE ADVANTAGE OF  $\sigma_{dc} \gg \sigma_{dt}$ . SEE EXAMPLE PROBLEM 8-10. TRIAL & ERROR SOLUTION REQ'D.



8-150 ASTM A36  $S_y = 36000 \text{ psi}$

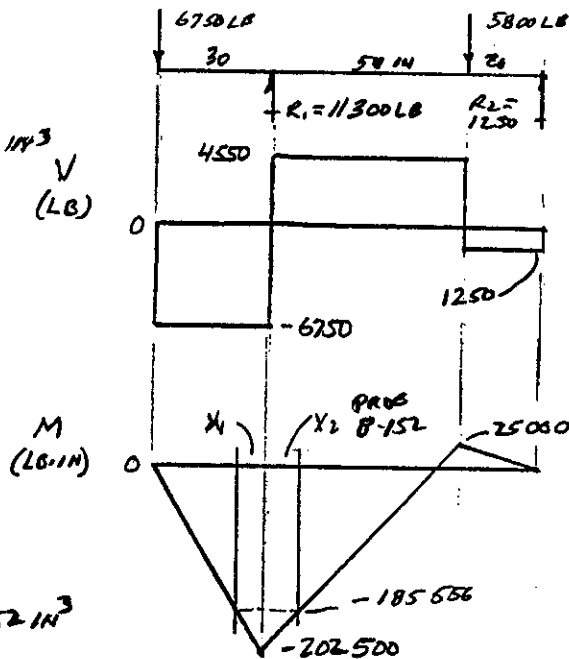
$$\sigma_a = 0.66(36000) = 23760 \text{ psi}$$

$$REQ'D S = \frac{M_{MAX}}{\sigma_a} = \frac{202500 \text{ LB-IN}}{23760 \text{ LB/IN}^2} = 8.52 \text{ IN}^3$$

$$\text{SPECIFY } W10 \times 12 \quad S = 10.9 \text{ IN}^3$$

CHECK BEAM FOR COMPACTNESS,  
WEB SHEAR, AND LATERAL  
SUPPORT REQUIRED.

$$\text{TOTAL WT} = \frac{12 \text{ LB}}{\text{FT}} \times \frac{1 \text{ FT}}{12 \text{ IN}} \times 100 \text{ IN} = 100 \text{ LB}$$



8-151 SHAPE IS IDENTICAL TO  
THAT IN PROBLEM 7-44.

$$I_x = 34.95 \text{ IN}^4$$

$$C_t = C_b = 3.50 \text{ IN}$$

$$S = I_x / C = 34.95 / 3.50 = 9.99 \text{ IN}^3 > 8.52 \text{ IN}^3$$

SAFE

$$6 \times 2 \times 1/4 \text{ TUBE WEIGHS } 12.2 \text{ LB/FT} \times 100 \text{ IN} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 101.7 \text{ LB.}$$

PLATES: VOL. IN 1.0 FT

$$2[(2(0.5) \text{ IN}^2) / 12 \text{ IN}] = 24 \text{ IN}^3$$

$$\text{WT} = 24 \text{ IN}^3 \times 6.283 \text{ LB/IN}^3 = 6.79 \text{ LB} ; 6.79 \text{ LB} \times 100 \text{ IN} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 56.6 \text{ LB}$$

$$\text{TOTAL WT} = 101.7 + 56.6 = 158.3 \text{ LB} \text{ MUCH HEAVIER THAN } W10 \times 12.$$

8-152 DESIGN PROBLEM - MANY POSSIBLE SOLUTIONS.

ONE POSSIBLE DESIGN: NOTE THAT BENDING MOMENT IS HIGH  
ONLY NEAR  $R_1$ . CONSIDER USING  $W8 \times 10$  BEAM WITH  $S = 7.8 \text{ IN}^3$   
AND ADDING COVER PLATES TO TOP AND/OR BOTTOM TO INCREASE  
SECTION MODULUS ONLY OVER THAT LENGTH WHERE REQUIRED  $S$  IS  
GREATER THAN  $7.8 \text{ IN}^3$ ,

$$\text{MAX. ALLOW. } M = \sigma_a \cdot S = (23760 \text{ LB/IN}^2)(7.8 \text{ IN}^3) = 185566 \text{ LB-IN}$$

$$\text{TO LEFT OF } R_1: X_1 = \frac{(202500 - 185566) \text{ LB-IN}}{6750 \text{ LB}} = 2.51 \text{ IN (SEE M-DIAGRAM)}$$

$$\text{TO RIGHT OF } R_1: X_2 = \frac{202500 - 185566}{4550 \text{ LB}} = 3.72 \text{ IN}$$

TOTAL LENGTH OF PLATES IS  $2.51 + 3.72 = 6.23 \text{ IN}$  QUITE SMALL  
USE  $1/4 \times 4.00 \text{ IN}$  A36 STEEL PLATES TOP AND BOTTOM FOR 6.50 INCHES

$I$  IS INCREASED FROM  $30.8 \text{ IN}^4$  TO  $63.92 \text{ IN}^4$

$S$  IS INCREASED FROM  $7.8 \text{ IN}^3$  TO  $15.24 \text{ IN}^3 > 8.52 \text{ IN}^3$  OK

WT OF BEAM IS  $10 \text{ LB/FT} \times (60/12) \text{ FT} = 83.3 \text{ LB}$

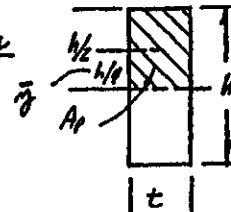
WT OF COVER PLATES IS 3.7 LB

TOTAL WT OF MODIFIED BEAM IS 87.0 LB. THIS IS 13 LB LESS  
THAN  $W10 \times 12$  BEAM,

# CHAPTER 9 Shearing Stresses in Beams

## GENERAL SHEAR FORMULA

$$\begin{aligned} \underline{9-1} \quad \tau &= \frac{VQ}{It} = \frac{(500 \text{ N})(2.5 \times 10^6 \text{ mm}^3)}{(33.3 \times 10^6 \text{ mm}^4)(50 \text{ mm})} = 1.125 \text{ N/mm}^2 = 1.125 \text{ MPa} \\ I &= \frac{bh^3}{12} = \frac{50(200)^3}{12} = 33.3 \times 10^6 \text{ mm}^4 \\ Q &= A_p \bar{y} = (100)(50)(50) = 2.5 \times 10^6 \text{ mm}^3 \end{aligned}$$



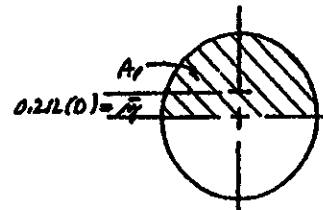
PROBLEMS 9-1  
THRU 9-4

$$\begin{aligned} \underline{9-2} \quad I &= (30 \times 180)^3 / 12 = 18.47 \times 10^6 \text{ mm}^4 \\ Q &= (90)(30)(45) = 1.539 \times 10^6 \text{ mm}^3 \\ \tau &= \frac{VQ}{It} = \frac{5000(1.539 \times 10^6)}{(18.47 \times 10^6)(30)} = 1.10 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \underline{9-3} \quad I &= (1.5)(7.25)^3 / 12 = 47.63 \text{ in}^4 ; Q = (3.625)(1.5)(1.813) = 9.86 \text{ in}^3 \\ \tau &= \frac{VQ}{It} = \frac{(12500 \text{ lb})(9.86 \text{ in}^3)}{(47.63 \text{ in}^4)(1.5 \text{ in})} = 1724 \text{ psi} \end{aligned}$$

$$\begin{aligned} \underline{9-4} \quad I &= (0.5)(11.25)^3 / 12 = 415 \text{ in}^4 ; Q = (5.625)(3.5)(2.813) = 55.37 \text{ in}^3 \\ \tau &= \frac{VQ}{It} = \frac{(20000 \text{ lb})(55.37 \text{ in}^3)}{(415 \text{ in}^4)(3.5 \text{ in})} = 762.0 \text{ psi} \end{aligned}$$

$$\begin{aligned} \underline{9-5} \quad I &= \pi D^4 / 64 = \pi (50^4) / 64 = 3.07 \times 10^5 \text{ mm}^4 \\ A_p &= \pi D^2 / 8 = \pi (50^2) / 8 = 982 \text{ mm}^2 \\ \bar{y} &= 0.212 D = 0.212(50) = 10.6 \text{ mm} \\ Q &= A_p \bar{y} = (982)(10.6) = 10407 \text{ mm}^3 \\ \tau &= \frac{VQ}{It} = \frac{(4500 \text{ N})(10407 \text{ mm}^3)}{(3.07 \times 10^5 \text{ mm}^4)(50 \text{ mm})} = 3.05 \text{ MPa} \end{aligned}$$



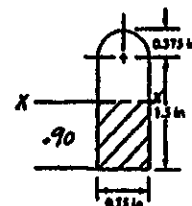
PROBLEMS 9-5  
THRU 9-8

$$\begin{aligned} \underline{9-6} \quad I &= \pi (38)^4 / 64 = 1.024 \times 10^5 \text{ mm}^4 ; Q = \frac{\pi (38)^2}{8} \cdot 0.212(38) = 4568 \text{ mm}^3 \\ \tau &= \frac{(2500 \text{ N})(4568 \text{ mm}^3)}{(1.024 \times 10^5 \text{ mm}^4)(38 \text{ mm})} = 2.94 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \underline{9-7} \quad I &= \pi (2.0)^4 / 64 = 0.785 \text{ in}^4 ; Q = [\pi (2.0)^2 / 8] [0.212(2.0)] = 0.666 \text{ in}^3 \\ \tau &= \frac{(7500 \text{ lb})(0.666 \text{ in}^3)}{(0.785 \text{ in}^4)(2.0 \text{ in})} = 3180 \text{ psi} \end{aligned}$$

$$\begin{aligned} \underline{9-8} \quad I &= \pi (0.63)^4 / 64 = 0.00773 \text{ in}^4 ; Q = [\pi (0.63)^2 / 8] [0.212(0.63)] = 0.0208 \text{ in}^3 \\ \tau &= \frac{VQ}{It} = \frac{(850 \text{ lb})(0.0208 \text{ in}^3)}{(0.00773 \text{ in}^4)(0.63 \text{ in})} = 3632 \text{ psi} \end{aligned}$$

$$\begin{aligned} \underline{9-9} \quad I_x &= 0.366 \text{ in}^4 ; \bar{y} = 0.90 \text{ in FROM P7-16.} \\ Q &= (0.75)(0.90)(0.45) = 0.304 \text{ in}^3 \\ \tau &= \frac{VQ}{It} = \frac{(9500 \text{ lb})(0.304 \text{ in}^3)}{(0.366 \text{ in}^4)(0.75 \text{ in})} = 1661 \text{ psi} \end{aligned}$$



8-150 ASTM A36  $S_y = 36000 \text{ psi}$

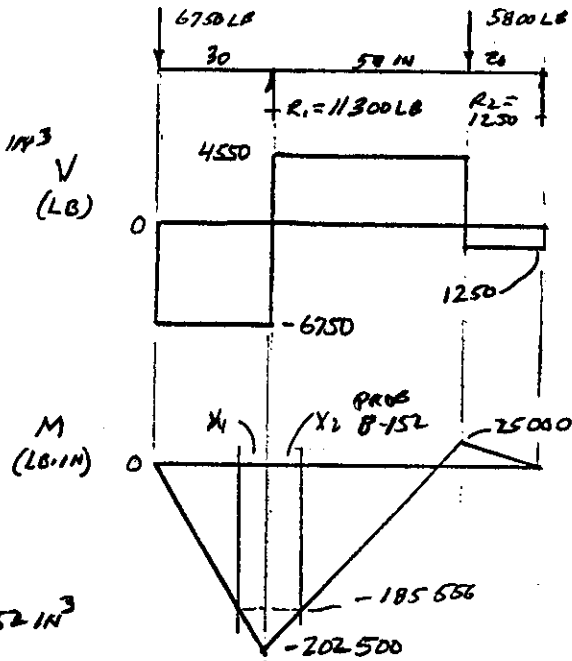
$$\sigma_b = 0.66(36000) = 23760 \text{ psi}$$

$$\text{REQ'D } S = \frac{M_{\text{MAX}}}{\sigma_b} = \frac{202500 \text{ LB-IN}}{23760 \text{ LB/IN}^2} = 8.52 \text{ IN}^3 \quad \checkmark$$

$$\text{SPECIFY } W10 \times 12 \quad S = 10.9 \text{ IN}^3$$

CHECK BEAM FOR COMPACTNESS,  
WEB SHEAR, AND LATERAL  
SUPPORT REQUIRED.

$$\text{TOTAL WT} = \frac{12 \text{ LB}}{\text{FT}} \times \frac{1 \text{ FT}}{12 \text{ IN}} \times 100 \text{ IN} = 100 \text{ LB}$$



8-151 SHAPE IS IDENTICAL TO  
THAT IN PROBLEM 7-44.

$$I_x = 34.95 \text{ IN}^4$$

$$C_t = C_b = 3.50 \text{ IN}$$

$$S = I/C = 34.95/3.50 = 9.99 \text{ IN}^3 > 8.52 \text{ IN}^3$$

SAFE

$$6 \times 2 \times 1/4 \text{ TUBE WEIGHS } 12.2 \text{ LB/FT} \times 100 \text{ IN} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 101.7 \text{ LB.}$$

PLATES: VOL. IN 1.0 FT

$$2[(2(0.5) \text{ IN}^2)(12 \text{ IN})] = 24 \text{ IN}^3$$

$$\text{WT} = 24 \text{ IN}^3 \times 6.283 \text{ LB/IN}^3 = 6.79 \text{ LB} ; 6.79 \text{ LB} \times 100 \text{ IN} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 56.6 \text{ LB}$$

$$\text{TOTAL WT} = 101.7 + 56.6 = 158.3 \text{ LB} \text{ MUCH HEAVIER THAN } W10 \times 12.$$

8-152 DESIGN PROBLEM - MANY POSSIBLE SOLUTIONS.

ONE POSSIBLE DESIGN: NOTE THAT BENDING MOMENT IS HIGH  
ONLY NEAR  $R_1$ . CONSIDER USING  $W8 \times 10$  BEAM WITH  $S = 7.8 \text{ IN}^3$   
AND ADDING COVER PLATES TO TOP AND/OR BOTTOM TO INCREASE  
SECTION MODULUS ONLY OVER THAT LENGTH WHERE REQUIRED  $S$  IS  
GREATER THAN  $7.8 \text{ IN}^3$ .

$$\text{MAX. ALLOW. } M = \sigma_b \cdot S = (23760 \text{ LB/IN}^2)(7.8 \text{ IN}^3) = 185568 \text{ LB-IN}$$

$$\text{TO LEFT OF } R_1: X_1 = \frac{(202500 - 185568) \text{ LB-IN}}{6750 \text{ LB}} = 2.51 \text{ IN (SEE M-DIAGRAM)}$$

$$\text{TO RIGHT OF } R_1: X_2 = \frac{202500 - 185568}{4550 \text{ LB}} = 3.72 \text{ IN}$$

TOTAL LENGTH OF PLATES IS  $2.51 + 3.72 = 6.23 \text{ IN}$  QUITE SMALL  
USE  $1/4 \times 4.00 \text{ IN}$  A36 STEEL PLATES TOP AND BOTTOM FOR 6.50 INCHES

$I$  IS INCREASED FROM  $30.8 \text{ IN}^4$  TO  $63.92 \text{ IN}^4$

$S$  IS INCREASED FROM  $7.8 \text{ IN}^3$  TO  $15.24 \text{ IN}^3 > 8.52 \text{ IN}^3$  OK

WT OF BEAM IS  $10 \text{ LB/FT} \times (100/12) \text{ FT} = 83.3 \text{ LB}$

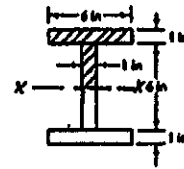
WT OF COVER PLATES IS  $3.7 \text{ LB}$

TOTAL WT OF MODIFIED BEAM IS  $87.0 \text{ LB}$ . THIS IS  $13 \text{ LB}$  LESS  
THAN  $W10 \times 12$  BEAM,

9-10  $I_x = 166 \text{ in}^4$  FROM P7-2.

$Q = A_1 y_1 + A_2 y_2 = 3(1.5) + 6(3.5) = 25.5 \text{ in}^3$

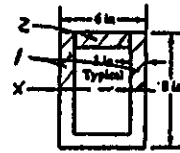
$\tau = \frac{VQ}{It} = \frac{(250)(25.5)}{(66)(1.0)} = 131 \text{ psi}$



9-11  $I_x = 184 \text{ in}^4$  FROM P7-3 :  $t = 2(1.0 \text{ in}) = 2.0 \text{ in}$

$Q = A_1 y_1 + A_2 y_2 = (2 \times 4)(2) + (4)(3.5) = 30.0 \text{ in}^3$

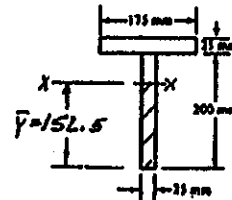
$\tau = \frac{(250)(30.0)}{(184)(2.0)} = 69.3 \text{ psi}$



9-12  $I_x = 4.64 \times 10^7 \text{ mm}^4$  FROM P7-4

$Q = A_1 \bar{y}_1 = (25 \times 152.5)(76.25) = 2.907 \times 10^5 \text{ mm}^3$

$\tau = \frac{(112 \times 10^3 \text{ N})(2.907 \times 10^5 \text{ mm}^3)}{(4.64 \times 10^7 \text{ mm}^4)(25 \text{ mm})} = 28.1 \text{ MPa}$

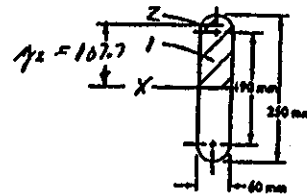


9-13  $I_x = 6.73 \times 10^7 \text{ mm}^4$  FROM P7-17

$Q = A_1 y_1 + A_2 y_2 = (60)(75)(47.5) + \frac{\pi(60)^2}{8}(107.7)$

$Q = 4.23 \times 10^5 \text{ mm}^3$

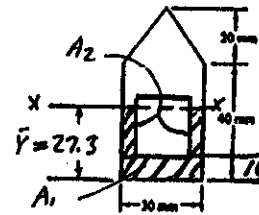
$\tau = \frac{(71.2 \times 10^3 \text{ N})(4.23 \times 10^5 \text{ mm}^3)}{(6.73 \times 10^7 \text{ mm}^4)(60)} = 7.46 \text{ MPa}$



9-14  $I_x = 3.08 \times 10^5 \text{ mm}^4$  FROM P7-18.  $t = 10 \text{ mm}$

$Q = A_1 \bar{y}_1 + A_2 \bar{y}_2 = (300)(22.3) + (172)(8.65) = 8186 \text{ mm}^3$

$\tau = \frac{(1780)(8186)}{(3.08 \times 10^5)(10)} = 4.73 \text{ MPa}$

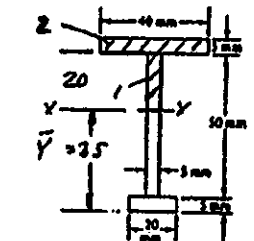


9-15  $I_x = 2.66 \times 10^5 \text{ mm}^4$  FROM P7-5

$Q = A_1 y_1 + A_2 y_2 = (20 \times 5)(10) + (40 \times 5)(22.5)$

$Q = 5500 \text{ mm}^3$

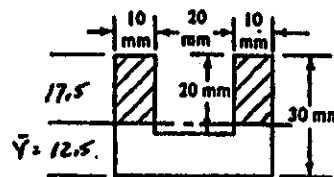
$\tau = \frac{(675)(5500)}{(2.66 \times 10^5)(5)} = 2.71 \text{ MPa}$



9-16  $I_x = 6.167 \times 10^4 \text{ mm}^4$  FROM P7-6

$Q = A_1 y_1 = (17.5 \times 20)(17.5/2) = 3063 \text{ mm}^3$

$\tau = \frac{(2500 \text{ N})(3063 \text{ mm}^3)}{(6.167 \times 10^4 \text{ mm}^4)(20 \text{ mm})} = 6.21 \text{ MPa}$



9-17  $I_x = 5.36 \times 10^4 \text{ mm}^4$  FROM P7-B.

$Q = A_1 y_1 + A_2 y_2$

$Q = (2)(5)(20)(12) + (30)(2.5)(12.5)$

$Q = 2094 \text{ mm}^3$

$\tau = \frac{VQ}{It} = \frac{(10500 \text{ N})(2094 \text{ mm}^3)}{(5.36 \times 10^4 \text{ mm}^4)(40 \text{ mm})} = 10.3 \text{ MPa}$

NOTE:  $\tau_{\text{max}} = 30.0 \text{ MPa}$  JUST ABOVE AREA WEB  
WHERE  $Q = 1531 \text{ mm}^3$  AND  $t = 10 \text{ mm}$ .

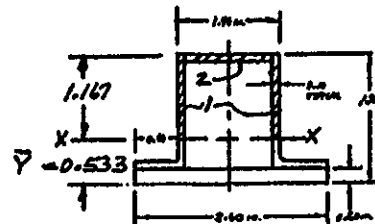
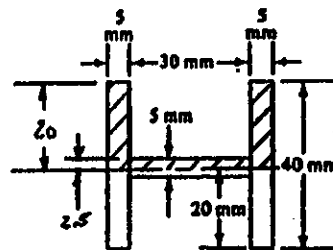
9-18  $I_x = 0.3672 \text{ in}^4$  FROM P7-14

$Q = A_1 y_1 + A_2 y_2$

$Q = (2)(1.0)(1.167)(1.167/2) + (0.1)(1.2)(1.117)$

$Q = 0.270 \text{ in}^3$

$\tau = \frac{(1200)(0.270)}{(0.3672)(0.20)} = 4416 \text{ psi}$

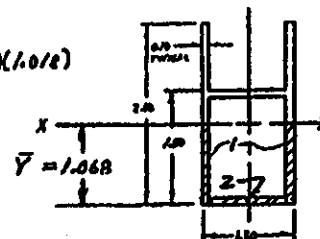


9-19  $I_x = 0.3572 \text{ in}^4$  FROM P7-15

$Q = A_1 y_1 + A_2 y_2 = (2)(1.0)(1.068)(1.068/2) + (0.1)(1.0)(1.0/2)$

$Q = 0.2159 \text{ in}^3$

$\tau = \frac{(775)(0.2159)}{(0.3572)(0.20)} = 2342 \text{ psi}$



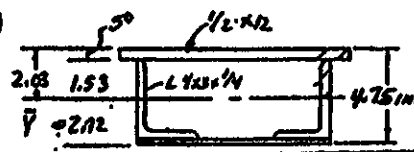
9-20  $I_x = 46.64 \text{ in}^4$  FROM P7-33.

$Q = A_1 y_1 + A_2 y_2$

$Q = (2)(0.25)(1.53)(1.53/2) + (0.5)(1.2)(1.10)$

$Q = 11.21 \text{ in}^3$

$\tau = \frac{(2500)(11.21)}{(46.64)(0.50)} = 1208 \text{ psi}$



PROBLEMS 9-21 THRU 9-30: GIVEN  $\tau = 70 \text{ psi} = \frac{VQ}{It}$

$V = \frac{\tau I t}{Q}$

9-21  $t = 1.50 \text{ in}$ ;  $h = 3.50 \text{ in}$ ;  $I = t^3/12 = 5.36 \text{ in}^4$ ;  $Q = t(h/2)(h/2) = 2.297 \text{ in}^3$

$V = \frac{(70)(5.36)(1.50)}{2.297} = 245 \text{ LB}$

9-22  $t = 3.5 \text{ in}$ ;  $h = 1.50 \text{ in}$ ;  $I = (3.5)(1.5)^3/12 = 0.984 \text{ in}^4$ ;  $Q = (3.5)(1.5/2)(1.5/4) = 0.984 \text{ in}^3$

$V = (70)(0.984)(3.5)/0.984 = 245 \text{ LB}$

9-23  $t = 1.50 \text{ in}$ ;  $h = 11.25 \text{ in}$ ;  $I = 178 \text{ in}^4$ ;  $Q = (1.50)(11.25/2)(11.25/4) = 23.73 \text{ in}^3$

$V = (70)(178)(1.50)/23.73 = 788 \text{ LB}$

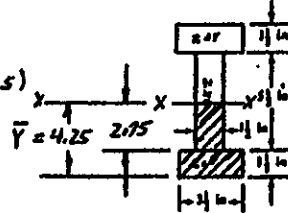
9-24  $t = 11.25 \text{ in}$ ;  $h = 1.50 \text{ in}$ ;  $I = 11.25(1.50)^3/12 = 3.16 \text{ in}^4$ ;  $Q = (11.25)(1.50/2)(1.50/4) = 3.16 \text{ in}^3$

$V = (70)(3.16)(11.25)/3.16 = 788 \text{ LB}$

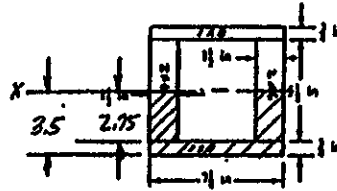
9-25  $t = 9.5 \text{ in} ; h = 11.5 \text{ in} ; I = 1204 \text{ in}^4 ; Q = (9.5)(11.5/2)(11.5/4) = 157.1 \text{ in}^3$   
 $V = (70)(1204)(9.5)/157 = 5098 \text{ LB}$

9-26  $t = 11.5 \text{ in} ; h = 9.5 \text{ in} ; I = 115(9.5)^3/12 = 822 \text{ in}^4$   
 $Q = (11.5)(9.5/2)(9.5/4) = 129.7 \text{ in}^3$   
 $V = (70)(822)(11.5)/129.7 = 5098 \text{ LB}$

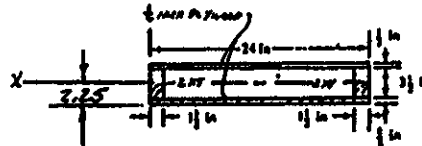
9-27  $I_x = 151.4 \text{ in}^4$  FROM P7-21.  
 $Q = A_1 y_1 + A_2 y_2 = (2.75)(1.5)(2.75/2) + (3.5)(1.5)(6.5)$   
 $Q = 24.05 \text{ in}^3$   
 $V = (70)(151.4)(1.50)/24.05 = 661 \text{ LB}$



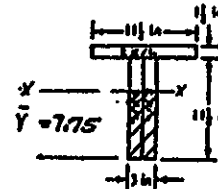
9-28  $I_x = 148.3 \text{ in}^4$  FROM P7-22.  
 $Q = A_1 y_1 + A_2 y_2$   
 $Q = (2)(1.5)(2.75)(2.75/2) + (7.5)(7.25)(3.125)$   
 $Q = 28.33 \text{ in}^3$   
 $V = (70)(148.3)(3.0)/28.33 = 1099 \text{ LB}$



9-29  $I_x = 107.2 \text{ in}^4$  FROM P7-23.  
 $Q = A_1 y_1 + A_2 y_2$   
 $Q = (2)(1.5)(1.75)(1.75/2) + (5)(2.75)(2.08)$   
 $Q = 28.59 \text{ in}^3$   
 $V = (70)(107.2)(3.0)/28.59 = 787 \text{ LB}$

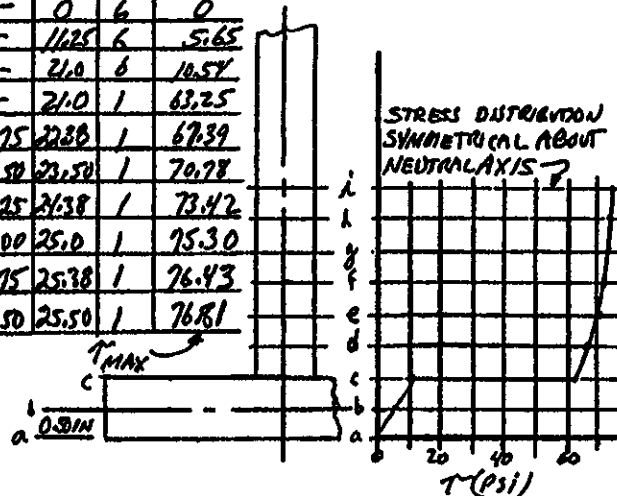


9-30  $I_x = 816.3 \text{ in}^4$  FROM P7-24.  
 $Q = A_1 y_1 = (7.75)(3.0)(7.75/2) = 90.09 \text{ in}^3$   
 $V = (70)(816.3)(3.0)/90.09 = 1903 \text{ LB}$



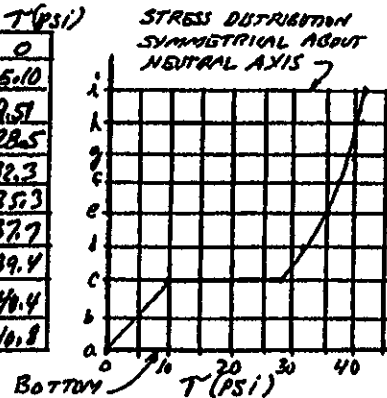
9-31  $I_x = 166 \text{ in}^4$  FROM P7-2 :  $V = 500 \text{ LB} ; \tau = \frac{VQ}{Ib}$

SECTION	A <sub>1</sub>	y <sub>1</sub>	A <sub>2</sub>	y <sub>2</sub>	Q	t	T (psi)
a-a	0	-	-	-	0	6	0
b-b	3.0	3.75	-	-	11.25	6	5.65
c-c	6.0	3.50	-	-	21.0	6	10.54
c-c'	6.0	3.5	-	-	21.0	1	63.25
d-d	6.0	3.5	0.50	2.75	22.88	1	67.39
e-e	6.0	3.5	1.0	2.50	23.50	1	70.78
f-f	6.0	3.5	1.5	2.25	24.38	1	73.42
g-g	6.0	3.5	2.0	2.00	25.0	1	75.30
h-h	6.0	3.5	2.5	1.75	25.38	1	76.43
i-i	6.0	3.5	3.0	1.50	25.50	1	76.81



9-32  $I_x = 184 \text{ in}^4$  FROM P7-3 :  $V = 500 \text{ LB}$  ;  $T = VQ/Ix$

SECTION	A <sub>1</sub>	y <sub>1</sub>	A <sub>2</sub>	y <sub>2</sub>	Q	t	T (psi)
a-a	0	—	—	—	0	6	0
b-b	3.0	3.75	—	—	11.25	6	5.10
c-c	6.0	3.50	—	—	21.0	6	9.57
c-c'	6.0	3.50	—	—	21.0	2	28.5
d-d	6.0	3.50	1.00	2.75	23.75	2	32.3
e-e	6.0	3.5	2.00	2.50	26.0	2	35.3
f-f	6.0	3.5	3.00	2.25	27.75	2	37.7
g-g	6.0	3.5	4.00	2.00	29.0	2	39.4
h-h	6.0	3.5	5.00	1.75	29.75	2	40.4
i-i	6.0	3.5	6.00	1.50	30.0	2	40.8

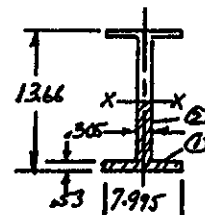


9-33  $W14 \times 43$  :  $I_x = 428 \text{ in}^4$

$Q = A_1 y_1 + A_2 y_2 = (7.995 \times .53)(6.57) + (6.30 \times 0.305)(3.15)$

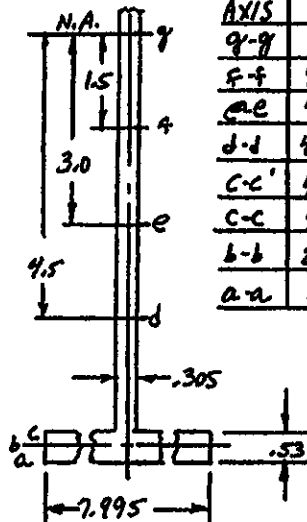
$Q = 33.89 \text{ in}^3$

$T = \frac{VQ}{It} = \frac{(500)(33.89)}{(428)(0.305)} = 8698 \text{ psi AT X-X}$

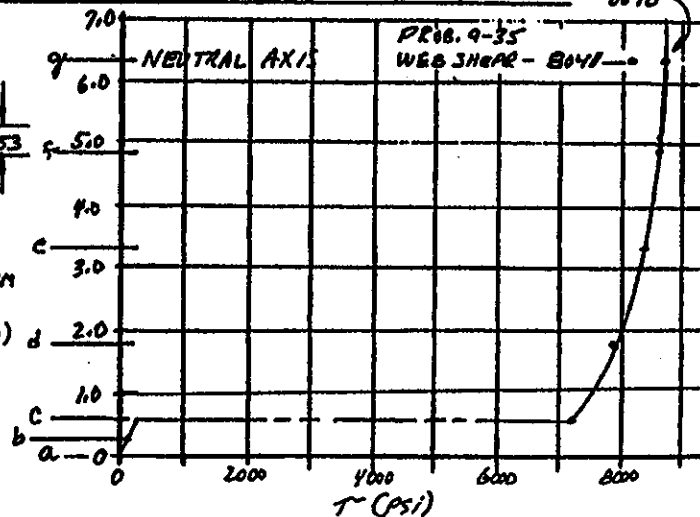


9-34  $W14 \times 43$  :  $I_x = 428 \text{ in}^4$

AXIS	A <sub>1</sub>	y <sub>1</sub>	A <sub>2</sub>	y <sub>2</sub>	Q	t	T
g-g	4.24	6.57	1.922	3.15	33.89	.305	8698
f-f	4.24	6.57	1.464	3.90	33.51	.305	8599
e-e	4.24	6.57	1.007	4.65	32.48	.305	8336
d-d	4.24	6.57	.549	5.40	30.76	.305	7895
c-c'	4.24	6.57	—	—	27.8	.305	7134
c-c	4.24	6.57	—	—	27.8	.7425	293
b-b	2.12	6.70	—	—	14.2	.7425	139
a-a	0	—	—	—	0	.7425	0



DISTANCE FROM  
BOTTOM OF  
SECTION (IN)



9-35 W14 X 43: FOR WEB SHEAR FORMULA:  $t = 0.305 \text{ in}$ ;  $h = 13.66 \text{ in}$

$$\tau_{ws} = \frac{V}{t h} = \frac{33500 \text{ LB}}{(0.305)(13.66) \text{ in}^2} = 8041 \text{ PSI} \quad \text{GRAPH ON PREVIOUS PAGE}$$

FROM PROB 9-34:  $\tau_{MAX} = 8698 \text{ PSI}$

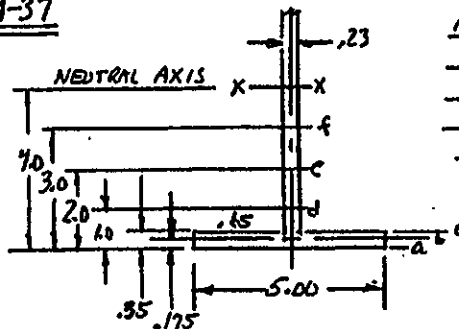
$$\tau_{ws} / \tau_{MAX} = 8041 / 8698 = 0.92$$

9-36 I B x 6.181 ALUMINUM I-BEAM:  $I_x = 59.69 \text{ in}^4$

$$Q = A_1 y_1 + A_2 y_2 = (5.00)(0.35)(2.825) + (3.65)(0.23)(1.825) = 8.226 \text{ in}^3$$

$$\tau = \frac{VQ}{I t} = \frac{(13500 \text{ LB})(8.226 \text{ in}^3)}{(59.69 \text{ in}^4)(0.23 \text{ in})} = 8089 \text{ PSI}$$

9-37



AXIS	A	y <sub>1</sub>	A <sub>2</sub>	y <sub>2</sub>	Q	t	τ
X-X	1.75	3.825	.840	1.825	8.226	.23	8089
F	1.75	3.825	.610	2.325	8.111	.23	7976
C	1.75	3.825	.380	2.825	7.767	.23	7638
D	1.75	3.825	.150	3.325	7.493	.23	7073
C'	1.75	3.825	-	-	6.694	.23	6592
G	1.75	3.825	-	-	6.694	5.00	303
B	.875	2.913	-	-	2.423	5.00	155
A	0	-	-	-	0	5.00	0

9-38

ALUMINUM I B X 6.181

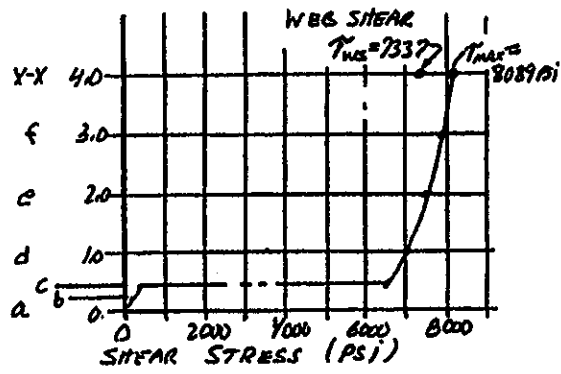
$t = 0.23 \text{ in}$ ;  $h = 8.00 \text{ in}$

WEB SHEAR FORMULA

$$\tau_{ws} = \frac{V}{t h} = \frac{13500 \text{ LB}}{(0.23)(8.00) \text{ in}^2}$$

$$\tau_{ws} = 7337 \text{ PSI}$$

$$\tau_{ws} / \tau_{MAX} = \frac{7337}{8089} = 0.907$$



9-39

$V_{MAX} = 10 \text{ K} = 10000 \text{ LB}$ ;  $M_{MAX} = 30 \text{ K-FT} = 3.6 \times 10^5 \text{ LB-IN}$ . SEE PROB. P6-4.

W10 X 15:  $I_x = 69.8 \text{ in}^4$ ;  $t = 0.23$ ;  $h = 9.99 \text{ in}$ ;  $S = 13.9 \text{ in}^3$

$$\tau_{ws} = \frac{V}{t h} = \frac{10000 \text{ LB}}{(0.23)(9.99) \text{ in}^2} = 4352 \text{ PSI}; \tau_a = .4 S_y = .4(36000) = 14400 \text{ PSI}; \text{SAFE}$$

$$\sigma = \frac{M}{S} = \frac{3.6 \times 10^5 \text{ LB-IN}}{13.9 \text{ in}^3} = 26100 \text{ PSI}; \sigma_a = 0.66 S_y = 23760 \text{ PSI}; \text{UNSAFE}$$

9-40

$M_{MAX} = 3.6 \times 10^5 \text{ LB-IN}$ ;  $\sigma_a = 23760 \text{ PSI}$  (PROB 9-39). SEE ALSO P6-4.

$$S_{req} = \frac{M}{\sigma_a} = \frac{3.6 \times 10^5 \text{ LB-IN}}{23760 \text{ LB/in}^2} = 15.15 \text{ in}^3; \text{W12 X 16}; S = 17.1 \text{ in}^3; t = .22 \text{ in}; h = 11.99 \text{ in}$$

$$\tau_{ws} = \frac{V}{t h} = \frac{10000 \text{ LB}}{(0.22)(11.99) \text{ in}^2} = 3791 \text{ PSI}; \tau_a = 0.4 S_y = 14400 \text{ PSI}; \text{SAFE}$$

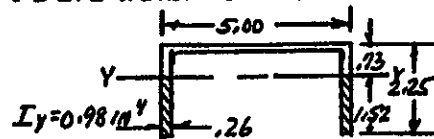


9-41 FROM PROB P6-52 :  $V_{max} = 44.5K = 44,500 \text{ LB}$  ;  $M_{max} = 148K \cdot \text{FT} = 1,776 \times 10^6 \text{ LB} \cdot \text{IN}$   
 $S_{min} = \frac{M}{\sigma_b} = \frac{1,776 \times 10^6 \text{ LB} \cdot \text{IN}}{23,760 \text{ LB}/\text{IN}^2} = 74.7 \text{ IN}^3$  ;  $W18 \times 55$  ;  $S = 98.3 \text{ IN}^3$  ;  $t = 0.390 \text{ IN}$  ;  $h = 18.1 \text{ IN}$   
 $T = \frac{V}{tA} = \frac{44,500 \text{ LB}}{(0.39)(18.1) \text{ IN}^2} = 6300 \text{ PSI}$  ;  $T_d = 0.4 S_y = 14,400 \text{ PSI}$  ; SAFE

9-42 FROM PROB P6-51 :  $V_{max} = 162.9 \text{ KN} \times \frac{1000 \text{ N}}{\text{KN}} = 162,900 \text{ N}$  ;  $M_{max} = 2228 \text{ LB} = 36,620 \text{ LB}$   
 $M_{max} = 228 \text{ KN} \cdot \text{m} \times \frac{1000 \text{ N}}{\text{KN}} \times \frac{0.857 \text{ LB} \cdot \text{IN}}{\text{N} \cdot \text{m}} = 2.018 \times 10^6 \text{ LB} \cdot \text{IN}$   
 $S_{min} = \frac{M}{\sigma_b} = \frac{2.018 \times 10^6 \text{ LB} \cdot \text{IN}}{23,760 \text{ LB}/\text{IN}^2} = 84.9 \text{ IN}^3$  ;  $W18 \times 55$  ;  $S = 98.3 \text{ IN}^3$  ;  $t = 0.390 \text{ IN}$  ;  $h = 18.1 \text{ IN}$   
 $T = \frac{V}{tA} = \frac{36,620 \text{ LB}}{(0.39)(18.1) \text{ IN}^2} = 5185 \text{ PSI}$  ;  $T_d = 0.4 S_y = 14,400 \text{ PSI}$  ; SAFE

9-43 FROM PROB P6-51 :  $V_{max} = 804 \text{ LB}$  ;  $M_{max} = 2528 \text{ LB} \cdot \text{IN}$   
 $\sigma_b = 54/3 = 18,000 \text{ PSI}/3 = 6000 \text{ PSI}$  ;  $T_d = 0.5 S_y/3 = 8000 \text{ PSI}$   
 $S_{min} = \frac{M}{\sigma_b} = \frac{2528 \text{ LB} \cdot \text{IN}}{16,000 \text{ LB}/\text{IN}^2} = 0.158 \text{ IN}^3$  ;  $1 \frac{1}{4} \text{ IN SCH 40 PIPE}$  ;  $S = 0.2346 \text{ IN}^3$   
 $T \approx \frac{2(V)}{A} = \frac{2(804 \text{ LB})}{(0.669) \text{ IN}^2} = 2404 \text{ PSI}$  ; SAFE ;  $A = 0.669 \text{ IN}^2$

9-44 FROM PROB P6-9 :  $V_{max} = 1557 \text{ LB}$  ;  $M_{max} = 6228 \text{ LB} \cdot \text{IN}$   
 $\sigma_b = 54/4 = 13,500 \text{ PSI}/4 = 3375 \text{ PSI}$  ;  $T_d = 0.5 S_y/4 = 5000 \text{ PSI}$   
 $S_{min} = \frac{M}{\sigma_b} = \frac{6228 \text{ LB} \cdot \text{IN}}{10,000 \text{ LB}/\text{IN}^2} = 0.623 \text{ IN}^3$  ;  $C5 \times 2.212 \text{ ALUM. CHANNEL}$   
 $T = \frac{VQ}{It} = \frac{(1557)(2)(2.212)(1.52)}{(0.98)(1.52)} = 1835 \text{ PSI}$  ; SAFE

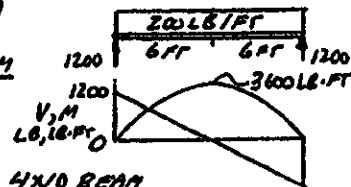


9-45 SHEAR:  $T = \frac{3V}{2A} = T_d = 75 \text{ PSI}$  (RECT. SECTION)

REQD A =  $\frac{3V}{2T_d} = \frac{3(1200 \text{ LB})}{2(75 \text{ LB}/\text{IN}^2)} = 24 \text{ IN}^2$  ; 4 X 8 BEAM

BENDING:  $\sigma = \frac{M}{S} = \sigma_b = 1150 \text{ PSI}$

REQD S =  $\frac{M}{\sigma_b} = \frac{(3600 \text{ LB} \cdot \text{FT})(12 \text{ IN}/\text{FT})}{1150 \text{ LB}/\text{IN}^2} = 37.6 \text{ IN}^3$  ; 4 X 10 BEAM



9-46 FROM PROB P6-53 :  $V_{max} = 2950 \text{ N}$  ;  $M_{max} = 3350 \text{ N} \cdot \text{m}$   
 SHEAR:  $A_{min} = \frac{3V}{2T_d} = \frac{(3)(2950 \text{ N})}{(2)(0.66 \text{ N}/\text{mm}^2)} = 6705 \text{ mm}^2$  ; 2 X 8 BEAM OR 4 X 10 BEAM

BENDING:  $S = \frac{M}{\sigma_b} = \frac{3350 \text{ N} \cdot \text{m} \times 10^3 \text{ mm}^3/\text{m}^3}{5.5 \text{ N}/\text{mm}^2} = 609 \times 10^3 \text{ mm}^3$  ; USE 4 X 10 BEAM

9-47 FROM PROB 9-28 :  $I_x = 148.3 \text{ IN}^4$  ;  $C = 3.50 \text{ IN}$  ;  $Q = 28.33 \text{ IN}^3$  . SEE ALSO P7-22.

$V_{max} = P$  ;  $M_{max} = P(3 \text{ FT}) = 36(P \text{ LB} \cdot \text{IN})$  ;  $T_d = 80 \text{ PSI}$  ;  $\sigma_{max} = 14,000 \text{ PSI}$

$T = \frac{VQ}{It}$  ;  $V_{max} = P_{max} = \frac{T_d I C}{Q} = \frac{(80)(148.3)(3.0)}{28.33} = 1256 \text{ LB} = P_{max}$

$\sigma = \frac{M C}{I} = \frac{36 P C}{I}$  ;  $P = \frac{\sigma_b I}{36 C} = \frac{(14,000)(148.3)}{(36)(3.50)} = 1648 \text{ LB}$

9-48 FROM PROB P6-8 :  $V_{MAX} = 21.36 \text{ kN} = 21360 \text{ N}$   
 $I \times 8.361 : c = 0.27 \text{ IN} (25.4 \text{ mm/IN}) = 6.86 \text{ mm} : h = 9.00 \text{ IN} (25.4) = 229 \text{ mm}$   
 $\tau_{\text{AV}} = \frac{V}{Z_A} = \frac{21360 \text{ N}}{(6.86)(229) \text{ mm}^2} = 1.36 \text{ MPa}$

9-49 FROM PROB P6-8 :  $M_{MAX} = 43.2 \text{ kN}\cdot\text{m}$   
 $I \times 8.361 : S = 22.67 \text{ IN}^3 \times \frac{(25.4 \text{ mm/IN})^3}{10^3} = 371495 \text{ mm}^3$   
 $\sigma = \frac{M}{S} = \frac{43.2 \times 10^3 \text{ N}\cdot\text{m}}{371495 \text{ mm}^3 \times \frac{10^3 \text{ mm}^3}{\text{m}^3}} = 116.3 \text{ MPa}$

9-50 TOTAL LOAD =  $W = wL = (80 \text{ LB/FT})(12 \text{ FT}) = 960 \text{ LB} : V_{MAX} = W/2 = 480 \text{ LB}$   
 $\tau = \frac{3V}{2A} = \frac{(3)(480 \text{ LB})}{2(10.87 \text{ IN}^2)} = 66.2 \text{ PSI} : \tau_d = 70 \text{ PSI OK}$   
 CHECK BENDING :  $M_{MAX} = wL^2/8 = \frac{(80)(12)^2}{8} = 1440 \text{ LB}\cdot\text{FT} (12 \text{ IN/FT}) = 17280 \text{ LB}\cdot\text{IN}$   
 $\sigma = \frac{M}{S} = \frac{17280 \text{ LB}\cdot\text{IN}}{13.14 \text{ IN}^3} = 1315 \text{ PSI} : \sigma_d = 1000 \text{ PSI UNSAFE}$

9-51 FROM PROB P6-10 :  $V_{MAX} = 1500 \text{ LB} : M_{MAX} = 9000 \text{ LB}\cdot\text{IN}$   
 (a) SHEAR :  $\tau = \frac{3V}{2A} = \frac{3(1500 \text{ LB})}{2(21.50)(4.0) \text{ IN}^2} = 1125 \text{ PSI}$   
 (b) BENDING :  $\sigma = \frac{M}{S} = \frac{9000 \text{ LB}\cdot\text{IN}}{(1.5)(4.0)^3/6 \text{ IN}^3} = 6750 \text{ PSI}$   
 (c)  $\tau_d = 0.5 S_y/3 : \text{REQD } S_y = 3\tau_d/0.5 = 3(1125)/0.5 = 6750 \text{ PSI}$   
 $\sigma_d = S_y/3 : \text{REQD } S_y = 3\sigma_d = 3(6750) = 20250 \text{ PSI ANY STEEL}$

9-52 FROM PROB P6-6 :  $V_{MAX} = 145 \text{ N} : M_{MAX} = 318 \text{ N}\cdot\text{m}$   
 (a) SHEAR :  $\tau = \frac{3V}{2A} = \frac{3(145 \text{ N})}{2(12)(60) \text{ mm}^2} = 2267 \text{ MPa}$   
 (b) BENDING :  $\sigma = \frac{M}{S} = \frac{318 \text{ N}\cdot\text{m} (10^3 \text{ mm}^3/\text{m})}{(6)(60)^3/6 \text{ mm}^3} = 35.1 \text{ MPa}$   
 (c)  $\tau_d = 0.5 S_y/3 : \text{REQD } S_y = \frac{3(\tau)}{0.5} = \frac{3(2267)}{0.5} = 13.6 \text{ MPa}$   
 $\sigma_d = S_y/3 : \text{REQD } S_y = 3\sigma = 3(32.1) = 99.3 \text{ MPa}$  6061-TY :  $S_y = 175 \text{ MPa}$  OR SEVERAL OTHERS

9-53 FROM PROB. P6-47 :  $V_{MAX} = 450 \text{ N} : M_{MAX} = 172.5 \text{ N}\cdot\text{m}$   
 $\sigma_d = S_y/N = 276 \text{ NPa}/4 = 69 \text{ MPa} = M/S :$   
 $\text{REQD } S = \frac{M}{\sigma_d} = \frac{172.500 \text{ N}\cdot\text{m}\cdot\text{m}}{69 \text{ N/m}^2} = 2500 \text{ mm}^3 = \frac{bh^2}{6}$   
 $\text{REQD } h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{6(2500) \text{ mm}^3}{12 \text{ mm}}} = 35.4 \text{ mm}$   
 $\tau = \frac{3V}{2A} = \frac{3(450 \text{ N})}{2(12)(35.4) \text{ mm}^2} = 1.59 \text{ MPa} = 0.5 S_y$   
 $N = \frac{0.5 S_y}{\tau} = \frac{0.5(276 \text{ MPa})}{1.59 \text{ MPa}} = 86.7$  SAFE. VERY HIGH N.

9-54 FROM PROB P6-48 :  $V_{max} = 1290 \text{ N}$  ;  $M_{max} = 370.8 \text{ N}\cdot\text{m}$

$$(a) \tau = \frac{4V}{3A} = \frac{4(1290 \text{ N})}{3(\pi)(40)^2/4 \text{ mm}^2} = 1.37 \text{ MPa} = \tau_{av} = \frac{0.5S_y}{4} = \frac{S_y}{8}$$

$$(b) \sigma = \frac{M}{S} = \frac{370.8 \times 10^3 \text{ N}\cdot\text{mm}}{\pi(40)^3/32} = 59.0 \text{ MPa} = \sigma_a = \frac{S_y}{4}$$

$$(c) \text{ FOR SHEAR : } \text{REDD } S_y = 8\tau = 8(1.37) = 11.0 \text{ MPa}$$

$$\text{FOR BENDING : } \text{REDD } S_y = 4\sigma = 4(59.0) = 236 \text{ MPa} \quad \text{AISI 1020 HR } S_y = 331 \text{ MPa}$$

9-55 FROM PROB P6-47 :  $M_{max} = 172.5 \text{ N}\cdot\text{m}$  ;  $V_{max} = 450 \text{ N}$

$$\text{REDD. } S = \frac{M}{\sigma_a} = \frac{(172.5 \text{ N}\cdot\text{m})(10^3 \text{ mm/m})}{120 \text{ N/mm}^2} = 1438 \text{ mm}^3 = \pi(D)^3/32$$

$$D = \left[ \frac{32S}{\pi} \right]^{1/3} = \left[ \frac{32(1438)}{\pi} \right]^{1/3} = 24.5 \text{ mm} ; A = \pi D^2/4 = 470 \text{ mm}^2$$

$$\tau = \frac{4V}{3A} = \frac{4(450 \text{ N})}{3(470 \text{ mm}^2)} = 1.28 \text{ MPa}$$

9-56  $\tau = 4V/3A$  ;  $V_{max} = \frac{3A\tau}{4} = \frac{3(\pi)(1.50)^2/4 \text{ in}^2 (70 \text{ lb/in}^2)}{(4)(4)} = 92.9 \text{ LB}$

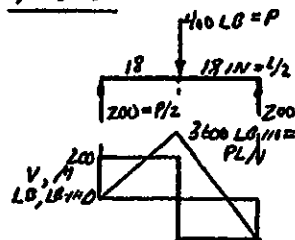
9-57  $\tau_a = \frac{0.5S_y}{6} = \frac{0.5(48000)}{6} = 4000 \text{ PSI} \approx \frac{2V}{A}$

$$\sigma_a = S_y/6 = 48000/6 = 8000 \text{ PSI} = M/S$$

$$\text{REDD } S = \frac{M}{\sigma_a} = \frac{3600 \text{ LB}\cdot\text{IN}}{8000 \text{ LB/IN}^2} = 0.45 \text{ IN}^3$$

$$A = 1.075 \text{ IN}^2 \quad \text{2-IN SCH 40 PIPE}$$

$$\tau = 2V/A = 2(280)/1.075 = 372 \text{ PSI ; OK}$$



9-58  $\sigma_a = S_y/4 = 48000/4 = 12000 \text{ PSI}$  ;  $\tau_a = 0.5S_y/4 = 6000 \text{ PSI}$  ;  $P = 2800 \text{ LB}$

$$V_{max} = P/2 = 1400 \text{ LB} ; M_{max} = PL/4 \text{ (PROB 9-57)} ; \text{REDD } S = M/\sigma_a ; \tau \approx 2V/A$$

L (IN)	M (LB-IN)	REDD S (IN <sup>3</sup> )	PIPE	A	$\tau \approx \frac{2V}{A}$
(a) 1.50	1050	0.0875	1	.494	5668 OK
(b) 3.00	2100	0.175	1 1/4	.669	4185 OK
(c) 4.50	3150	0.263	1 1/2	.799	3504 OK
(d) 6.00	4200	0.350	2	1.075	2605 OK

9-59 SHEAR FLOW AT JOINT =  $q = VQ/I$   
FROM FIG P7-14 :  $I = 0.3672 \text{ IN}^4$  ;  $\bar{Y} = 0.533 \text{ IN}$

$$Q = A_p \bar{y} = (2.60)(0.20)(1.533 - 0.10) = 0.225 \text{ IN}^3$$

$$q = (1200 \text{ LB})(0.225 \text{ IN}^3)/0.3672 \text{ IN}^4 = 736 \text{ LB/IN}$$

$$\text{ON 1.0 IN OF LENGTH, AREA OF GLUE} = (1.0 \text{ IN})(2)(0.7 \text{ IN}) = 1.40 \text{ IN}^2 = A_s$$

$$\tau = \frac{736 \text{ LB}}{1 \text{ IN}} \times \frac{1 \text{ IN}}{1.40 \text{ IN}^2} = 526 \text{ PSI}$$

9-60 FROM PROB P7-26 :  $I = 469.4 \text{ IN}^4$  ;  $V = 7.90 \text{ IN}$  ; FLANGE WIDTH = 5.477 IN  
 $Q = A_p \bar{y} = (7.35)(3.81) = 28.0 \text{ IN}^3$  FOR ENTIRE CHANNEL

$$q = VQ/I = (2500)(28.0)/469.4 = 149 \text{ LB/IN}$$

$$A_s = (1.0 \text{ IN})(5.477 \text{ IN}) = 5.477 \text{ IN}^2/\text{IN}$$

$$\tau = q/A_s = 149 \text{ LB/IN} / 5.477 \text{ IN}^2 = 27.2 \text{ PSI}$$

9-61 FROM PROB P7-33 :  $I = 46.6 \text{ in}^4$   
 $\sigma_a = s_y/4 = 2100/4 = 5250 \text{ psi}$   
 $T_d = 0.5 s_y/4 = 2625 \text{ psi}$

BENDING:  $\sigma = M c / I$

$$M_{\text{allow}} = \frac{\sigma_a I}{c} = \frac{(5250)(46.6)}{2.72} = 89945 \text{ LB-IN}$$

$$w = \frac{8M}{L^2} = \frac{8(89945 \text{ LB-IN})}{(120 \text{ IN})^2} = 50.0 \text{ LB/IN}$$

SHEAR AT NEUTRAL AXIS:

$$Q = (0.5 \times 12)(2.03 - 0.25) + 2(1.53)(0.25)(0.532)$$

$$Q = 11.27 \text{ IN}^3 ; c = 2(0.25) = 0.50 \text{ IN}$$

$$T = \frac{VQ}{Ic} ; V = \frac{T_d I c}{Q} = \frac{(2625)(46.6)(0.50)}{11.27} = 5429 \text{ LB}$$

$$w = \frac{2(V)}{L} = \frac{2(5429 \text{ LB})}{120 \text{ IN}} = 90.5 \text{ LB/IN}$$

SHEAR FLOW AT NEZOS:  $q = \frac{VQ}{I} ; V = \frac{q I}{Q} = \frac{(11800)(46.6)}{(6.0 \times 2.03 - 0.25)} = 7851 \text{ LB}$

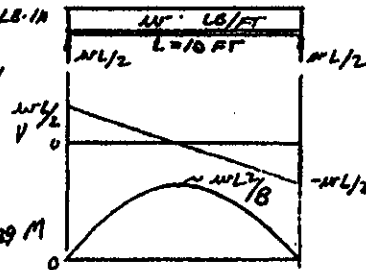
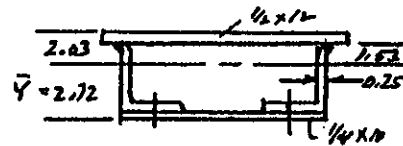
$$w = 2V/L = 2(7851)/120 = 131 \text{ LB/IN}$$

RIVETS:  $S = \text{SPACING} = 4.0 \text{ IN} ; F_{sd} = 2(600) = 1200 \text{ LB} = S q_d$

$$q_d = \frac{F_{sd}}{S} = \frac{1200 \text{ LB}}{4.0 \text{ IN}} = 300 \text{ LB/IN}$$

$$q = \frac{VQ}{I} ; V = \frac{q I}{Q} = \frac{(300 \text{ LB/IN})(46.6 \text{ IN}^4)}{(2.50 \times 2.72 - 0.25 \text{ IN}^3)} = 2155 \text{ LB}$$

$$w = 2V/L = 2(2155 \text{ LB})/120 \text{ IN} = 35.9 \text{ LB/IN} \quad \text{LIMITING VALUE} \quad (12 \text{ IN/FT}) = 431 \text{ LB/FT}$$



9-62 FROM PROB P7-24 :  $I = 816.3 \text{ in}^4 ; \gamma = 7.75 \text{ IN}$

#3 SOUTHERN PINE

BENDING:  $\sigma = M c / I$

$$\sigma_d = 650 \text{ psi}$$

$$M = \frac{\sigma_a I}{c} = \frac{(650)(816.3)}{7.75} = 68464 \text{ LB-IN} = M L^2/8$$

$$T_d = 70 \text{ psi}$$

$$w = \frac{8M}{L^2} = \frac{8(68464)}{(120)^2} = 38.0 \text{ LB/IN}$$

SHEAR AT NEUTRAL AXIS:  $Q = A \bar{y} = (3.0)(7.75)(7.75/2) = 90.09 \text{ IN}^3$

$$T = \frac{VQ}{Ic} ; V = \frac{T_d I c}{Q} = \frac{(70)(816.3)(3.0)}{90.09} = 1903 \text{ LB} = w L/2$$

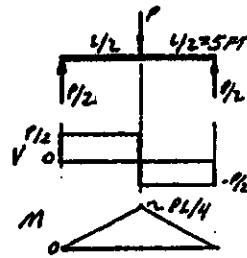
$$w = \frac{2V}{L} = \frac{2(1903)}{120} = 31.7 \text{ LB/IN}$$

NAILS:  $S = 6.0 \text{ IN} ; F_{sd} = 2(160) = 320 \text{ LB} = S q_d ; q_d = \frac{F_{sd}}{S} = \frac{320 \text{ LB}}{6.0 \text{ IN}} = 53.3 \text{ LB/IN}$

$$q = \frac{VQ}{I} ; V = \frac{q I}{Q} = \frac{(53.3)(816.3)}{(0.5)(11.25)(12.75 - 2.75 - 0.75)} = 607 \text{ LB}$$

$$w = \frac{2V}{L} = \frac{2(607)}{120} = 10.1 \text{ LB/IN} \quad (12 \text{ IN/FT}) = 121 \text{ LB/FT} \quad \text{LIMITING}$$

9-63 FROM PROB P7-21 :  $I = 151.4 \text{ in}^4$  ;  $\bar{y} = 4.25 \text{ in}$   
 BENDING :  $\sigma = Mc/I$  ;  $\sigma_a = 1450 \text{ psi}$   
 $M = \frac{\sigma_a I}{c} = \frac{(1450)(151.4)}{4.25} = 51654 \text{ lb}\cdot\text{in} = P L/4$   
 $P = \frac{4M}{L} = \frac{4(51654 \text{ lb}\cdot\text{in})}{120 \text{ in}} = 1722 \text{ lb}$  LIMITING



SHEAR :  $T = VQ/I\bar{c}$  ;  $V = T\bar{c}I\bar{c}/Q$  ;  $T_a = 95 \text{ psi}$   
 AT NEUTRAL AXIS :  $Q = (3.5 \times 1.5)(3.5) + (2.75 \times 1.5)(2.75/2)$   
 $Q = 24.05 \text{ in}^3$   
 $V = \frac{T_a I \bar{c}}{Q} = \frac{(95)(151.4)(4.25)}{24.05} = 897 \text{ lb} = \frac{P}{2}$  ;  $P = 2V = 1794 \text{ lb}$

GLUE AT TOP OR WELD :

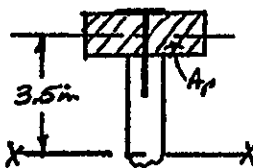
$q = \frac{VQ}{I} = \frac{PQ}{2L} = \frac{P(3.5)(1.5)(3.5)}{2(151.4)} = 0.0668 P \text{ lb/in}$

$T_a = \frac{q}{A_s}$  ;  $q_a = T_a A_s = \frac{800 \text{ lb}}{\text{in}^2} \times \frac{0.5 \times 1.5 \text{ in}^2}{\text{in}} = 1200 \text{ lb/in}$

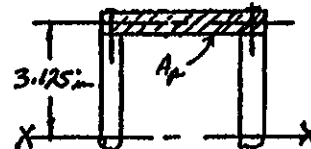
$P = \frac{q_a}{0.0668} = \frac{1200}{0.0668} = 19775 \text{ lb}$

9-64  $I = 151.4 \text{ in}^4$  ;  $Q = (3.5)(1.5)(3.5) = 18.375 \text{ in}^3$   
 $q = \frac{VQ}{I} = \frac{(300)(18.375)}{151.4} = 36.4 \text{ lb/in}$   
 $S = \frac{Fsd}{q} = \frac{180 \text{ lb}}{36.4 \text{ lb/in}} = 4.94 \text{ in}$

FIGURE P7-21

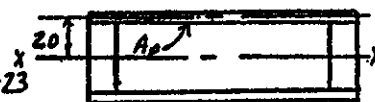


9-65  $I = 148.3 \text{ in}^4$  ;  $Q = (7.25)(0.75)(3.125) = 17.0 \text{ in}^3$   
 $q = VQ/I = (600)(17.0)/148.3 = 68.8 \text{ lb/in}$   
 $S = Fsd/q = 2(150)/68.8 = 4.36 \text{ in}$



9-66  $I = 107.2 \text{ in}^4$  ;  $Q = (24)(0.5)(2.0) = 24.0 \text{ in}^3$   
 $q = VQ/I = (500)(24.0)/107.2 = 112 \text{ lb/in}$

FIGURE P7-23



9-67  $V = 175 \text{ kN} = 175000 \text{ N}$  ;  $V = \frac{175000 \text{ N}}{4.448 \text{ N/lb}} = 39344 \text{ lb}$

FROM PROB P7-25 :  $I = 1155 \text{ in}^4$  ;  $\bar{y} = 7.33 \text{ in}$

$Q = (0.5)(14.5)(7.08) = 51.3 \text{ in}^3$

$q = VQ/I = (39344)(51.3)/1155 = 1749 \text{ lb/in}$

$S = \frac{Fsd}{q} = \frac{0.2(2650) \text{ lb}}{1749 \text{ lb/in}} = 3.03 \text{ in}$

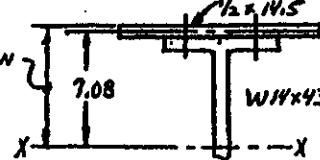


FIGURE P7-25

9-68  $V = 50000 \text{ N} (110/4.448 \text{ N}) = 11241 \text{ lb}$   
 FROM PROB P7-26 :  $I = 469.4 \text{ in}^4$   
 $Q = A\bar{y} = (7.35)(3.81) = 28.0 \text{ in}^3$   
 $q = \frac{VQ}{I} = \frac{(11241)(28.0)}{469.4} = 671 \text{ lb/in}$   
 $S = \frac{Fsd}{q} = \frac{2(1750)}{671} = 5.22 \text{ in}$

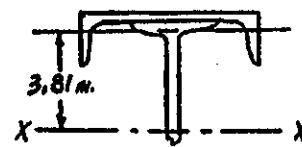


FIGURE P7-26

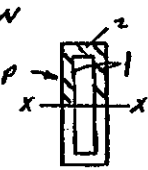
9-69 W18x55:  $t = 0.390 \text{ in}$ ,  $h = 18.11 \text{ in}$ , ASTM A36  $S_y = 36000 \text{ psi}$   
 $\tau = \frac{V}{t h} = \frac{36600 \text{ lb}}{(0.39)(18.11) \text{ in}^2} = 5182 \text{ psi}$  SAFE  
 $T_d = 0.40 S_y = 0.40(36000 \text{ psi}) = 14400 \text{ psi}$  OK

9-70 W18x40:  $t = 0.315 \text{ in}$ ,  $h = 17.90 \text{ in}$ , ASTM A242  $S_y = 50000 \text{ psi}$   
 $\tau = \frac{V}{t h} = \frac{36600 \text{ lb}}{(0.315)(17.90) \text{ in}^2} = 6491 \text{ psi}$  SAFE  
 $T_d = 0.40 S_y = 0.40(50000 \text{ psi}) = 20000 \text{ psi}$  OK

9-71 W14x26:  $t = 0.255 \text{ in}$ ,  $h = 13.91 \text{ in}$ , ASTM A242  $S_y = 50000 \text{ psi}$   
 $\tau = \frac{V}{t h} = \frac{10000 \text{ lb}}{(0.255)(13.91) \text{ in}^2} = 2819 \text{ psi}$  SAFE  
 $T_d = 20000 \text{ psi}$  (PROB 9-70)

9-72 6I x 4.692 ALUMINUM:  $t = 0.21 \text{ in}$ ,  $h = 6.00 \text{ in}$   
 $\tau = \frac{V}{t h} = \frac{10000 \text{ lb}}{(0.21)(6.00)} = 7937 \text{ psi}$  SAFE  
 AL 6061-T6,  $S_y = 40000 \text{ psi}$ ;  $T_d = \frac{S_y}{2N} = \frac{S_y}{2(2)} = 0.25 S_y$   
 $T_d = (0.25)(40000 \text{ psi}) = 10000 \text{ psi}$  OK.

9-73 W10x13:  $t = 0.190 \text{ in}$ ,  $h = 9.82 \text{ in}$ , ASTM A36,  $T_d = 14400 \text{ psi}$  (PROB 9-69)  
 $\tau = \frac{V}{t h} = \frac{6750 \text{ lb}}{(0.190)(9.82) \text{ in}^2} = 3599 \text{ psi}$  SAFE

9-74 HSS 6x2x1/4.  $I = 13.8 \text{ in}^4$ ,  $t = 2(0.25) = 0.50 \text{ in}$   
 $Q = A_1 y_1 + A_2 y_2 = (6.50)(2.75)(1.375) + (0.25)(2.00)(2.875) A_p$   
 $Q = 1.8906 + 1.4375 = 3.328 \text{ in}^3$   
 $\tau = \frac{VQ}{It} = \frac{(6750 \text{ lb})(3.328 \text{ in}^3)}{(13.8 \text{ in}^4)(0.50 \text{ in})} = 3256 \text{ psi}$  SAFE  
 $T_d = \frac{S_y}{2N} = \frac{50000 \text{ psi}}{2(2)} = 12500 \text{ psi}$  OK  
  
 ASTM A500, GRA C  
 $S_y = 50000 \text{ psi}$

9-75 2x8 WOOD BEAM:  $A = 10.87 \text{ in}^2$ , No. 2 SOUTH PINE,  $T_d = 70.0 \text{ psi}$   
 $\tau = \frac{3V}{2A} = \frac{3(480 \text{ lb})}{2(10.87 \text{ in}^2)} = 66.2 \text{ psi}$  SAFE

9-76 FIG. P9-29:  $Q = 28.59 \text{ in}^3$  FROM PROB 9-29,  $I_x = 107.2 \text{ in}^4$   
 $\tau = \frac{VQ}{It} = \frac{(750 \text{ lb})(28.59 \text{ in}^3)}{(107.2 \text{ in}^4)(3.0 \text{ in})} = 66.7 \text{ psi}$  SAFE  
 No. 2 DOUGLAS FIR,  $T_d = 95 \text{ psi}$  OK

9-77 HSS 8x2 x 1/4;  $I = 30.1 \text{ in}^4$ ,  $t = 2(0.25) = 0.50 \text{ in}$ .

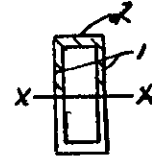
$$Q = A_1 y_1 + A_2 y_2 = (6.50)(3.75)(1.875) + (6.25)(2.00)(3.875)$$

$$Q = 3.516 + 1.9375 = 5.454 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(12000 \text{ lb})(5.454 \text{ in}^3)}{(30.1 \text{ in}^4)(0.50 \text{ in})} = 4350 \text{ psi SAFE}$$

$$\tau_d = S_y/2N = 36000 \text{ psi}/2(2) = 9000 \text{ psi OK}$$

ASTM A501  
 $S_y = 36000 \text{ psi}$



9-78 FROM PROBLEM 7-42,  $I = 63.52 \text{ in}^4$ ,  $t = 0.280 \text{ in}$  (W4x13)

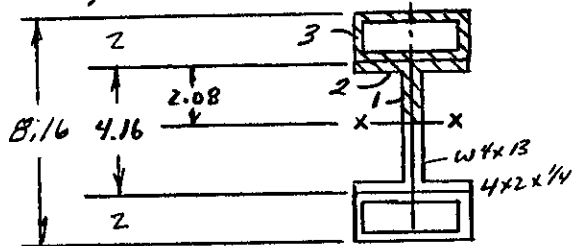
$$Q = A_1 y_1 + A_2 y_2 + A_3 y_3$$

	A	y	Ay
1	.486	.868	.422
2	1.401	1.908	2.673
3	2.59	3.080	7.977

$$Q = 11.072 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(1800 \text{ lb})(11.072 \text{ in}^3)}{(63.52 \text{ in}^4)(0.280 \text{ in})} = 1121 \text{ psi SAFE}$$

$$\tau_d = S_y/2N = 36000 \text{ psi}/2(2) = 9000 \text{ psi OK}$$



9-79 C10x6.136 ALUMINUM CHANNEL

$$I_y = 6.33 \text{ in}^4, t_f = 0.41 \text{ in}$$

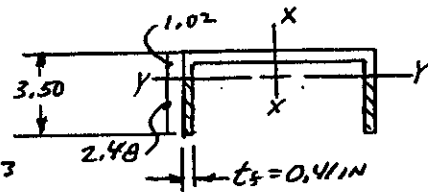
Q FOR LOWER PART OF FLANGES

$$Q = A y = 2(0.41)(2.48)(2.48/2) = 2.522 \text{ in}^3$$

$$\tau = \frac{VQ}{It} = \frac{(430 \text{ lb})(2.522 \text{ in}^3)}{(6.33 \text{ in}^4)(2 \times 0.41 \text{ in})} = 209 \text{ psi SAFE}$$

6061-T6 ALUM.  
 $S_y = 40000 \text{ psi}$

$$\tau_d = S_y/2N = 40000/2(2) = 10000 \text{ psi OK}$$



9-80 DATA OF PROB. 9-78. SHEAR FLOW  $q = \frac{VQ}{I}$

$$I = 63.52 \text{ in}^4, V = 1800 \text{ lb}$$

Q = Ay FOR ONE 4x2 x 1/4 TUBE

$$Q = (2.59 \text{ in}^2)(3.08 \text{ in}) = 7.977 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(1800 \text{ lb})(7.977 \text{ in}^3)}{63.52 \text{ in}^4} = 226 \text{ lb/in}$$

9-81 FIG. P9-29:  $I = 107.2 \text{ in}^4$

Q = Ay FOR TOP PANEL

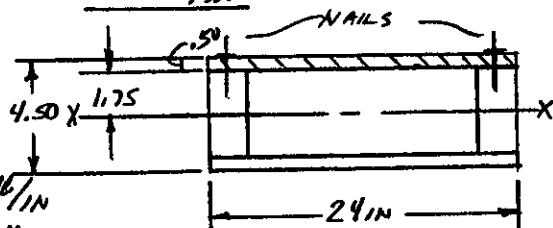
$$Q = (0.50)(24)(2.00) = 24.0 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(500 \text{ lb})(24.0 \text{ in}^3)}{107.2 \text{ in}^4} = 112.16 \text{ lb/in}$$

$$F_{sd} = 135 \text{ lb/NAIL} \times 2 \text{ NAILS} = 270 \text{ lb}$$

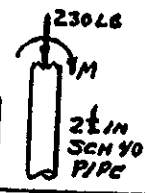
$$\text{MAX. SPACING} = S_{\text{max}} = F_{sd}/q = 270 \text{ lb}/112.16 \text{ lb/in} = 2.41 \text{ in MAXIMUM}$$

SPECIFY  $S = 2.25 \text{ in}$



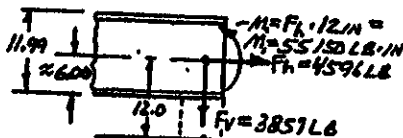
# CHAPTER 10 Special Cases of Combined Stresses

10-1  $M = (230 \text{ LB} \times 48 \text{ IN}) = 11040 \text{ LB} \cdot \text{IN}$ ;  $A = 1.704 \text{ IN}^2$ ;  $S = 1.064 \text{ IN}^3$   
 $\sigma = \frac{-F}{A} - \frac{M}{S} = \frac{-230 \text{ LB}}{1.704 \text{ IN}^2} - \frac{11040 \text{ LB} \cdot \text{IN}}{1.064 \text{ IN}^3} = -10,510 \text{ PSI}$



10-2  $F_h = F \sin 30^\circ = 1.70 \text{ kN}$   $M = F_v \cdot 350 \text{ mm} = 1.029 \times 10^6 \text{ N} \cdot \text{mm}$   
 $F = 3.4 \text{ kN}$   $\text{AREA} = bh = (18)(75) = 1350 \text{ mm}^2$   
 $F_v = F \cos 30^\circ = 2.94 \text{ kN}$   $S = bh^3/6 = 16875 \text{ mm}^3$   
 $\sigma_{\text{MAX}} = \frac{F_h}{A} + \frac{M}{S} = \frac{1700 \text{ N}}{1350 \text{ mm}^2} + \frac{1.029 \times 10^6 \text{ N} \cdot \text{mm}}{16875 \text{ mm}^3} = 62.2 \text{ MPa}$   
 MAX OCCURS ON TOP OF BRACKET AT WALL

10-3  $F_h = F \cos 40^\circ = 4596 \text{ LB}$   $W12 \times 16 \text{ BEAM}$ ;  $A = 4.71 \text{ IN}^2$ ;  $S = 17.1 \text{ IN}^3$   
 $F = 6000 \text{ LB}$   $M = F_h \cdot 12 \text{ IN} = 55150 \text{ LB} \cdot \text{IN}$   
 $F_v = F \sin 40^\circ = 3857 \text{ LB}$   $M_2 = F_v \cdot 52 \text{ IN} = 2.006 \times 10^5 \text{ LB} \cdot \text{IN}$   
 MOMENT DUE TO  $F_v$ :  $M_2 = F_v \cdot 52 \text{ IN}$   
 AT N:  $\sigma_N = \frac{F_h}{A} + \frac{M_2}{S} - \frac{M_1}{S} = \frac{4596}{4.71} + \frac{2.006 \times 10^5}{17.1} - \frac{55150}{17.1} = 9180 \text{ PSI}$   
 AT M:  $\sigma_M = \frac{F_h}{A} - \frac{M_2}{S} + \frac{M_1}{S} = -7530 \text{ PSI}$

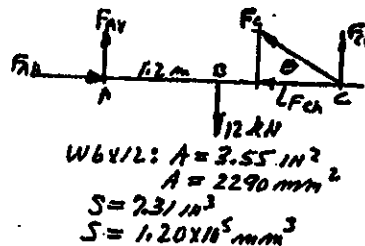


10-4  $M = 6000 \text{ LB} \cdot 52 \text{ IN} = 3.12 \times 10^5 \text{ LB} \cdot \text{IN}$   
 $\sigma = \frac{M}{S} = \frac{3.12 \times 10^5 \text{ LB} \cdot \text{IN}}{17.1 \text{ IN}^3} = 18246 \text{ PSI}$  TENSION AT N; COMP. AT M

10-5 LOADING IS SAME AS 11-3 EXCEPT  $F_h$  ACTS TO LEFT AND  $M_1$  IS OPPOSITE DIRECTION.  
 AT N:  $\sigma_N = \frac{-F_h}{A} + \frac{M_2}{S} + \frac{M_1}{S} = \frac{-4596}{4.71} + \frac{2.006 \times 10^5}{17.1} + \frac{55150}{17.1} = 13980 \text{ PSI}$   
 AT M:  $\sigma_M = \frac{-F_h}{A} - \frac{M_2}{S} - \frac{M_1}{S} = -915 - 11731 - 3225 = -15931 \text{ PSI}$

10-6  $M = (125 \text{ N})(145 \text{ mm}) = 18125 \text{ N} \cdot \text{mm}$ ;  $S = \frac{bh^3}{6} = \frac{(4)(16^3)}{6} = 66.67 \text{ mm}^3$   
 $\sigma = \frac{-F}{A} - \frac{M}{S} = \frac{-125 \text{ N}}{4 \text{ mm}^2} - \frac{18125 \text{ N} \cdot \text{mm}}{66.67 \text{ mm}^3} = -3.125 - 271.9 = -275 \text{ MPa}$

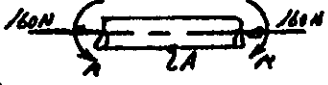
10-7  $F_{AV} = F_{CV} = 12 \text{ kN}/2 = 6.0 \text{ kN}$   
 $\tan \theta = 1.5/2.4 = 0.625$   
 $F_{CH} = F_{AH} = \frac{F_{CV}}{\tan \theta} = \frac{6.0 \text{ kN}}{0.625} = 9.6 \text{ kN}$   
 $M = F_{AV} \cdot 1.2 \text{ m} = 7.2 \text{ kN} \cdot \text{m}$  AT B  
 $\sigma = \frac{-F_h}{A} - \frac{M}{S} = \frac{-9600 \text{ N}}{2290 \text{ mm}^2} - \frac{7.2 \times 10^6 \text{ N} \cdot \text{mm}}{1.20 \times 10^5 \text{ mm}^3}$   
 $\sigma = -4.19 - 60.1 = -64.3 \text{ MPa}$



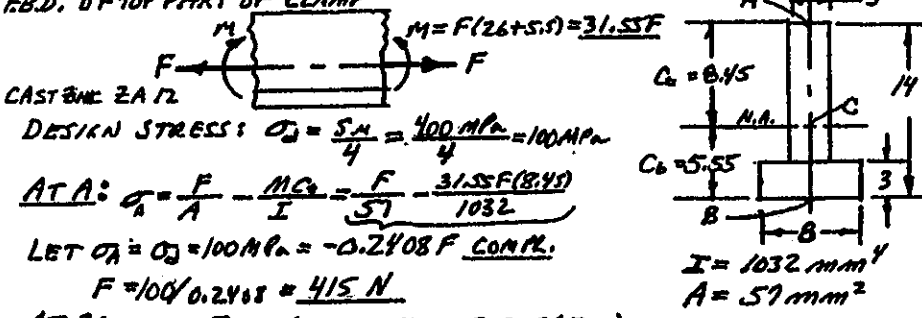
COMPRESSION ON TOP OF BEAM



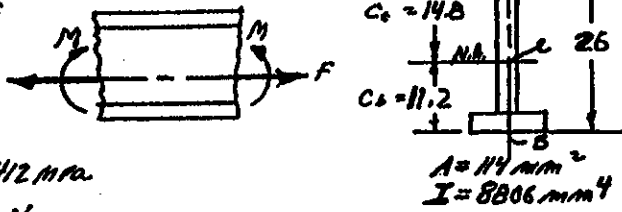
10-8  $M = 160 \text{ N} \cdot 80 \text{ mm} = 12800 \text{ N}\cdot\text{mm}$ ;  $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(12^2 - 10^2) = 34.6 \text{ mm}^2$   
 $S = \frac{\pi}{32} \frac{(D^4 - d^4)}{D} = \frac{\pi(12^4 - 10^4)}{32(12)} = 87.8 \text{ mm}^3$   
 AT POINT A:  $\sigma = \frac{-F}{A} - \frac{M}{S}$   
 $\sigma = \frac{-160 \text{ N}}{34.6 \text{ mm}^2} - \frac{12800 \text{ N}\cdot\text{mm}}{87.8 \text{ mm}^3} = 150.4 \text{ MPa}$



10-9 F.B.D. OF TOP PART OF CLAMP  
 CAST IRON 2A12  
 DESIGN STRESS:  $\sigma_d = \frac{S_u}{4} = \frac{400 \text{ MPa}}{4} = 100 \text{ MPa}$   
 AT A:  $\sigma_A = \frac{F}{A} - \frac{M C_A}{I} = \frac{F}{57} - \frac{31.55 F(8.45)}{1032}$   
 LET  $\sigma_A = \sigma_d = 100 \text{ MPa} = 0.2408 F \text{ COMP.}$   
 $F = 100 / 0.2408 = 415 \text{ N}$   
 AT B:  $\sigma_B = \frac{F}{A} + \frac{M C_B}{I} = \frac{F}{57} + \frac{31.55 F(5.55)}{1032} = 0.1872 F \text{ TENSION}$   
 LET  $\sigma_B = \sigma_d = 100 \text{ MPa} = 0.1872 F$   
 $F = 100 / 0.1872 = 534 \text{ N}$   
 LIMITING VALUE OF  $P = 415 \text{ N}$  IF  $S_{uL}$  IS EQUAL IN TENSION AND COMPRESSION. NOTE THAT  $S_{uL} > S_{uT}$  FOR SOME CAST ALLOYS.



10-10 ASTM A220 45008:  $S_{uT} = 448 \text{ MPa}$ ,  $S_{uL} = 1650 \text{ MPa}$   
 $\sigma_{dL} = 448/4 = 112 \text{ MPa}$ ;  $\sigma_{dC} = 1650/4 = 412 \text{ MPa}$   
 $M = F(42 + 11.2) = 53.2 F$   
 AT A:  $\sigma_A = \frac{F}{A} - \frac{M C_A}{I}$   
 $\sigma_A = \frac{F}{114} - \frac{53.2 F(14.8)}{8806}$   
 $\sigma_A = -0.0806 F = \sigma_{dC} = 412 \text{ MPa}$   
 $F = 412 / 0.0806 = 5109 \text{ N}$   
 AT B:  $\sigma_B = \frac{F}{A} + \frac{M C_B}{I} = \frac{F}{114} + \frac{53.2 F(11.2)}{8806} = 0.0764 F = \sigma_{dL} = 112 \text{ MPa}$   
 $F = 112 / 0.0764 = 1465 \text{ N}$  LIMITING VALUE



10-11 FOR M16x2 THREADED ROD: STRESS AREA =  $157 \text{ mm}^2$  (APP A3)  
 COMPUTE SECTION MODULUS BASED ON PROJ:  $S = \frac{\pi D^3}{32} = \frac{\pi(13.2)^3}{32} = 226 \text{ mm}^3$   
 $\sigma = K_t \left[ \frac{F}{A} + \frac{M}{S} \right] = 3.0 \left[ \frac{1200}{157} + \frac{(1200)(4)}{226} \right] = 724 \text{ MPa}$  TENSION  
 FOR  $\sigma_d = S_y/2$ ;  $S_y = 2\sigma = 2(724) = 1448 \text{ MPa}$   
 USE AISI 4140 Q&T 700,  $S_y = 1462 \text{ MPa}$  - ONE POSSIBLE CHOICE

10-12  $M_A = F_V(30) = 5850 \text{ N}\cdot\text{mm}$

$M_B = F_V(375) = 73125 \text{ N}\cdot\text{mm}$

$S_A = \frac{bh^2}{2} = \frac{10(14)^2}{2} = 327 \text{ mm}^3$ ;  $A_A = bh = 140 \text{ mm}^2$

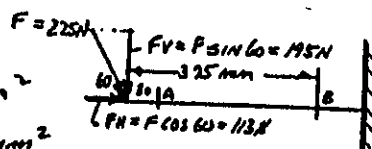
$S_B = \frac{10(25)^2}{2} = 1042 \text{ mm}^3$ ;  $A_B = bh = 250 \text{ mm}^2$

ASSUME  $S_{ut} \ll S_{uc}$  - TENSION GOVERNS DESIGN FACTOR.  $S_{ut} = 552 \text{ MPa}$

AT A:  $\sigma_A = \frac{-F_h}{A_A} + \frac{M_A}{S_A} = \frac{-113}{140} + \frac{5850}{327} = 17.08 \text{ MPa}$

AT B:  $\sigma_B = \frac{-F_h}{A_B} + \frac{M_B}{S_B} = \frac{-113}{250} + \frac{73125}{1042} = 69.7 \text{ MPa}$

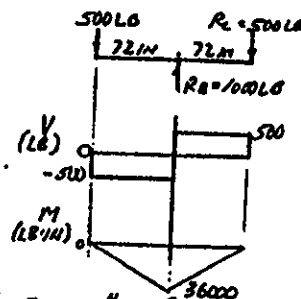
$N = \frac{S_{uc}}{\sigma_B} = \frac{552 \text{ MPa}}{69.7 \text{ MPa}} = 7.9$



10-13 FOR  $S3 \times 5.7$ :  $A = 1.67 \text{ in}^2$ ;  $S = 1.68 \text{ in}^3$

AT B-TOP:  $\sigma = \frac{F}{A} + \frac{M}{S} = \frac{4600}{1.67} + \frac{36000}{1.68} = 24183 \text{ PSI}$   
TENSION

AT B-BOTTOM:  
 $\sigma = \frac{F}{A} - \frac{M}{S} = \frac{4600}{1.67} - \frac{36000}{1.68} = -18674 \text{ PSI}$   
COMPRESSION



10-14 FOR TUBE:  $A = 2256 \text{ mm}^2$ ;  $I = 5.74 \times 10^6 \text{ mm}^4$ ;  $S = \frac{I}{c} = 7.65 \times 10^4 \text{ mm}^3$

LOAD:  $w = m \cdot g = 1000 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 9810 \text{ N}$

$\Sigma M_A = 0 = 9810(1.6) - B_V(0.6)$

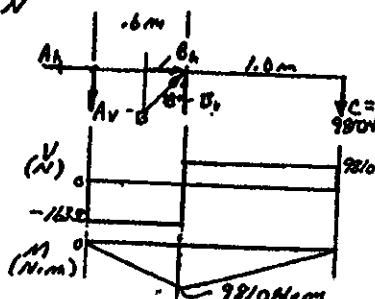
$B_V = 26160 \text{ N} = B_H = A_H$

$A_V = B_V - C_V = 26160 - 9810 = 16350 \text{ N}$

NEAR B-TO LEFT:

$\sigma = \frac{B_h}{A} + \frac{M_B}{S} = \frac{26160}{2256} + \frac{9810 \times 10^3}{7.65 \times 10^4}$

$\sigma = 140 \text{ MPa}$



10-15  $\sigma_{\text{MAX}}$  TO LEFT OF B (SEE 11-14).

$M_B = (1.0 \text{ m})(F) = (1000 \text{ mm})F$

$B_V = B_H = \frac{1.6}{0.6} F = 2.67F$

$\sigma_B = \frac{F}{S} = \frac{414 \text{ MPa}}{3} = 138 \text{ MPa}$

$\sigma = \frac{B_H}{A} + \frac{M_B}{S} = \frac{2.67F}{2256} + \frac{1000 F}{7.65 \times 10^4} = 0.0143 F = 138 \text{ MPa} = \sigma_B$

$F = 138 / 0.0143 = 9650 \text{ N}$ ;  $m = \frac{F}{g} = \frac{9650 \text{ kg}\cdot\text{m/s}^2}{9.81 \text{ m/s}^2} = 983 \text{ kg}$

10-16

$\sigma_B = 0.65 \sigma = 0.6(36000) = 21600 \text{ PSI}$

$\sigma = \frac{P}{A} + \frac{M}{S}$  USE TRIAL AND ERROR

FOR BENDING ONLY:

$S = \frac{M}{\sigma} = \frac{28800}{21600} = 1.33 \text{ in}^3$

FOR AXIAL TENSION:

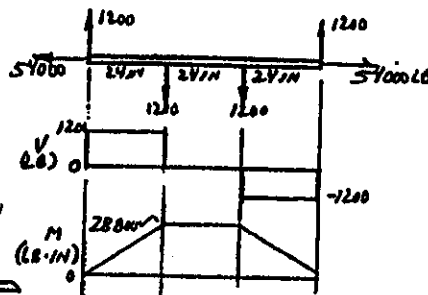
$A = \frac{P}{\sigma} = \frac{54000}{21600} = 2.50 \text{ in}^2$

TRY  $1\frac{1}{2} \times 1\frac{1}{2}$  (TWO)

$A = 2(1.94) = 3.88 \text{ in}^2$

$I = 2I_x = 2(3.04) = 6.08 \text{ in}^4$ ;  $C_b = \gamma = 1.09 \text{ in}$ ;  $C_t = 4.0 - 1.09 = 2.91 \text{ in}$

AT BOTTOM:  $\sigma = \frac{P}{A} + \frac{M C_b}{I} = \frac{54000}{3.88} + \frac{28800(1.09)}{6.08} = 19081 \text{ PSI}$



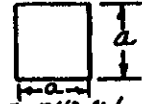
10-17FROM PROBLEM P6-77:  $M = 120 \text{ N}\cdot\text{m}$ ;  $P = 800 \text{ N}$  AXIAL TENSION

$$\sigma = \frac{P}{A} + \frac{M}{S} = \frac{P}{a^2} + \frac{M}{a^3/6} = \frac{P}{a^2} + \frac{6M}{a^3}$$

MULTIPLY BY  $a^3$ 

$$\sigma a^3 = Pa + 6M: \sigma a^3 - Pa - 6M = 0: \text{LET } \sigma = 42 \text{ MPa} = 42 \text{ N/mm}^2$$

$$42 a^3 - 800a - 6(120000) = 0: \text{THEN } a = 26 \text{ mm}$$

10-18FROM P6-78:  $M = 344 \text{ LB}\cdot\text{IN}$ ;  $P = 250 \text{ LB}$  AXIAL COMP.

$$\sigma = \frac{P}{A} - \frac{M}{S} = \frac{P}{a^2} - \frac{M}{a^3/6} = \frac{P}{a^2} - \frac{6M}{a^3}$$

$$\sigma a^3 = -Pa - 6M: \sigma a^3 + Pa + 6M = 0: \text{LET } \sigma = -6000 \text{ PSI (COMP.)}$$

$$-6000 a^3 + 250a + 6(344) = 0: \text{THEN } a = 0.720 \text{ IN}$$

10-19FROM P6-79:  $M = 42200 \text{ N}\cdot\text{mm}$ ;  $P = 1200 \text{ N}$  AXIAL TENSION

$$\text{FROM 11-17: } \sigma a^3 - Pa - 6M = 0: 42 a^3 - 1200 - 6(42200) = 0: a = 18.7 \text{ mm}$$

10-20FROM P6-80:  $M = 520 \text{ LB}\cdot\text{IN}$ ;  $P = 400 \text{ LB}$  AXIAL TENSION

$$\text{FROM 11-17: } \sigma a^3 - Pa - 6M = 0: 6000 a^3 - 400a - 6(520) = 0: a = 0.832 \text{ IN}$$

10-21

$$A = \pi(40)^2/4 = 1257 \text{ mm}^2: z_p = \pi(40)^3/16 = 12566 \text{ mm}^3$$

$$\sigma = \frac{P}{A} = \frac{150000 \text{ N}}{1257 \text{ mm}^2} = 119 \text{ MPa}: \tau = \frac{T}{z_p} = \frac{500000 \text{ N}\cdot\text{mm}}{12566 \text{ mm}^3} = 39.8 \text{ MPa}$$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{119}{2}\right)^2 + 39.8^2} = 71.6 \text{ MPa}$$

10-22

$$A = \pi(2.25)^2/4 = 3.98 \text{ IN}^2: z_p = \pi(2.25)^3/16 = 2.24 \text{ IN}^3$$

$$\sigma = \frac{P}{A} = \frac{47000 \text{ LB}}{3.98 \text{ IN}^2} = 11809 \text{ PSI}: \tau = \frac{T}{z_p} = \frac{8500 \text{ LB}\cdot\text{IN}}{2.24 \text{ IN}^3} = 3795 \text{ PSI}$$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{11809}{2}\right)^2 + 3795^2} = 7019 \text{ PSI}$$

10-23

$$A = \pi(4.00)^2/4 = 12.57 \text{ IN}^2: z_p = \pi(4.00)^3/16 = 12.57 \text{ IN}^3$$

$$\sigma = \frac{P}{A} = \frac{-40000 \text{ LB}}{12.57 \text{ IN}^2} = -3183 \text{ PSI}: \tau = \frac{T}{z_p} = \frac{25000 \text{ LB}\cdot\text{IN}}{12.57 \text{ IN}^3} = 1989 \text{ PSI}$$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{3183}{2}\right)^2 + 1989^2} = 2548 \text{ PSI}$$

10-24

$$\text{FOR 12 IN PIPE: } A = 15.7 \text{ IN}^2: z_p = 94.18 \text{ IN}^3$$

$$\sigma = \frac{T}{A} = \frac{-250000 \text{ LB}}{15.7 \text{ IN}^2} = -15883 \text{ PSI}: \tau = \frac{T}{z_p} = \frac{180000 \text{ LB}\cdot\text{IN}}{94.18 \text{ IN}^3} = 1911 \text{ PSI}$$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{15883}{2}\right)^2 + 1911^2} = 8168 \text{ PSI}$$

10-25

$$\text{FOR 3 IN PIPE: } A = 2.228 \text{ IN}^2: z_p = 3.448 \text{ IN}^3$$

$$\sigma = \frac{P}{A} = \frac{-25000 \text{ LB}}{2.228 \text{ IN}^2} = -11221 \text{ PSI}: \tau = \frac{T}{z_p} = \frac{15500 \text{ LB}\cdot\text{IN}}{3.448 \text{ IN}^3} = 4495 \text{ PSI}$$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{11221}{2}\right)^2 + 4495^2} = 7189 \text{ PSI}$$

10-26

$$T = (20 \text{ LB})(8 \text{ FT})(12 \text{ IN/FT}) = 1920 \text{ LB}\cdot\text{IN}: M = (20 \text{ LB})(15 \text{ FT})(12 \text{ IN/FT}) = 3600 \text{ LB}\cdot\text{IN}$$

$$\tau_c = \sqrt{\tau^2 + \tau_n^2} = \sqrt{920^2 + 3600^2} = 4080 \text{ LB/IN}$$

$$z_p = \frac{\pi}{16} \frac{D^4 - d^4}{D} = \frac{\pi(1.50^4 - 1.375^4)}{16(1.50)} = 0.195 \text{ IN}^3$$

$$\tau = \frac{\tau_c}{z_p} = \frac{4080 \text{ LB}\cdot\text{IN}}{0.195 \text{ IN}^3} = 20920 \text{ PSI}$$

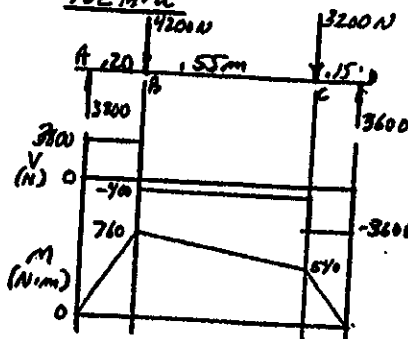
$$\text{LET } T_d = SY/2N: N = \frac{SY}{2\tau} = \frac{40000}{2(20920)} = 0.96 \text{ UNSAFE}$$

10-27 NEAR SUPPORT:  $T = 300 \text{ F} = 300(1200) = 3.6 \times 10^5 \text{ N}\cdot\text{mm}$   
 $M = 450 \text{ F} = 450(1200) = 5.4 \times 10^5 \text{ N}\cdot\text{mm}$   
 $T_c = \sqrt{T^2 + M^2} = 6.49 \times 10^5 \text{ N}\cdot\text{mm}$ ;  $z_p = \frac{\pi}{16} (40)^3 = 12566 \text{ mm}^3$   
 $T = \frac{T_c}{z_p} = \frac{6.49 \times 10^5 \text{ N}\cdot\text{mm}}{12566 \text{ mm}^3} = 51.6 \text{ MPa}$

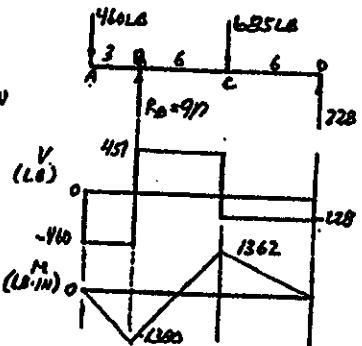
10-28 TOTAL DOWNWARD LOAD =  $200 + 200 + 300 + 600 = 1300 \text{ LB}$   
 $M = (1300 \text{ LB})(3 \text{ ft}) = 46800 \text{ LB}\cdot\text{ft}$  AT SUPPORT  
NET TORQUE =  $600(40) + 300(20) - 200(20) - 200(20) = 18000 \text{ LB}\cdot\text{ft}$   
 $T_c = \sqrt{T^2 + M^2} = \sqrt{18000^2 + 46800^2} = 50142 \text{ LB}\cdot\text{ft}$   
REQD  $z_p = \frac{T_c}{T_s} = \frac{50142 \text{ LB}\cdot\text{ft}}{8000 \text{ LB}/\text{ft}^2} = 6.27 \text{ in}^3$  USE 4 IN SCH 40 PIPE

10-29  $M_B = 2400 \text{ N}(150 \text{ mm}) = 360000 \text{ N}\cdot\text{mm}$ ;  $S = \frac{\pi D^3}{32} = \frac{\pi (20)^3}{32} = 785 \text{ mm}^3$   
 $z_p = \frac{\pi D^3}{16} = \frac{\pi (20)^3}{16} = 1571 \text{ mm}^3$ ;  $T = \frac{M}{z_p} = \frac{360000 \text{ N}\cdot\text{mm}}{1571 \text{ mm}^3} = 9.55 \text{ MPa}$   
 $\sigma = \frac{M}{S} = \frac{360000 \text{ N}\cdot\text{mm}}{785 \text{ mm}^3} = 459 \text{ MPa}$   
 $T_{\text{MAX}} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + T^2} = \sqrt{\left(\frac{459}{2}\right)^2 + 9.55^2} = 98.2 \text{ MPa}$

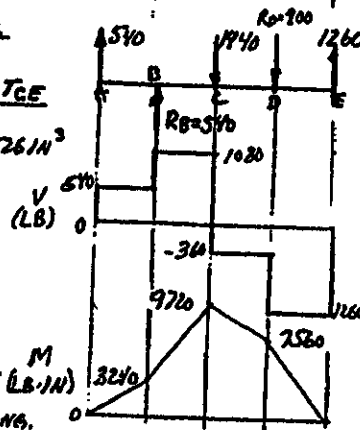
10-30  $S = \frac{\pi D^3}{32} = \frac{\pi (25)^3}{32} = 1534 \text{ mm}^3$   
 $z_p = \frac{\pi D^3}{16} = 25 = 3068 \text{ mm}^3$   
 $\sigma = \frac{M}{S} = \frac{760000 \text{ N}\cdot\text{mm}}{1534 \text{ mm}^3} = 495 \text{ MPa}$   
 $T = \frac{I}{z_p} = \frac{458000}{3068} = 1467 \text{ MPa}$   
 $T_{\text{MAX}} = \sqrt{\left(\frac{495}{2}\right)^2 + 1467^2} = 1488 \text{ MPa}$



10-31  $T = \frac{63000 \text{ (F)}}{M} = \frac{63000(25)}{1150} = 1370 \text{ LB}\cdot\text{ft}$   
AT B:  $T_c = \sqrt{T^2 + M^2} = \sqrt{1370^2 + 1380^2} = 1945 \text{ LB}\cdot\text{ft}$   
 $z_p = \frac{\pi D^3}{16} = \frac{\pi (1.6)^3}{16} = 0.196 \text{ in}^3$   
 $T = \frac{T_c}{z_p} = \frac{1945}{0.196} = 9923 \text{ psi} = T_d = \frac{S_y}{2N}$   
REQD  $S_y = 2NT = 2(16)(9923) = 119000 \text{ psi}$   
AISI 1141 OQT 900  $S_y = 129 \text{ ksi}$ , 15% ELONG.



10-32 (a)  $T_A = (150 - 90)(6) = 2160 \text{ LB}\cdot\text{ft}$  CCW =  $T_{AC}$   
 $T_C = (200 - 240)(4) = 3840 \text{ LB}\cdot\text{ft}$  CW  
 $T_E = (650 - 210)(2) = 1680 \text{ LB}\cdot\text{ft}$  CCW =  $T_{CE}$   
(b)  $z_p = \frac{\pi D^3}{16} = \frac{\pi (1.75)^3}{16} = 1.05 \text{ in}^3$ ;  $S = \frac{\pi D^3}{32} = 0.526 \text{ in}^3$   
AT C:  $T = \frac{2160(1.6)}{1.05} = 3291 \text{ psi}$   
 $\sigma = \frac{9720(1.6)}{0.526} = 29567 \text{ psi}$



(c)  $T_{\text{MAX}} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + T^2} = 15145 \text{ psi} = T_d = \frac{S_y}{2N}$   
LET  $N = 4$   
REQD  $S_y = 2NT = 2(4)(15145) = 121160 \text{ psi}$  (LB-IN)  
AISI 1141 OQT 900,  $S_y = 129 \text{ ksi}$ , 15% ELONG.

10-33

$$T_B = (500 - 200)(50) = 60000 \text{ N}\cdot\text{mm}$$

$$T_C = (600 - 120)(125) = 60000 \text{ N}\cdot\text{mm}$$

$$A_{TB} : Z_P = \frac{\pi(28)^3}{16} = 4310 \text{ mm}^3 ; A = \frac{\pi(28)^2}{4} = 616 \text{ mm}^2$$

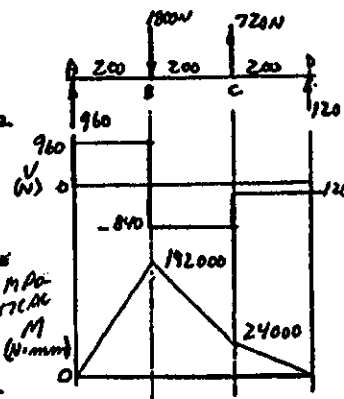
$$S = \frac{\pi(28)^3}{32} = 2155 \text{ mm}^3$$

$$T = T/2 = \frac{60000}{2} = 30000 \text{ N}\cdot\text{mm}$$

$$\sigma = T/S = \frac{30000}{2155} = 13.9 \text{ MPa}$$

$$\sigma_c = -P/A = \frac{-6200}{616} = -10.1 \text{ MPa}$$

$$T_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + T^2} = \sqrt{\left(\frac{13.9}{2}\right)^2 + (10.1)^2} = 12.6 \text{ MPa}$$



10-34

READ  $\sigma = 2N(67.1) = 58 \text{ MPa}$  - USE THIS 1040 1100  
 $N = 9$

10-35

B-32 UNC THREAD:  $D_M = 0.164 \text{ in}$ ;  $A_t = 0.0140 \text{ in}^2$   
 IN THREADS:  $P = \sigma A_t = (5000 \text{ lb/in}^2)(0.0140 \text{ in}^2) = 70 \text{ lb}$   
 FOR  $D_M$ :  $A_s = A_M = \pi D_M^2/4 = \pi(0.164)^2/4 = 0.0211 \text{ in}^2$   
 $\sigma = P/A_s = 70/0.0211 = 3317 \text{ psi}$ ;  $T = F_s/A_t = 120/0.0140 = 8571 \text{ psi}$   
 $T_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + T^2} = \sqrt{\left(\frac{3317}{2}\right)^2 + 8571^2} = 8758 \text{ psi}$

10-36

FOR 1/4-20 UNC THREAD:  $D_M = 0.250 \text{ in}$ ;  $A_t = 0.0318 \text{ in}^2$   
 $P = \sigma A_t = (5000)(0.0318) = 159 \text{ lb}$   
 FOR  $D_M$ :  $A_R = A_S = \pi(0.25)^2/4 = 0.0491 \text{ in}^2$   
 $\sigma = P/A_R = 159/0.0491 = 3238 \text{ psi}$ ;  $T = F_s/A_t = 775/0.0318 = 24371 \text{ psi}$   
 $T_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + T^2} = \sqrt{\left(\frac{3238}{2}\right)^2 + 24371^2} = 24499 \text{ psi}$

10-37

FOR 4-48 UNF THREAD:  $D_M = 0.112 \text{ in}$ ;  $A_t = 0.00661 \text{ in}^2$   
 $P = \sigma A_t = 15000(0.00661) = 99.2 \text{ lb}$   
 FOR  $D_M$ :  $A_R = A_S = \pi(0.112)^2/4 = 0.00985 \text{ in}^2$   
 $\sigma = P/A_R = 99.2/0.00985 = 10069 \text{ psi}$ ;  $T = F_s/A_t = 50/0.00661 = 7564 \text{ psi}$   
 $T_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + T^2} = \sqrt{\left(\frac{10069}{2}\right)^2 + 7564^2} = 7749 \text{ psi}$

10-38

FOR 1/4-12 THREAD:  $D_M = 1.250 \text{ in}$ ;  $A_t = 1.073 \text{ in}^2$   
 $P = \sigma A_t = 15000(1.073) = 16095 \text{ lb}$   
 FOR  $D_M$ :  $A_R = A_S = \pi(1.25)^2/4 = 1.227 \text{ in}^2$   
 $\sigma = P/A_R = 16095/1.227 = 13115 \text{ psi}$ ;  $T = F_s/A_t = 2500/1.073 = 2330 \text{ psi}$   
 $T_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + T^2} = \sqrt{\left(\frac{13115}{2}\right)^2 + 2330^2} = 6867 \text{ psi}$

10-39

FOR M16x2:  $D_M = 16 \text{ mm}$ ;  $A_t = 157 \text{ mm}^2$   
 $P = \sigma A_t = (120 \text{ N/mm}^2)(157 \text{ mm}^2) = 18840 \text{ N}$   
 FOR  $D_M$ :  $A_R = A_S = \pi(16)^2/4 = 201 \text{ mm}^2$   
 $\sigma = P/A_R = 18840/201 = 93.7 \text{ MPa}$ ;  $T = F_s/A_t = 8000/157 = 50.9 \text{ MPa}$   
 $T_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + T^2} = \sqrt{\left(\frac{93.7}{2}\right)^2 + 50.9^2} = 61.5 \text{ MPa}$

10-40

FOR M48x5:  $D_M = 48 \text{ mm}$ ;  $A_t = 1473 \text{ mm}^2$   
 $P = \sigma A_t = (120 \text{ N/mm}^2)(1473 \text{ mm}^2) = 1.77 \times 10^5 \text{ N}$   
 FOR  $D_M$ :  $A_R = A_S = \pi(48)^2/4 = 1810 \text{ mm}^2$   
 $\sigma = P/A_R = \frac{1.77 \times 10^5}{1810} = 97.7 \text{ MPa}$ ;  $T = \frac{F_s}{A_t} = \frac{80000}{1473} = 54.3 \text{ MPa}$   
 $T_{MAX} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + T^2} = \sqrt{\left(\frac{97.7}{2}\right)^2 + 54.3^2} = 65.9 \text{ MPa}$

10-41

AT B:  $V = 2750 \text{ LB}$ ;  $M = 82500 \text{ LB}\cdot\text{IN}$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

FOR BEAM CROSS SECTION:

$$A = bh = (2)(6) = 12.00 \text{ IN}^2$$

$$S = bh^2/6 = (2)(6)^2/6 = 12.00 \text{ IN}^3$$

$$I = bh^3/12 = (2)(6)^3/12 = 36.00 \text{ IN}^4$$

AT a:  $\tau = 0$ ;  $\sigma = M/S$

$$\sigma = \frac{82500 \text{ LB}\cdot\text{IN}}{12.00 \text{ IN}^3} = 6875 \text{ psi}$$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{\sigma}{2} = 3438 \text{ psi}$$

AT b: SAME AS (a)

AT c:  $\sigma = 0$ ;  $\tau = \tau_{\text{MAX}} = 3V/2A$

$$\tau_{\text{MAX}} = \frac{3(2750)}{2(12)} = 344 \text{ psi}$$

AT d:  $\sigma = \frac{Mx}{I} = \frac{(82500)(2.0)}{36.0} = 4583 \text{ psi}$

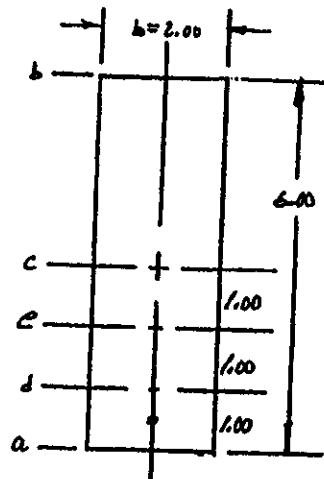
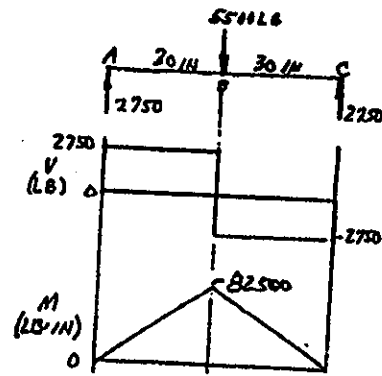
$$\tau = \frac{VQ}{Ib} = \frac{(2750)(2.0)(1.0)(2.5)}{(36.0)(2.0)} = 191 \text{ psi}$$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{4583}{2}\right)^2 + 191^2} = 2300 \text{ psi}$$

AT e:  $\sigma = \frac{Mx}{I} = \frac{(82500)(1.0)}{36.0} = 2292 \text{ psi}$

$$\tau = \frac{VQ}{Ib} = \frac{(2750)(2.0)(2.0)(2.0)}{(36.0)(2.0)} = 306 \text{ psi}$$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{2292}{2}\right)^2 + 306^2} = 1186 \text{ psi}$$



10-42

FROM 11-35:  $V = 2750 \text{ LB}$ ;  $M = 82500 \text{ LB}\cdot\text{IN}$

FOR ALUM  $I 6 \times 4.692$ :  $A = 3.990 \text{ IN}^2$

$$S = 8.50 \text{ IN}^3$$

$$I = 25.50 \text{ IN}^4$$

AT a AND b:  $\tau = 0$ ;  $\sigma = M/S$

$$\sigma = \frac{82500 \text{ LB}\cdot\text{IN}}{8.50 \text{ IN}^3} = 9706 \text{ psi}$$

$$\tau_{\text{MAX}} = \frac{\sigma}{2} = 4853 \text{ psi}$$

AT c:  $\sigma = 0$ ;  $\tau = \tau_{\text{MAX}} = \frac{VQ}{Ib}$

$$Q = (.35)(4.00)(2.825) + (.21)(2.65)(1.825)$$

$$Q = 3.955 + 0.737 = 4.692 \text{ IN}^3$$

$$\tau_{\text{MAX}} = \frac{(2750)(4.692)}{(25.50)(.21)} = 2410 \text{ psi}$$

AT d:

$$\sigma = \frac{Mx}{I} = \frac{(82500)(2.0)}{25.50} = 6471 \text{ psi}$$

$$\tau = \frac{VQ}{Ib} = \frac{(2750)(3.999)}{(25.50)(0.21)} = 8213 \text{ psi}$$

$$Q = 3.955 + (.21)(0.65)(2.325) = 3.999 \text{ IN}^3$$

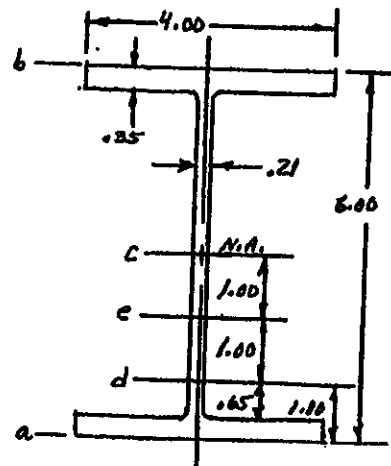
$$\tau_{\text{MAX}} = \sqrt{\left(\frac{6471}{2}\right)^2 + 8213^2} = 8827 \text{ psi}$$

AT e:

$$\sigma = \frac{Mx}{I} = \frac{82500(1.0)}{25.50} = 3235 \text{ psi}; Q = 3.955 + (0.21)(1.65)(2.825) = 4.241 \text{ IN}^3$$

$$\tau = \frac{VQ}{Ib} = \frac{(2750)(4.241)}{(25.50)(0.21)} = 2178 \text{ psi}$$

$$\tau_{\text{MAX}} = \sqrt{\left(\frac{3235}{2}\right)^2 + 2178^2} = 2713 \text{ psi}$$



10-43 SEE 11-41 FOR BEAM SECTION PROPERTIES

AT MIDDLE OF BEAM - B:

AT A AND b:  $\sigma = \frac{M}{S} = \frac{45000}{12.00} = 3750 \text{ PSI}$

$T = 0$ ;  $T_{\text{MAX}} = \frac{V}{2} = 1875 \text{ PSI}$

AT c:  $\sigma = 0$ ;  $T = 0$ ;  $T_{\text{MAX}} = 0$

AT d:  $\sigma = \frac{Mx}{I} = \frac{45000(2.0)}{36.0} = 2500 \text{ PSI}$

$T = 0$ ;  $T_{\text{MAX}} = \frac{V}{2} = 1250 \text{ PSI}$

AT e:  $\sigma = \frac{Mx}{I} = \frac{45000(1.0)}{36.0} = 1250 \text{ PSI}$ ;  $T = 0$ ;  $T_{\text{MAX}} = \frac{V}{2} = 625 \text{ PSI}$

AT SUPPORTS A AND C:  $V = 3000 \text{ LB}$ ;  $M = 0$ ;  $\sigma = 0$

AT a AND b:  $T = 0$ ;  $T_{\text{MAX}} = 0$

AT c:  $T = 3V/2A = 3(3000)/(2)(12) = 375 \text{ PSI} = T_{\text{MAX}}$

AT d:  $T = T_{\text{MAX}} = \frac{VQ}{It} = \frac{(3000)(2.0)(1.0)(2.5)}{(36.0)(2.0)} = 208 \text{ PSI}$

AT e:  $T = T_{\text{MAX}} = \frac{VQ}{It} = \frac{(3000)(1.0)(2.0)(2.0)}{(36.0)(2.0)} = 333 \text{ PSI}$

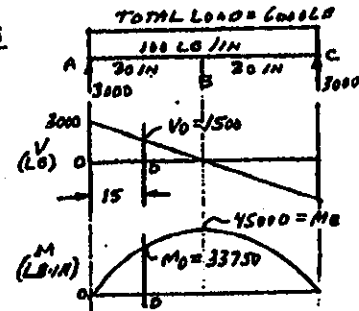
AT D - 15IN FROM A:  $V = 1500 \text{ LB}$ ;  $M = 33750 \text{ LB}\cdot\text{IN}$

AT a AND b:  $T = 0$ ;  $\sigma = \frac{M}{S} = \frac{33750}{12.0} = 2813 \text{ PSI}$ ;  $T_{\text{MAX}} = \frac{V}{2} = 1406 \text{ PSI}$

AT c:  $\sigma = 0$ ;  $T = \frac{3V}{2A} = \frac{3(1500)}{2(12)} = 188 \text{ PSI} = T_{\text{MAX}}$

AT d:  $\sigma = \frac{Mx}{I} = \frac{(33750)(2.0)}{36.0} = 1875 \text{ PSI}$   
 $T = \frac{VQ}{It} = \frac{(1500)(2.0)(1.0)(2.5)}{(36.0)(2.0)} = 104 \text{ PSI}$  }  $T_{\text{MAX}} = \sqrt{\left(\frac{1875}{2}\right)^2 + 104^2} = 943 \text{ PSI}$

AT e:  $\sigma = \frac{Mx}{I} = \frac{(33750)(1.0)}{36.0} = 938 \text{ PSI}$   
 $T = \frac{VQ}{It} = \frac{(1500)(2.0)(2.0)(2.0)}{(36.0)(2.0)} = 167 \text{ PSI}$  }  $T_{\text{MAX}} = \sqrt{\left(\frac{938}{2}\right)^2 + 167^2} = 494 \text{ PSI}$



10-44

FROM FIG 5-25:  $Q = 0.20(25)^2 = 0.208(25)^2 = 32.50 \text{ mm}^3$

$T = T/Q = \frac{245000 \text{ N}\cdot\text{mm}}{32.50 \text{ mm}^3} = 75.4 \text{ MPa}$ ;  $\sigma = \frac{P}{A} = \frac{75000 \text{ N}}{(25)^2 \text{ mm}^2} = 120 \text{ MPa}$

$T_{\text{MAX}} = \sqrt{\left(\frac{120}{2}\right)^2 + 75.4^2} = 96.4 \text{ MPa}$

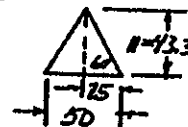
10-45

FROM FIG 5-25:  $Q = \frac{bh^2}{3 + 1.8(h/b)} = \frac{50(30)^2}{3 + 1.8(90/50)} = 11029 \text{ mm}^3$

$T = \frac{T}{Q} = \frac{525000 \text{ N}\cdot\text{mm}}{11029 \text{ mm}^3} = 47.6 \text{ MPa}$ ;  $\sigma = \frac{P}{A} = \frac{175000}{(30)(50)} = 116.7 \text{ MPa}$

$T_{\text{MAX}} = \sqrt{\left(\frac{116.7}{2}\right)^2 + 47.6^2} = 75.3 \text{ MPa}$

10-46 FROM FIG. S-25:  $Q = 0.050 a^3 = 0.050 (50)^3 = 6250 \text{ mm}^3$   
 $\tau = \frac{I}{Q} = \frac{775000 \text{ N}\cdot\text{mm}}{6250 \text{ mm}^3} = 124 \text{ MPa}$ ;  $\sigma = \frac{P}{A} = \frac{115000}{\frac{1}{2}(50)(13.3)} = 106 \text{ MPa}$   
 $\tau_{\text{MAX}} = \sqrt{\left(\frac{106}{2}\right)^2 + 124^2} = \underline{135 \text{ MPa}}$



10-47  $S_y = 50 \text{ ksi}$ ;  $\sigma_a = \frac{S_y}{3} = \frac{50000}{3} = 16667 \text{ psi} = P/A$   
 (a)  $P = \sigma \cdot A = (16667 \text{ lb/in}^2)(2.59 \text{ in}^2) = 43167 \text{ lb}$   
 $\tau_{\text{MAX}} = \frac{\sigma}{2} = \underline{8333 \text{ psi}}$

(b) FROM FIG. S-25:  $Q = 2t(a-t)(b-t) = 2(0.25)(2.75)(2.75) = 3.78 \text{ in}^3$   
 $\tau = \frac{I}{Q} = \frac{(750)(12)(10.1 \text{ in})}{3.78 \text{ in}^3} = 3015 \text{ psi}$

$\tau_{\text{MAX}} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{16667}{2}\right)^2 + 3015^2} = \underline{8862 \text{ psi}} = \frac{S_y}{2N}$

$N = \frac{S_y}{2\tau_{\text{MAX}}} = \frac{50000}{2(8862)} = \underline{2.82}$



10-48

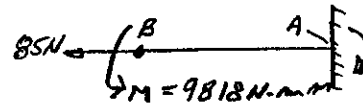
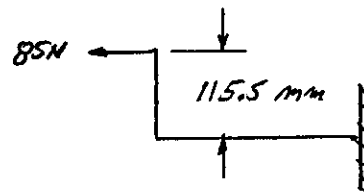
TENSILE: TOP FROM B TO A

$$\sigma = \frac{85N}{33.0 \text{ mm}^2} + \frac{(9818 \text{ N}\cdot\text{mm})(2.5 \text{ mm})}{128 \text{ mm}^4}$$

$$\sigma = 2.58 + 191.7 = 194.3 \text{ N/mm}^2 = 194 \text{ MPa}$$

COMPRESSIVE: BOTTOM FROM B TO A

$$\sigma = 2.58 - \frac{(9818 \text{ N}\cdot\text{mm})(4.5 \text{ mm})}{128 \text{ mm}^4} = -343 \text{ MPa}$$

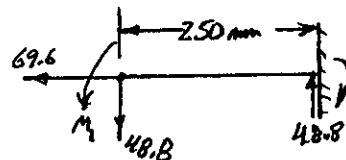
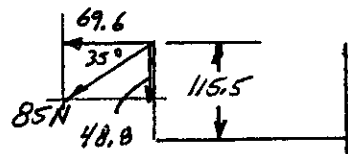


10-49 TENSILE: TOP AT A

$$\sigma = \frac{69.6 \text{ N}}{33.0 \text{ mm}^2} + \frac{(20239 \text{ N}\cdot\text{mm})(2.5 \text{ mm})}{128 \text{ mm}^4} = 397 \text{ MPa}$$

COMPRESSIVE: BOTTOM

$$\sigma = 2.11 \text{ MPa} - \frac{(20239)(4.5)}{128} = -709 \text{ MPa}$$



$$M = 69.6(115.5) = 8039 \text{ N}\cdot\text{mm}$$

$$M_2 = 48.8(250) = 12200 \text{ N}\cdot\text{mm}$$

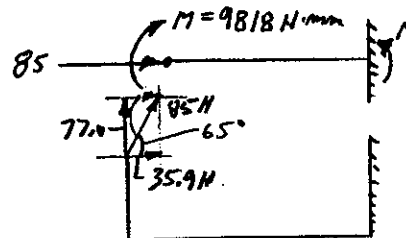
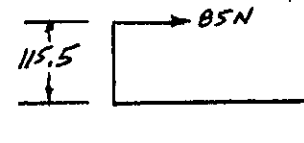
$$M_1 + M_2 = 20239 \text{ N}\cdot\text{mm}$$

10-50 TENSION: BOTTOM

$$\sigma = \frac{-85 \text{ N}}{33.0 \text{ mm}^2} + \frac{(9818 \text{ N}\cdot\text{mm})(4.5 \text{ mm})}{128 \text{ mm}^4} = 343 \text{ MPa}$$

COMPRESSIVE: TOP

$$\sigma = \frac{-85 \text{ N}}{33.0 \text{ mm}^2} - \frac{(9818)(2.5)}{128} = -194 \text{ MPa}$$



10-51 TENSION: BOTTOM

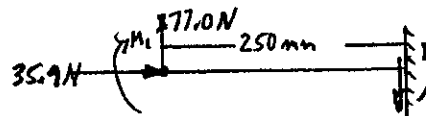
$$\sigma = \frac{-25.9 \text{ N}}{33.0 \text{ mm}^2} + \frac{(23396 \text{ N}\cdot\text{mm})(4.5 \text{ mm})}{128 \text{ mm}^4}$$

$$\sigma = -1.09 + 822.5 = 821 \text{ MPa}$$

COMPRESSIVE: TOP

$$\sigma = \frac{-35.9}{33.0} - \frac{(23396)(2.5)}{128} = -1.09 - 457.0$$

$$\sigma = -458 \text{ MPa}$$



$$M_1 = (35.9)(115.5) = 4146 \text{ N}\cdot\text{mm}$$

$$M_2 = (77.0)(250) = 19250 \text{ N}\cdot\text{mm}$$

$$M_1 + M_2 = 23396 \text{ N}\cdot\text{mm}$$

10-52 2x3x1/4 STEEL TUBING,  $S = 2.10 \text{ in}^3$ ,  $A = 2.59 \text{ in}^2$

$$M_1 = (615.6 \text{ lb})(15.0 \text{ in}) = 9234 \text{ lb}\cdot\text{in}$$

$$M_2 = (169 \text{ lb})(7.5 \text{ in}) = 12683 \text{ lb}\cdot\text{in}$$

AXIAL LOAD = 169 LB  $\downarrow$  TENSION

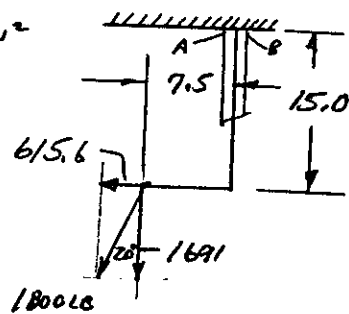
NET BENDING MOMENT =  $M_2 - M_1$

$$M_{\text{NET}} = 12683 - 9234 = 3449 \text{ lb}\cdot\text{in}$$

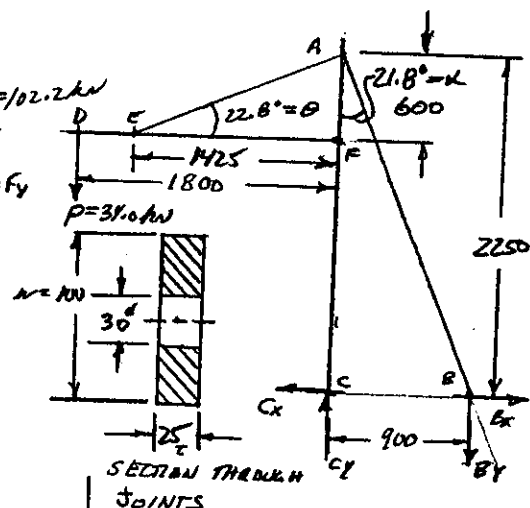
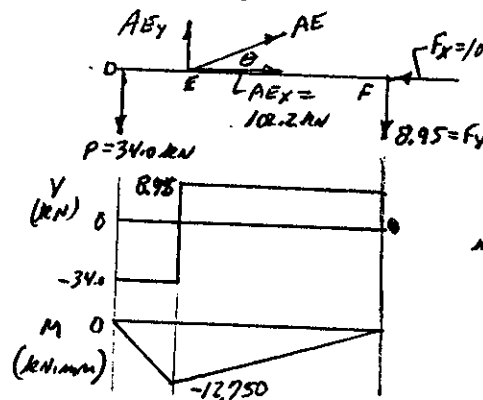
PRODUCES TENSION AT A, COMP. AT B

$$\sigma_A = \frac{(169 \text{ lb})}{2.59 \text{ in}^2} + \frac{(3449 \text{ lb}\cdot\text{in})}{2.10 \text{ in}^3} = 653 + 1642 = 2295 \text{ psi TENSION}$$

$$\sigma_B = 653 - 1642 = -989 \text{ psi COMPRESSION}$$



10-53 LOAD =  $P = 34.0 \text{ kN}$



$$\sum M_F = 0 = (34.0 \text{ kN})(1800 \text{ mm}) - AE_y(1425 \text{ mm})$$

$$AE_y = 42.95 \text{ kN}$$

$$\sum F_y = 0 = 42.95 - 34.0 - F_y; F_y = 8.95 \text{ kN} \downarrow$$

$$AE = AE_y / \sin \theta = 42.95 \text{ kN} / \sin 22.8^\circ = 110.8 \text{ kN}$$

$$AE_x = AE \cos \theta = (110.8 \text{ kN}) \cos 22.8^\circ = 102.2 \text{ kN}$$

AT E: FLAT PLATE WITH CENTRAL HOLE  
APP. A-21-4, CURVE C - BENDING

$$\sigma_b = \frac{M K_{tc} C}{I_{\text{NET}}} = \frac{K_{tc} M_{\text{NET}}}{(100^3 - d^3)(t)} = \frac{(12750 \text{ kN}\cdot\text{mm})(100 \text{ mm})}{(100^3 - 30^3)(25 \text{ mm})} \times \frac{1000 \text{ N}}{\text{kN}} = 52.4 \text{ MPa}$$

$$d/t = 30/100 = 0.30 \Rightarrow K_{tc} = 1.0$$

$$\text{DIRECT COMPRESSION } \sigma_c = \frac{K_c AE_x}{A_{\text{NET}}} = \frac{(3.70)(102.2 \text{ kN})}{(100 - 30)(25 \text{ mm})} \times \frac{1000 \text{ N}}{\text{kN}} = 216 \text{ MPa}$$

COMBINATION OF  $\sigma_b$  AND  $\sigma_c$  IS PROBLEMATIC BECAUSE  $\sigma_{b \text{ max}}$  OCCURS NEAR TOP OR BOTTOM OF SECTION WHILE  $\sigma_{c \text{ max}}$  OCCURS NEAR CENTERLINE NEAR HOLE.

$\sigma_{\text{max TENSILE}}$  IS AT E TO LEFT WHERE  $\sigma_b = 52.4 \text{ MPa}$  AND NO COMP. STRESS.  
 $\sigma_{\text{max COMP.}}$  IS TO RIGHT OF E WITH A VALUE BETWEEN 216 MPa AND  $216 + 52.4 = 268 \text{ MPa}$  ON LOWER PART OF THE CROSS SECTION.

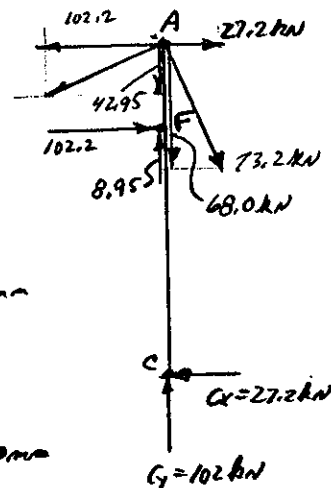
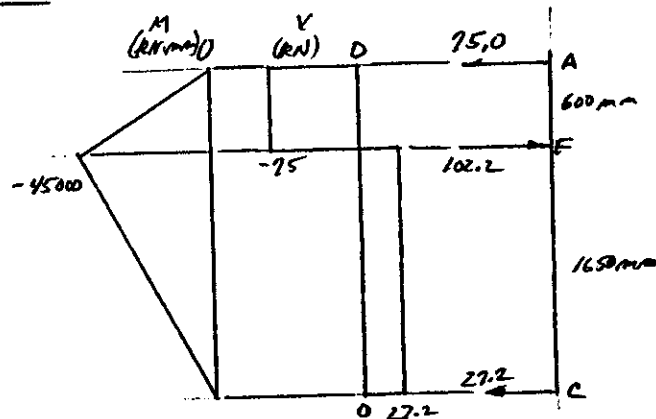
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10-53 (CONTINUED)

MEMBER AFC - LOADS FROM FBD OF WHOLE STRUCTURE

AXIAL LOAD:  $AF = 42.95 + 68.0 = 110.95 \text{ kN COMP.}$

$CF = 110.95 - 8.95 = 102 \text{ kN COMP.}$



BENDING STRESS AT F (SAME SECTION PROPERTIES AS AT E) ( $K_t = 1.0$ )

$$\sigma_{bf} = \frac{M K_t C}{I_{NET}} = \frac{(75000 \text{ kN-mm})}{(100^3 - 30^3)(25) \text{ mm}^4} = \frac{(75000 \text{ kN-mm})(100 \text{ mm})}{(100^3 - 30^3)(25) \text{ mm}^4} \times 1000 \text{ N/kN}$$

$\sigma_{bf} = 308 \text{ MPa}$  TENSILE ON RIGHT SIDE; COMP. ON LEFT.

COMPRESSION NEAR F USE  $P_F = 110.95 \text{ kN}$  ABOVE F.  $K_t = 3.70$  FOR  $d/w = 0.30$

$$\sigma_{cf} = \frac{-P_F K_t}{A_{NET}} = \frac{-(110.95 \text{ kN})(3.70)}{(100 - 30)(25) \text{ mm}^2} \times 1000 \text{ N/kN} = -233.6 \text{ MPa NEAR HOLE.}$$

COMBINED  $\sigma_b$  AND  $\sigma_c$  IS PROBLEMATIC AS AT E ON MEMBER DEF.

BECAUSE EFFECT OF  $K_t$  IS LOCALIZED NEAR HOLE,  $\sigma_c$  IS LIKELY TO BE MUCH LESS NEAR OUTSIDE SURFACES WHERE  $\sigma_b$  IS MAXIMUM.

$$\text{ASSUME } \sigma_c = \frac{-P_F}{A_{NET}} = \frac{-(110.95 \text{ kN})(1000 \text{ N/kN})}{(100 - 30)(25) \text{ mm}^2} = -63.1 \text{ MPa NEAR OUTSIDE EDGE ABOVE F.}$$

$$\sigma_c = \frac{(-102 \text{ kN})(1000 \text{ N/kN})}{(100 - 20)(25) \text{ mm}^2} = -58.3 \text{ MPa BELOW F.}$$

COMBINED STRESS - TENSION - BELOW F.

$$\sigma_T = +308 \text{ MPa} - 58.3 = 249.7 \text{ MPa ON RIGHT SIDE.}$$

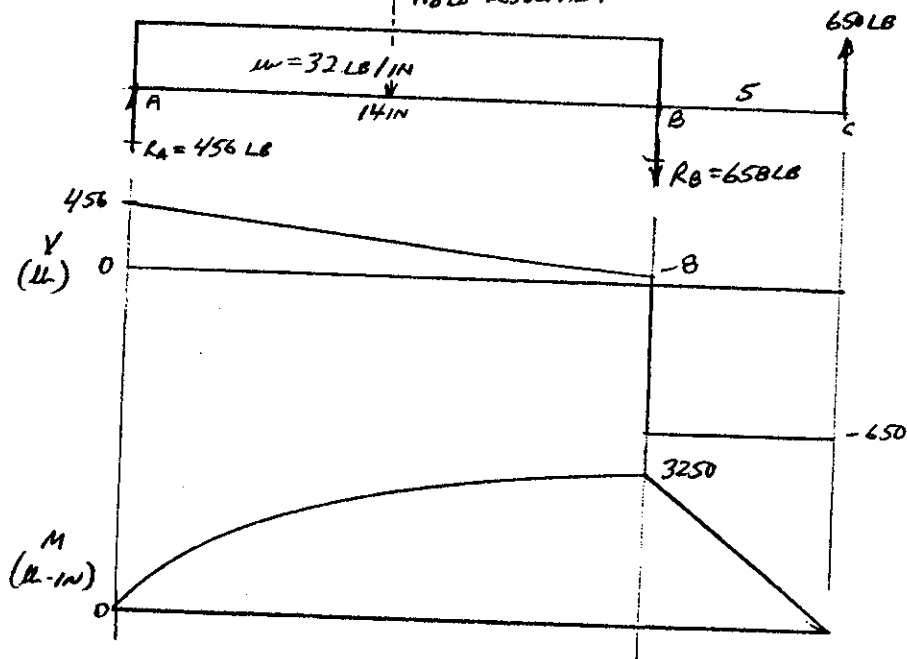
COMBINED STRESS - COMPRESSION - ABOVE F.

$$\sigma_c = -308 \text{ MPa} - 63.1 = -371 \text{ MPa ON LEFT SIDE.}$$

10-54 SHAFT: POWER = 30.5 HP;  $n = 320$  RPM

$$T = \frac{63,000(30.5 \text{ HP})}{320 \text{ RPM}} = 6064 \text{ LB-IN}$$

448 LB-RESULTANT



MAXIMUM BENDING AND TORSION OCCUR NEAR B.  
USE EQUIVALENT TORQUE METHOD, EQ (10-8).  
MAX STRESS OCCURS AT STEP IN 1.0 IN DIA. SHAFT

$$Z_P = \frac{\pi D^3}{16} = \frac{\pi (1.0 \text{ IN})^3}{16} = 0.196 \text{ IN}^3$$

$$\frac{r}{d} = \frac{0.03}{1.0} = 0.03; \frac{D}{d} = \frac{1.25}{1.0} = 1.25; K_{t \text{ BEND}} = 2.40; K_{t \text{ TORS}} = 1.77$$

$$T_e = \sqrt{(K_{t \text{ BEND}} M)^2 + (K_{t \text{ TORS}} T)^2} = \sqrt{(2.40(3250))^2 + (1.77(6064))^2} = 13,268 \text{ LB-IN}$$

$$\tau_{\text{MAX}} = \frac{T_e}{Z_P} = \frac{13,268 \text{ LB-IN}}{0.196 \text{ IN}^3} = 67,694 \text{ PSI}$$

$$\text{LET } N = 4; \tau_d = \frac{S_y}{2N} = \frac{S_y}{8}$$

$$\text{REQ'D } S_y = 8 \tau_{\text{MAX}} = 8(67,694) = 541,550 \text{ PSI} = 542 \text{ KSI}$$

SPECIFY AISI 1040 OQT 1160  $S_y = 552 \text{ KSI}$ , 24% ELONGATION  
OTHER STEELS COULD BE USED WITH  $S_y > 542 \text{ KSI}$  AND  
GOOD DUCTILITY.

10-55 FIND STRESS ON ELEMENTS M AND N,  $P = 450 \text{ N}$

FORCE  $P$  ACTS  $18 \text{ mm}$  TO RIGHT OF CENTER LINE AND  $8 \text{ mm}$  ABOVE CL.

AT M:

$$\text{AXIAL STRESS} = \frac{P}{A} = \frac{450 \text{ N}}{(20)(28) \text{ mm}^2} = 0.804 \text{ MPa TENSION}$$

$$\text{BENDING MOMENT: } M_1 = (450 \text{ N})(8 \text{ mm}) = 3600 \text{ N}\cdot\text{mm}$$

$$S = \frac{b h^2}{6} = \frac{(20)(28)^2}{6} = 2613 \text{ mm}^3$$

$$\sigma_{M_1} = \frac{M_1}{S} = \frac{3600 \text{ N}\cdot\text{mm}}{2613 \text{ mm}^3} = 1.377 \text{ MPa TENSION AT M.}$$

$$\sigma_{M \text{ TOTAL}} = 0.804 + 1.377 = \underline{2.181 \text{ MPa TENSION AT M}}$$

AT N:

$$\text{AXIAL STRESS} = 0.804 \text{ MPa AS AT M. (TENSION)}$$

$$\text{BENDING MOMENT: } M_2 = (450 \text{ N})(18 \text{ mm}) = 8100 \text{ N}\cdot\text{mm}$$

$$S = \frac{h(b)^2}{6} = \frac{(28)(20)^2}{6} = 1867 \text{ mm}^3$$

$$\sigma_{M_2} = \frac{M_2}{S} = \frac{8100 \text{ N}\cdot\text{mm}}{1867 \text{ mm}^3} = 4.339 \text{ MPa COMPRESSION AT N}$$

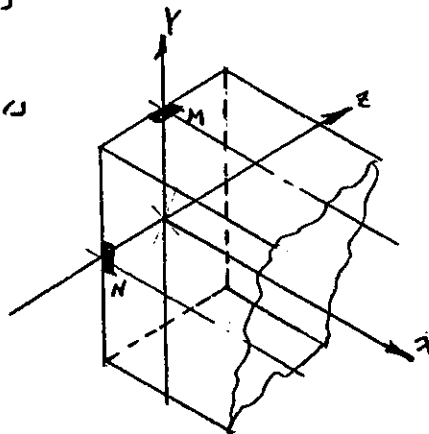
$$\sigma_{N \text{ TOTAL}} = 0.804 - 4.339 = \underline{-3.535 \text{ MPa COMPRESSION AT N.}}$$

ELEMENT M IS ON NEUTRAL AXIS

FOR BENDING ABOUT Y-AXIS.

ELEMENT N IS ON NEUTRAL AXIS

FOR BENDING ABOUT Z-AXIS



## CHAPTER 11    The General Case of Combined Stress and Mohr's Circle

### ANSWERS FOR CHAPTER 11 PROBLEMS

NOTE: The complete solution for problems 1-50 require the construction of the complete Mohr's circle and the drawing of the principal stress element and the maximum shear stress element. Listed below are the significant numerical results. Following the list are representative examples of the complete solutions. Note that the problems fall into groups of similar forms as described below.

- A. For problems 1-4, the x-axis on the Mohr's circle lies in the first quadrant.

For problems 5-8, the x-axis on the Mohr's circle lies in the second quadrant.

For problems 9-12, the x-axis on the Mohr's circle lies in the third quadrant.

For problems 13-16, the x-axis on the Mohr's circle lies in the fourth quadrant.

For problems 17-24, the x-axis on the Mohr's circle could lie in any quadrant. No pattern exists.

For problems 25-28, the x-axis on the Mohr's circle lies along the original x-axis and the principal stresses are the same as the normal stresses on the given element.

- B. For problems 29-40, the Mohr's circle from the given data results in both principal stresses having the same sign. For this class of problems, the supplementary circle is drawn using the procedures discussed in Section 10-11 of the text. The results include three principle stresses where  $\sigma_1 > \sigma_2 > \sigma_3$ . Also, the maximum shear stress is found from the radius of the circle containing  $\sigma_1$  and  $\sigma_3$ , and is equal to  $\frac{1}{2}\sigma_1$  or  $\frac{1}{2}\sigma_3$ , whichever has the greatest magnitude. Angles of rotation of the resulting elements are not requested.
- C. For problems 41-50, the Mohr's circles from earlier problems are used to find the stress condition on the element at some specified angle of rotation. The listed results include the two normal stresses and the shear stress on the specified element.
- D. For problems 51-54, specific numerical answers are requested.

CHAPTER // - PROBLEMS 1-28.

Prob. No.	$\sigma_1$	$\sigma_2$	$\theta$ (deg)	$\tau_{max}$	$\sigma_{avg}$	$\theta'$ (deg)
1	315.4 MPa	-115.4 MPa	10.9 cw	215.4 MPa	100.0 MPa	34.1 ccw
2	255.2 MPa	-55.2 MPa	7.5 cw	155.2 MPa	100.0 MPa	37.5 ccw
3	110.0 MPa	-40.0 MPa	26.6 cw	75.0 MPa	35.0 MPa	18.4 ccw
4	202.1 MPa	-42.1 MPa	27.5 cw	122.1 MPa	80.0 MPa	17.5 ccw
5	23.5 ksi	-8.5 ksi	19.3 ccw	16.0 ksi	7.5 ksi	64.3 ccw
6	42.8 ksi	-29.8 ksi	14.9 ccw	36.3 ksi	6.5 ksi	59.9 ccw
7	79.7 ksi	-9.7 ksi	31.7 ccw	44.7 ksi	35.0 ksi	76.7 ccw
8	36.6 ksi	-54.6 ksi	13.0 ccw	45.6 ksi	-9.0 ksi	58.0 ccw
9	677.6 kPa	-977.6 kPa	77.5 ccw	827.6 kPa	-150.0 kPa	57.5 cw
10	137.8 kPa	-587.8 kPa	84.0 ccw	362.8 kPa	-225.0 kPa	51.0 cw
11	327.0 kPa	-1202.0 kPa	60.9 ccw	764.5 kPa	-437.5 kPa	74.1 cw
12	79.9 kPa	-354.9 kPa	74.8 ccw	217.4 kPa	-137.5 kPa	60.2 cw
13	570.0 psi	-2070.0 psi	71.3 cw	1320.0 psi	-750.0 psi	26.3 cw
14	1476.1 psi	-6676.1 psi	81.7 cw	4176.1 psi	-2500.0 psi	36.7 cw
15	4180.0 psi	-5180.0 psi	71.6 cw	4680.0 psi	-500.0 psi	26.6 cw
16	8600.7 psi	-150.7 psi	89.5 cw	4375.7 psi	4225.0 psi	44.5 cw
17	360.2 MPa	-100.2 MPa	27.8 ccw	230.2 MPa	130.0 MPa	72.8 ccw
18	1827.1 kPa	-377.1 kPa	24.4 cw	1102.1 kPa	725.0 kPa	20.6 ccw
19	23.9 ksi	-1.9 ksi	15.9 cw	12.9 ksi	11.0 ksi	29.1 ccw
20	7971.2 psi	-1221.2 psi	21.4 ccw	4596.2 psi	3375.0 psi	66.4 ccw
21	4.4 ksi	-32.4 ksi	20.3 cw	18.4 ksi	-14.0 ksi	24.7 ccw
22	527.6 MPa	-87.6 MPa	67.8 cw	307.6 MPa	220.0 MPa	22.8 cw
23	321.0 MPa	-61.0 MPa	66.4 ccw	191.0 MPa	130.0 MPa	68.6 cw
24	344.5 kPa	-1904.5 kPa	23.0 ccw	1124.5 kPa	-780.0 kPa	68.0 ccw

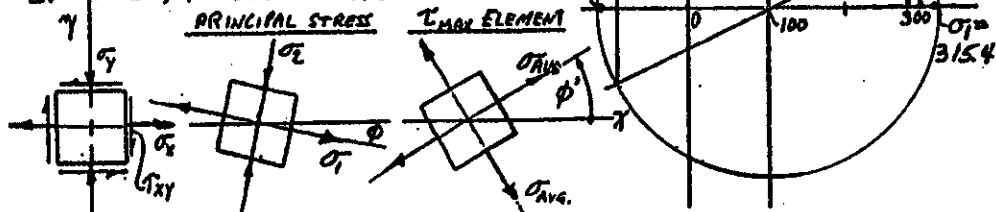
Prob. No.	$\sigma_1$	$\sigma_2$	$\phi$ (deg)	$\tau_{\max}$	$\sigma_{\text{avg}}$	$\phi'$ (deg)
25	225.0 MPa	-85.0 MPa	0.0	155.0 MPa	70.0 MPa	45.0 ccw
26	6250.0 psi	-875.0 psi	0.0	3562.5 psi	2687.5 psi	45.0 ccw
27	775.0 kPa	-145.0 kPa	0.0	460.0 kPa	315.0 kPa	45.0 ccw
28	38.6 ksi	-13.4 ksi	0.0	26.0 ksi	12.6 ksi	45.0 ccw

CHAPTER - PROBLEMS 29-40

Prob. No.	$\sigma_1$	$\sigma_2$	$\sigma_3$	$\tau_{\max}$
29	328.1 MPa	71.9 MPa	0.0 MPa	164.0 MPa
30	264.0 MPa	136.0 MPa	0.0 MPa	132.0 MPa
31	214.5 MPa	75.5 MPa	0.0 MPa	107.2 MPa
32	161.1 MPa	68.9 MPa	0.0 MPa	80.5 MPa
33	35.0 ksi	10.0 ksi	0.0 ksi	17.5 ksi
34	41.8 ksi	21.2 ksi	0.0 ksi	20.9 ksi
35	55.6 ksi	14.4 ksi	0.0 ksi	27.8 ksi
36	62.9 ksi	19.1 ksi	0.0 ksi	31.5 ksi
37	0.0 kPa	-307.9 kPa	-867.1 kPa	433.5 kPa
38	0.0 kPa	-37.5 kPa	-337.5 kPa	168.8 kPa
39	0.0 psi	-295.7 psi	-1804.3 psi	902.1 psi
40	0.0 psi	-2167.6 psi	-6832.4 psi	3416.2 psi

PROB. 11-1 DETAILED SOLUTION

GIVEN:  $\sigma_x = 300 \text{ MPa}$ ;  $\sigma_y = -100 \text{ MPa}$ ;  $\tau_{xy} = 80 \text{ MPa}$  cw  
 $\sigma_1 = 315.4 \text{ MPa}$ ;  $\sigma_2 = -115.4 \text{ MPa}$ ;  $\tau_{\max} = 215.4 \text{ MPa}$   
 $\sigma_{\text{avg}} = 100 \text{ MPa}$ ;  $2\phi = 21.8^\circ$ ;  $\phi = 10.9^\circ$  CW FROM X.  
 $2\phi' = 68.2^\circ$ ;  $\phi' = 34.1^\circ$  CCW FROM X  $\sigma_2 = -115.4$





# CHAPTER 11 - PROBLEMS 41-50

Prob. No.	$\sigma_A$	$\sigma_{A'}$	$\tau_A$
41	130.7 MPa	69.3 MPa	213.2 MPa cw
42	269.3 MPa	-69.3 MPa	133.2 MPa ccw
43	-37.9 MPa	197.9 MPa	31.6 MPa ccw
44	19.1 ksi	-6.1 ksi	34.0 ksi ccw
45	3.6 ksi	-21.6 ksi	43.9 ksi cw
46	-300.0 kPa	-150.0 kPa	355.0 kPa cw
47	-2010.3 psi	510.3 psi	392.6 psi cw
48	-765.5 psi	-234.5 psi	4672.5 psi cw
49	8363.5 psi	86.5 psi	1421.2 psi cw
50	894.8 kPa	555.2 kPa	1088.9 kPa ccw

## CHAPTER - PROBLEMS 51-58

51  $\tau_{\max} = \sqrt{(\sigma/2)^2 + (\tau)^2} = \sqrt{(260/2)^2 + (190)^2} = \underline{230.2 \text{ MPa}}$

Using a similar technique:

52  $\tau_{\max} = 1102.1 \text{ kPa}$       53  $\tau_{\max} = 12.9 \text{ ksi}$       54  $\tau_{\max} = 4596.2 \text{ psi}$

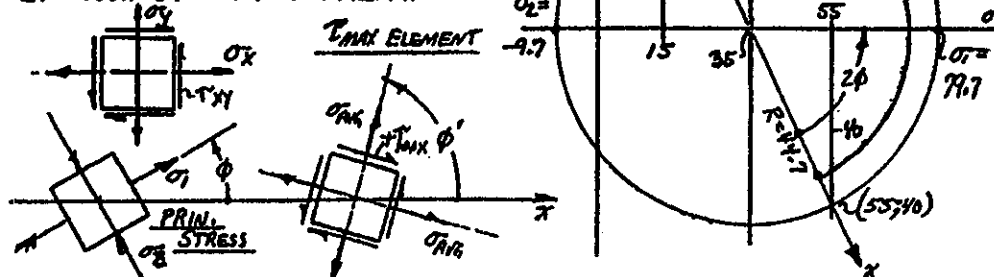
### PROB 11-7 DETAILED SOLUTION

GIVEN:  $\sigma_x = 55 \text{ ksi}$ ;  $\sigma_y = 15 \text{ ksi}$ ;  $\tau_{xy} = 44 \text{ ksi ccw}$

$\sigma_1 = 77.7 \text{ ksi}$ ;  $\sigma_2 = -9.7 \text{ ksi}$ ;  $\tau_{\max} = 44.7 \text{ ksi}$

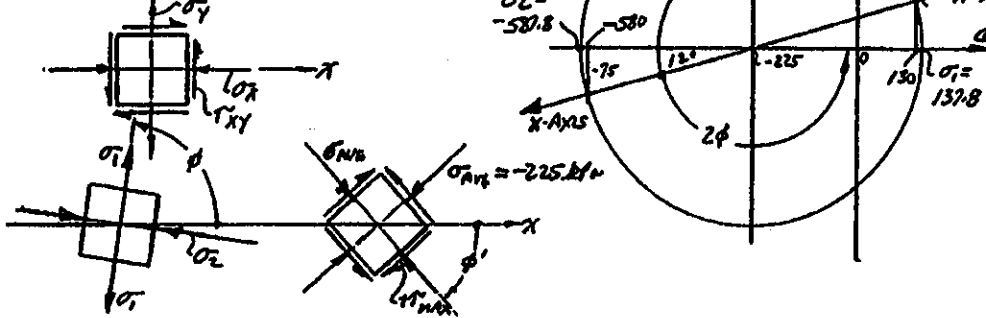
$2\phi = 63.4^\circ$ ;  $\phi = 31.7^\circ \text{ CCW FROM } x$

$2\phi' = 153.4^\circ$ ;  $\phi' = 76.7^\circ \text{ CCW FROM } x$



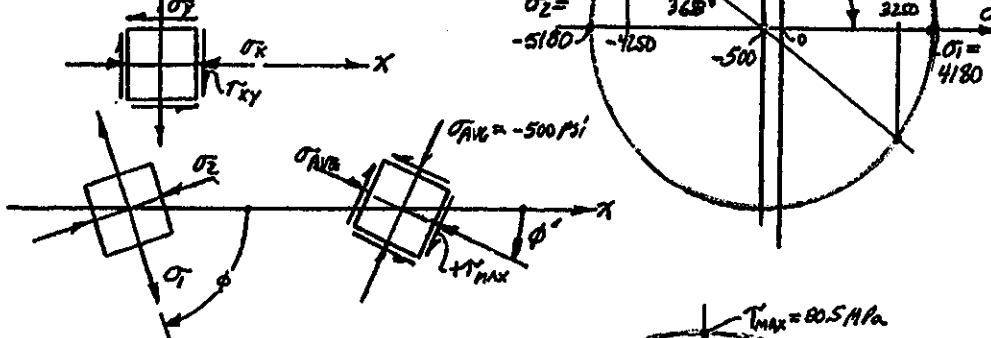
PROB. 11-10 DETAILED SOLUTION

GIVEN:  $\sigma_x = -580 \text{ kPa}$ ;  $\sigma_y = 130 \text{ kPa}$ ;  $\tau_{xy} = 25 \text{ kPa ccw}$   
 $\sigma_1 = 137.8 \text{ kPa}$ ;  $\sigma_2 = -587.8 \text{ kPa}$ ;  $\tau_{MAX} = 362.8 \text{ kPa}$   
 $2\phi = 168^\circ$ ;  $\phi = 84^\circ$  CCW FROM X-AXIS  
 $2\phi' = 102^\circ$ ;  $\phi' = 51^\circ$  CW FROM X-AXIS



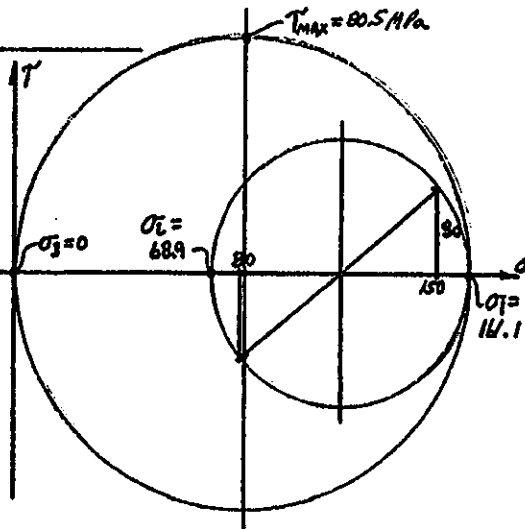
PROB. 11-15 DETAILED SOLUTION

GIVEN:  $\sigma_x = -4250 \text{ psi}$ ;  $\sigma_y = 3250 \text{ psi}$   
 $\tau_{xy} = 2800 \text{ psi cw}$   
 $\sigma_1 = 4180 \text{ psi}$ ;  $\sigma_2 = -5180 \text{ psi}$ ;  $\tau_{MAX} = 4680 \text{ psi}$   
 $2\phi = 143.2^\circ$ ;  $\phi = 71.6^\circ$  CW FROM X-AXIS  
 $2\phi' = 53.2^\circ$ ;  $\phi' = 26.6^\circ$  CW FROM X-AXIS



PROB. 11-32 DETAILED SOLUTION

GIVEN:  $\sigma_x = 150 \text{ MPa}$ ;  $\sigma_y = 80 \text{ MPa}$   
 $\tau_{xy} = 30 \text{ MPa cw}$   
 PRIMARY MOHR'S CIRCLE GIVES  
 $\sigma_1 = 161 \text{ MPa}$ ,  $\sigma_2 = 68.9 \text{ MPa}$ ;  
 BECAUSE BOTH HAVE THE SAME  
 SIGN, SECONDARY CIRCLE IS  
 DRAWN WITH  $\sigma_3 = 0$ . THEN  
 $\tau_{MAX} = \sigma_1/2 = 80.5 \text{ MPa}$



PROB 11-41 DETAILED SOLUTION

BASIC MOHR'S CIRCLE FROM PROB 11-1.

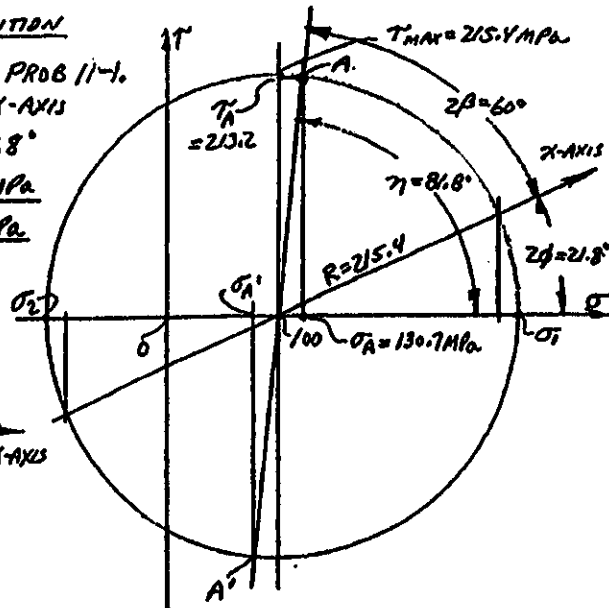
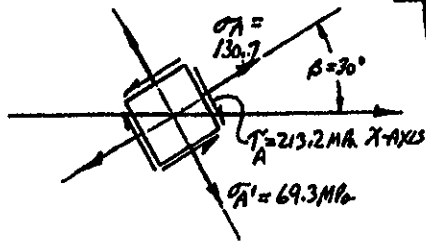
FOR  $\beta = 30^\circ$  CCW FROM  $\pi$ -AXIS

$$\eta = 2\phi + 2\beta = 21.8 + 60 = 81.8^\circ$$

$$\sigma_A = 100 + R \cos \eta = 130.7 \text{ MPa}$$

$$\sigma_{A'} = 100 - R \cos \eta = 69.3 \text{ MPa}$$

$$\tau_A = R \sin \eta = 213.2 \text{ MPa}$$



**SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES  
FROM STRAIN GAGE ROSETTE OUTPUT DATA**

Refer to Section 11-11 for method.

Input data in shaded elements

Aluminum 6061-T6

**Material Properties**      *SI Metric Units*

Modulus of Elasticity     $69.0 \times 10^9$  Pa

Poisson's Ratio        0.33

**Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]**

**Problem 11-55.**

Strain from Gage 1     $1480 \times 10^{-6}$  m/m

Strain from Gage 2     $165 \times 10^{-6}$  m/m

Strain from Gage 3     $428 \times 10^{-6}$  m/m

**Results:**

Max Principal Strain     $1902 \times 10^{-6}$  m/m

Min Principal Strain     $6 \times 10^{-6}$  m/m

Angle  $\beta$     -28.2 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress    147 Mpa

Min Principal Stress    49.1 Mpa

Max Shear Strain    1897 radians [Dimensionless]

Max Shear Stress    49.2 MPa [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress    73.7 MPa

**Delta [0, 60, 120 degree] Rosette Data [Uses Equations 11-18 to 11-20]**

**Problem 11-63**

Strain from Gage 1     $1480 \times 10^{-6}$  m/m

Strain from Gage 2     $165 \times 10^{-6}$  m/m

Strain from Gage 3     $428 \times 10^{-6}$  m/m

**Results:**

Max Principal Strain     $1494 \times 10^{-6}$  m/m

Min Principal Strain     $-112 \times 10^{-6}$  m/m

Angle  $\beta$     -5.4 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress    113 Mpa

Min Principal Stress    29.5 MPa

Max Shear Strain    1607 radians [Dimensionless]

Max Shear Stress    41.7 MPa [in plane of initial element]

\*\*\*Only when Max and Min principal stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress    56.4 MPa

# SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.

Input data in shaded elements

Aluminum 7075-T6

Material Properties SI Metric Units

Modulus of Elasticity  $71.7 \times 10^9$  Pa

Poisson's Ratio 0.33

## Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

### Problem 11-56.

Strain from Gage 1  $853 \times 10^{-6}$  m/m

Strain from Gage 2  $406 \times 10^{-6}$  m/m

Strain from Gage 3  $641 \times 10^{-6}$  m/m

#### Results:

Max Principal Strain  $1104 \times 10^{-6}$  m/m

Min Principal Strain  $390 \times 10^{-6}$  m/m

Angle  $\beta$  -36.4 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 99.2 Mpa

Min Principal Stress 60.7 Mpa

Max Shear Strain  $714 \times 10^{-6}$  radians [Dimensionless]

Max Shear Stress 19.3 MPa [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 49.6 MPa

## Delta [0, 60, 120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

### Problem 11-64.

Strain from Gage 1  $853 \times 10^{-6}$  m/m

Strain from Gage 2  $406 \times 10^{-6}$  m/m

Strain from Gage 3  $641 \times 10^{-6}$  m/m

#### Results:

Max Principal Strain  $892 \times 10^{-6}$  m/m

Min Principal Strain  $375 \times 10^{-6}$  m/m

Angle  $\beta$  -15.9 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 81.7 Mpa

Min Principal Stress 53.9 MPa

Max Shear Strain  $516 \times 10^{-6}$  radians [Dimensionless]

Max Shear Stress 13.9 MPa [in plane of initial element]

\*\*\*Only when Max and Min principal stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 40.8 MPa

# SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.

Input data in shaded elements

AISI 1040 cold drawn steel

Material Properties SI Metric Units

Modulus of Elasticity  $207.0 \times 10^9$  Pa

Poisson's Ratio 0.29

## Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

### Problem 11-57.

Strain from Gage 1  $389 \times 10^{-6}$  m/m

Strain from Gage 2  $737 \times 10^{-6}$  m/m

Strain from Gage 3  $-290 \times 10^{-6}$  m/m

#### Results:

Max Principal Strain  $816 \times 10^{-6}$  m/m

Min Principal Strain  $-717 \times 10^{-6}$  m/m

Angle  $\beta$  31.9 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 137.5 Mpa

Min Principal Stress -108.6 Mpa

Max Shear Strain 1534 radians [Dimensionless]

Max Shear Stress 123.0 MPa [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 68.7 MPa

## Delta [0, 60, 120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

### Problem 11-65.

Strain from Gage 1  $389 \times 10^{-6}$  m/m

Strain from Gage 2  $737 \times 10^{-6}$  m/m

Strain from Gage 3  $-290 \times 10^{-6}$  m/m

#### Results:

Max Principal Strain  $882 \times 10^{-6}$  m/m

Min Principal Strain  $-324 \times 10^{-6}$  m/m

Angle  $\beta$  39.7 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 178.0 Mpa

Min Principal Stress -15.5 MPa

Max Shear Strain 1206 radians [Dimensionless]

Max Shear Stress 96.8 MPa [in plane of initial element]

\*\*\*Only when Max and Min principal stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 89.0 MPa

# SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.

Input data in shaded elements

AISI 4140 OQT 900 steel

Material Properties      SI Metric Units

Modulus of Elasticity       $207.0 \times 10^9$  Pa

Poisson's Ratio      0.29

## Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

### Problem 11-58.

Strain from Gage 1       $925 \times 10^{-6}$  m/m

Strain from Gage 2       $-631 \times 10^{-6}$  m/m

Strain from Gage 3       $552 \times 10^{-6}$  m/m

#### Results:

Max Principal Strain       $2121 \times 10^{-6}$  m/m

Min Principal Strain       $-644 \times 10^{-6}$  m/m

Angle  $\beta$       -41.1 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress      437.1 Mpa

Min Principal Stress      -6.5 Mpa

Max Shear Strain      2764 radians [Dimensionless]

Max Shear Stress      221.8 MPa [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress      218.5 MPa

## Delta [0, 60, 120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

### Problem 11-66.

Strain from Gage 1       $925 \times 10^{-6}$  m/m

Strain from Gage 2       $-631 \times 10^{-6}$  m/m

Strain from Gage 3       $552 \times 10^{-6}$  m/m

#### Results:

Max Principal Strain       $1220 \times 10^{-6}$  m/m

Min Principal Strain       $-656 \times 10^{-6}$  m/m

Angle  $\beta$       -23.4 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress      232.7 Mpa

Min Principal Stress      -68.3 MPa

Max Shear Strain      1876 radians [Dimensionless]

Max Shear Stress      150.5 MPa [in plane of initial element]

\*\*\*Only when Max and Min principal stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress      116.4 MPa

# SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.

Input data in shaded elements

Copper C14500 hard

**Material Properties** U.S. Customary Unit System

Modulus of Elasticity  $17.0 \times 10^6$  psi

Poisson's Ratio 0.33

## Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

### Problem 11-59

Strain from Gage 1  $169 \times 10^{-6}$  in/in

Strain from Gage 2  $-266 \times 10^{-6}$  in/in

Strain from Gage 3  $543 \times 10^{-6}$  in/in

#### Results:

Max Principal Strain  $1006 \times 10^{-6}$  in/in

Min Principal Strain  $-294 \times 10^{-6}$  in/in

Angle  $\beta$  36.6 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 17335 psi

Min Principal Stress 731 psi

Max Shear Strain 1299 radians [Dimensionless]

Max Shear Stress 8302 psi [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 8667 psi

## Delta [0, 60, 120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

### Problem 11-67

Strain from Gage 1  $169 \times 10^{-6}$  in/in

Strain from Gage 2  $-266 \times 10^{-6}$  in/in

Strain from Gage 3  $543 \times 10^{-6}$  in/in

#### Results:

Max Principal Strain  $616 \times 10^{-6}$  in/in

Min Principal Strain  $-319 \times 10^{-6}$  in/in

Angle  $\beta$  -43.8 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 9748 psi

Min Principal Stress -2204 psi

Max Shear Strain 935 radians [Dimensionless]

Max Shear Stress 5976 psi [in plane of initial element]

\*\*\*Only when Max and Min principal stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 4874 psi



**SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES  
FROM STRAIN GAGE ROSETTE OUTPUT DATA**

Refer to Section 11-11 for method.

Input data in shaded elements

Titanium Ti-6Al-4V, aged

**Material Properties** U.S. Customary Unit System

Modulus of Elasticity  $16.5 \times 10^6$  psi

Poisson's Ratio 0.3

**Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]**

**Problem 11-60.**

Strain from Gage 1  $775 \times 10^{-6}$  in/in

Strain from Gage 2  $369 \times 10^{-6}$  in/in

Strain from Gage 3  $-318 \times 10^{-6}$  in/in

**Results:**

Max Principal Strain  $793 \times 10^{-6}$  in/in

Min Principal Strain  $-336 \times 10^{-6}$  in/in

Angle  $\beta$  7.2 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 12548 psi

Min Principal Stress -1776 psi

Max Shear Strain 1129 radians [Dimensionless]

Max Shear Stress 7162 psi [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 6274 psi

**Delta [0, 60, 120 degree] Rosette Data [Uses Equations 11-18 to 11-20]**

**Problem 11-68.**

Strain from Gage 1  $775 \times 10^{-6}$  in/in

Strain from Gage 2  $369 \times 10^{-6}$  in/in

Strain from Gage 3  $-318 \times 10^{-6}$  in/in

**Results:**

Max Principal Strain  $913 \times 10^{-6}$  in/in

Min Principal Strain  $-363 \times 10^{-6}$  in/in

Angle  $\beta$  19.2 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 14587 psi

Min Principal Stress -1607 psi

Max Shear Strain 1276 radians [Dimensionless]

Max Shear Stress 8097 psi [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 7294 psi

# SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.

Input data in shaded elements

Ductile Iron, ASTM A536, 80-55-6

Material Properties U.S. Customary Unit System

Modulus of Elasticity  $24.0 \times 10^6$  psi

Poisson's Ratio 0.27

## Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

### Problem 11-61.

Strain from Gage 1  $389 \times 10^{-6}$  in/in

Strain from Gage 2  $737 \times 10^{-6}$  in/in

Strain from Gage 3  $-290 \times 10^{-6}$  in/in

#### Results:

Max Principal Strain  $816 \times 10^{-6}$  in/in

Min Principal Strain  $-717 \times 10^{-6}$  in/in

Angle  $\beta$  31.9 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 16117 psi

Min Principal Stress -12863 psi

Max Shear Strain 1534 radians [Dimensionless]

Max Shear Stress 14490 psi [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 8059 psi

## Delta [0, 60, 120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

### Problem 11-69.

Strain from Gage 1  $389 \times 10^{-6}$  in/in

Strain from Gage 2  $737 \times 10^{-6}$  in/in

Strain from Gage 3  $-290 \times 10^{-6}$  in/in

#### Results:

Max Principal Strain  $882 \times 10^{-6}$  in/in

Min Principal Strain  $-324 \times 10^{-6}$  in/in

Angle  $\beta$  39.7 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 20559 psi

Min Principal Stress -2236 psi

Max Shear Strain 1206 radians [Dimensionless]

Max Shear Stress 11397 psi [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 10280 psi

# SPREADSHEET FOR COMPUTING PRINCIPAL STRAINS AND STRESSES FROM STRAIN GAGE ROSETTE OUTPUT DATA

Refer to Section 11-11 for method.

Input data in shaded elements

Stainless Steel, AISI 501 OQT 1000

Material Properties U.S. Customary Unit System

Modulus of Elasticity  $29.0 \times 10^6$  psi

Poisson's Ratio 0.30

## Rectangular [0, 45, 90 degree] Rosette Data [Uses Equations 11-15 to 11-17]

### Problem 11-62.

Strain from Gage 1  $1592 \times 10^{-6}$  in/in

Strain from Gage 2  $-226 \times 10^{-6}$  in/in

Strain from Gage 3  $-893 \times 10^{-6}$  in/in

#### Results:

Max Principal Strain  $2688 \times 10^{-6}$  in/in

Min Principal Strain  $-263 \times 10^{-6}$  in/in

Angle  $\beta$  -38.7 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 83147 psi

Min Principal Stress 17317 psi

Very high stress:  $s_y = 135$  ksi

$N = 1.62$  Low

Max Shear Strain  $2951 \times 10^{-6}$  radians [Dimensionless]

Max Shear Stress 32915 psi [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 41574 psi

## Delta [0, 60, 120 degree] Rosette Data [Uses Equations 11-18 to 11-20]

### Problem 11-70.

Strain from Gage 1  $532 \times 10^{-6}$  in/in

Strain from Gage 2  $-226 \times 10^{-6}$  in/in

Strain from Gage 3  $-893 \times 10^{-6}$  in/in

#### Results:

Max Principal Strain  $1761 \times 10^{-6}$  in/in

Min Principal Strain  $-296 \times 10^{-6}$  in/in

Angle  $\beta$  -19.5 degrees

[From the axis of gage 1 to the nearer principal axis]

Max Principal Stress 53289 psi

Min Principal Stress 7390 psi

Max Shear Strain  $2058 \times 10^{-6}$  radians [Dimensionless]

Max Shear Stress 22949 psi [in plane of initial element]

\*\*\*Only when Max and Min Principal Stresses have the same sign\*\*\*

[Assuming stress = 0 perpendicular to plane of initial element]

True Max Shear Stress 28644 psi

12-1

$$A22-(a): \eta = -\frac{P L^3}{48 E I} = \frac{(3000)(700)^3}{(48)(207 \times 10^3)(6.147 \times 10^4)} = -2.01 \text{ mm}$$

$$E = 207 \times 10^3 \frac{\text{N}}{\text{mm}^2} \times \frac{1 \text{ m}^2}{(10^3 \text{ mm})^2} = 207 \times 10^3 \text{ N/mm}^2$$

$$I = \frac{\pi D^4}{64} = \frac{\pi (32 \text{ mm})^4}{64} = 5.147 \times 10^4 \text{ mm}^4$$

12-2

$$E = 69 \text{ GPa} = 69 \times 10^3 \text{ N/mm}^2; \eta \text{ INVERSELY PROPORTIONAL TO } E$$

$$\eta = -2.01 \text{ mm} \times \frac{207}{69} = -6.03 \text{ mm}$$

12-3

$$A24-(e):$$

$$\eta = \frac{-P L^3}{192 E I} = \frac{(-3000)(700)^3}{192 (207 \times 10^3) (5.147 \times 10^4)} = -0.503 \text{ mm}$$

12-4

$$\eta = \frac{-P L^3}{48 E I} = \frac{-3000 (350)^3}{48 (207 \times 10^3) (5.147 \times 10^4)} = -0.252 \text{ mm}$$

12-5

$$I = \pi D^4 / 64 = \pi (25)^4 / 64 = 19175 \text{ mm}^4$$

$$\eta = -\frac{3000 (700)^3}{48 (207 \times 10^3) (19175)} = -5.40 \text{ mm}$$

12-6

$$A22-(b):$$

$$\text{AT LOA0: } \eta = \frac{-P a^2 b^2}{3 E I L} = \frac{-3000 (175)^2 (525)^2}{3 (207 \times 10^3) (5.147 \times 10^4) (700)} = -1.13 \text{ mm}$$

$$\text{AT CENTER: } \eta = \frac{-P b x}{6 E I L} (L^2 - x^2 - b^2), \text{ USE } x = 350 \text{ mm}$$

$$\eta = \frac{-3000 (175) (350)}{6 (207 \times 10^3) (5.147 \times 10^4) (700)} [(700)^2 - (350)^2 - (175)^2]$$

$$\eta = -1.38 \text{ mm}$$

12-7

$$A22-(c): W12 \times 16 \quad ; \quad I = 103 \text{ in}^4; \text{ LOADING IN P6-4.}$$

$$P = 10000 \text{ lb}; \quad a = 36 \text{ in}; \quad L = 168 \text{ in}; \quad E = 30 \times 10^6 \text{ psi}$$

$$\text{AT LOA0S: } \eta = \frac{-P a^2}{6 E I} (3L - 4a) = \frac{-(10000)(36)^2}{6 (30 \times 10^6) (103)} [(3)(168) - 4(36)]$$

$$\eta = -0.251 \text{ in}$$

$$\text{AT CENTER: } \eta = \frac{-P a}{24 E I} (3L^2 - 4a^2) = \frac{-(10000)(36)}{24 (30 \times 10^6) (103)} [(3)(168)^2 - 4(36)^2]$$

$$\eta = -0.385 \text{ in}$$

12-8

$$A22-(g): I = 0.310 \text{ in}^4$$

$$\eta = \frac{-P\ell^3}{48EI} = \frac{-650(28)^3}{48(30 \times 10^6)(0.310)} = -0.032 \text{ in}$$

12-9

$$A22-(d): I = 59.69 \text{ in}^4; E = 10 \times 10^6 \text{ psi}$$

$$W = (1125 \text{ lb/ft})(10 \text{ ft}) = 11250 \text{ lb}$$

$$\ell = 10 \text{ ft} \times 12 \text{ in/ft} = 120 \text{ in}$$

$$\eta = \frac{5W\ell^3}{384EI} = \frac{-5(11250)(120)^3}{384(10 \times 10^6)(59.69)} = -0.424 \text{ in}$$

12-10

$$A22-(d): x = 3.5 \text{ ft} (12 \text{ in/ft}) = 42 \text{ in}$$

$$W = (1125 \text{ lb/ft})(1 \text{ ft}/12 \text{ in}) = 93.75 \text{ lb/in}$$

$$\eta = \frac{-Wx}{24EI} (L^3 - 2Lx^2 + x^3) = \frac{-(93.75)(42)}{24(10 \times 10^6)(59.69)} (120^3 - 2(120)(42)^2 + 42^3)$$

$$\eta = -0.379 \text{ in}$$

12-11

LOADING IN FIGURE P6-12.

$$I = 238 \text{ in}^4; A22-(g); a = 48 \text{ in}; \ell = 120 \text{ in}; P = 15000 \text{ lb}$$

$$\eta = \frac{-Pa^2}{3EI} (a + \ell) = \frac{-15000(48)^2}{3(30 \times 10^6)(238)} (48 + 120) = -0.271 \text{ in}$$

12-12

$$A22-(g): a = 24 \text{ in}; L = 144 \text{ in}$$

$$\eta = \frac{-Pa^2}{3EI} (a + L) = \frac{-15000(24)^2}{3(30 \times 10^6)(238)} (24 + 144) = -0.0678 \text{ in}$$

12-13

$$+\eta_{\text{max}} \text{ at } x = 0.577 (\ell) = 0.577 (120) = 69.2 \text{ in from A}$$

$$\eta = \frac{0.06415 Pa\ell^2}{EI} = \frac{(0.06415)(15000)(48)(120)^2}{(30 \times 10^6)(238)} = 0.093 \text{ in (UPWARD)}$$

12-14

$$A23-(a): \eta = \frac{-P\ell^3}{3EI} = \frac{-120(8)^3}{3(30 \times 10^6)(0.08734)} = -0.0078 \text{ in}$$

12-15

A22-(a)

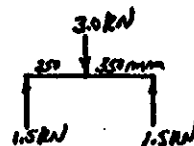
$$\eta = \frac{-P\ell^3}{48EI} \therefore \text{REQ'D } I = \frac{-P\ell^3}{48E\eta} = \frac{-3000(700)^3}{48(207 \times 10^3)(2.10)} = 8.63 \times 10^5 \text{ mm}^4$$

$$I = \frac{\pi D^4}{64} \therefore D = \left[ \frac{64I}{\pi} \right]^{1/4} = \left[ \frac{64(8.63 \times 10^5)}{\pi} \right]^{1/4} = 64.8 \text{ mm}$$

12-16

$$\sigma = \frac{Mc}{I} = \frac{(1500 \text{ N})(350 \text{ mm})(324 \text{ mm})}{8.63 \times 10^5 \text{ mm}^4} = 19.7 \text{ MPa}$$

$$\text{REQ'D } S_x = 8(19.7 \text{ MPa}) = 158 \text{ MPa (ANY STEEL)}$$

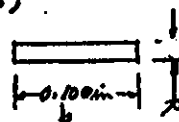


12-17

A23-(a)

$$\eta_{\text{max}} = \frac{-P\ell^3}{3EI} \therefore \text{REQ'D } I = \frac{-P\ell^3}{3E\eta} = \frac{-0.52(1120)^3}{3(30 \times 10^6)(0.15)} = 6.658 \times 10^8 \text{ in}^4$$

$$I = \frac{bh^3}{12} \therefore x = \left[ \frac{12I}{b} \right]^{1/3} = \left[ \frac{12(6.658 \times 10^8)}{0.100} \right]^{1/3} = 0.020 \text{ in}$$



12-18

$$I = \frac{bh^3}{12} = \frac{(1.50)(9.25)^3}{12} = 98.93 \text{ in}^4$$

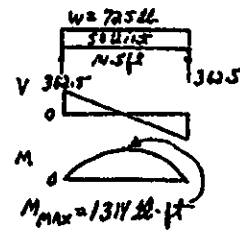
$$S = \frac{bh^2}{6} = \frac{(1.50)(9.25)^2}{6} = 21.39 \text{ in}^3$$

$$\gamma_{max} = \frac{-5WL^3}{384EI} = \frac{-5(725)(174)^3}{384(1.76 \times 10^6)(98.93)} = -0.286 \text{ in}$$

$$\sigma = \frac{M}{S} = \frac{1314 \text{ lb-ft} \cdot (12 \text{ in/ft})}{21.39} = 737 \text{ psi}$$

$$\tau = \frac{3V}{2A} = \frac{3(362.5)}{2(1.50)(9.25)} = 39.2 \text{ psi}$$

OK PER TABLE A-18



12-19

COMBINE A22(a) AND A22(b) SUPERPOSITION

$$\gamma_B = \gamma_{B1} + \gamma_{B2} + \gamma_{B3}$$

$$\gamma_C = \gamma_{C1} + \gamma_{C2} + \gamma_{C3}$$

$$\gamma_D = \gamma_{D1} + \gamma_{D2} + \gamma_{D3}$$

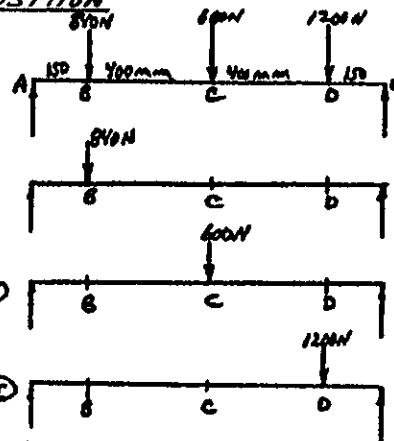
Ⓐ - A22-2

$$\gamma_{B1} = \frac{-Pa^2b^2}{3EIL} ; b = 150, a = 950$$

$$EI = (69 \times 10^3 \text{ N/mm}^2)(0.001 \text{ m}^4)$$

$$EI = 1.283 \times 10^{10}$$

$$EIL = (1.283 \times 10^{10})(1100) = 1.412 \times 10^{13}$$



$$\gamma_{B1} = \frac{(-840)(150)^2(950)^2}{3(1.412 \times 10^{13})} = -0.403 \text{ mm}$$

$$\gamma_{C1} = \frac{-Pbx}{6EIL} (L^2 - x^2 - b^2) = \frac{-840(150)(550)}{6(1.412 \times 10^{13})} (1100^2 - 550^2 - 150^2) = -0.724 \text{ mm}$$

$$\gamma_{D1} = \frac{-840(150)(150)}{6(1.412 \times 10^{13})} (1100^2 - 550^2 - 150^2) = -0.260 \text{ mm}$$

Ⓑ - A22-1(a)

$$\gamma_{B2} = \gamma_{B2} = \frac{-Px}{48EI} (3L^2 - 4x^2) = \frac{-600(150)}{48(1.283 \times 10^{10})} (3(1100)^2 - 4(150)^2) = -0.517 \text{ mm}$$

$$\gamma_{C2} = \frac{-Px^3}{48EI} = \frac{-600(1100)^3}{48(1.283 \times 10^{10})} = -1.297 \text{ mm}$$

Ⓒ - A22-1(b)

$$\gamma_{B3} = \frac{-Pbx}{6EIL} (L^2 - x^2 - b^2) = \frac{-1200(150)(550)}{6(1.412 \times 10^{13})} (1100^2 - 550^2 - 150^2) = -0.371 \text{ mm}$$

$$\gamma_{D3} = \frac{-1200(150)(150)}{6(1.412 \times 10^{13})} (1100^2 - 550^2 - 150^2) = -1.034 \text{ mm}$$

$$\gamma_{D3} = \frac{-Pa^2b^2}{3EIL} = \frac{-1200(150)^2(950)^2}{3(1.412 \times 10^{13})} = -0.576 \text{ mm}$$

TOTAL DEFLECTIONS -

$$\gamma_B = \gamma_{B1} + \gamma_{B2} + \gamma_{B3} = -1.291 \text{ mm}$$

$$\gamma_C = \gamma_{C1} + \gamma_{C2} + \gamma_{C3} = -3.055 \text{ mm}$$

$$\gamma_D = \gamma_{D1} + \gamma_{D2} + \gamma_{D3} = -1.353 \text{ mm}$$

12-20

$$E = 73 \text{ GPa} = 73 \times 10^9 \text{ N/m}^2 = 73 \times 10^3 \text{ N/mm}^2$$

$$EI = (73 \times 10^3)(16956) = 1.238 \times 10^9 \text{ N}\cdot\text{mm}^2$$

$$EI_L = (1.238 \times 10^9)(1200) = 1.485 \times 10^{12} \text{ N}\cdot\text{mm}^3$$

(I) A22-(a) [I FROM P7-12]

$$\eta_{B1} = -\frac{Px}{48EI} (3l^2 - 4x^2)$$

$$\eta_{B1} = \frac{-400(300)}{48(1.238 \times 10^9)} (3(1200)^2 - 4(300)^2) = -8.000 \text{ mm}$$

$$\eta_{C1} = -\frac{Pl^3}{48EI} = \frac{-400(1200)^3}{48(1.238 \times 10^9)} = -11.632 \text{ mm}$$

(II) A22-(b)

$$\eta_{B2} = \frac{-Pa^2b^2}{3EI_L} = \frac{-500(300)^2(900)^2}{3(1.485 \times 10^{12})} = -8.182 \text{ mm}$$

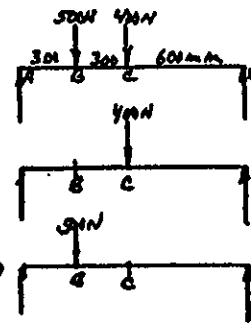
$$\eta_{C2} = \frac{-Pbx}{6EI_L} (l^2 - x^2 - b^2) = \frac{-500(300)(600)}{6(1.485 \times 10^{12})} (1200^2 - 600^2 - 300^2) = -10.000 \text{ mm}$$

TOTAL DEFLECTIONS

$$\eta_B = \eta_{B1} + \eta_{B2} = -8.000 \text{ mm} - 8.182 \text{ mm} = -16.182 \text{ mm}$$

$$\eta_C = \eta_{C1} + \eta_{C2} = -11.632 \text{ mm} - 10.000 \text{ mm} = -21.632 \text{ mm}$$

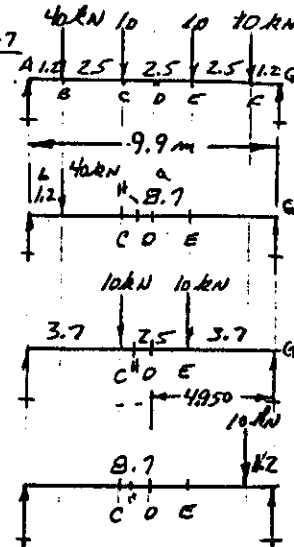
LARGE DEFLECTIONS



12-21

LOADING IN FIGURE P6-7. FIGURE P6-7  
W18-55 STEEL BEAM  
USE SUPERPOSITION

- (I) 40 kN LOAD ONLY AT B, SUPERPOSITION  
CASE A22(b):  $L = 9900 \text{ mm}$   
 $a = 8700 \text{ mm}$ ,  $b = 1200 \text{ mm}$
- (II) TWO 10 kN LOADS AT C AND E,  
CASE A-22(c):  $L = 9900 \text{ mm}$   
 $a = 3700 \text{ mm}$
- (III) 10 kN LOAD ONLY AT F,  
 $a = 8700 \text{ mm}$ ,  $b = 1200 \text{ mm}$   
CASE A22(b):  $L = 9900 \text{ mm}$



POINT OF MAXIMUM DEFLECTION IS NOT OBVIOUS BECAUSE EACH CASE PRODUCES A MAXIMUM DEFLECTION AT A DIFFERENT POINT. DEFLECTION AT C, D, AND E ARE COMPUTED FOR EACH LOADING, THEN SUMMED. THE MAXIMUM DEFLECTION FOR CASE I OCCURS BETWEEN C AND D AT THE POINT CALLED H, 4226 mm FROM A. DEFLECTION COMPUTED THERE ALSO.

PRODUCT OF EI APPEARS IN ALL EQUATIONS,  
 $E = 207 \times 10^3 \text{ N/mm}^2$   
 $I = 890 \text{ IN}^4 \times 4.162 \times 10^5 \text{ mm}^4 / \text{IN}^4$   
 $I = 3.70 \times 10^8 \text{ mm}^4$   
 $EI = 7.66 \times 10^{13} \text{ N} \cdot \text{mm}^2$

SUMMARY OF RESULTS:

I	$\eta_C = -3.802 \text{ mm}$	II	$\eta_C = -4.438 \text{ mm}$	III	$\eta_C = -0.809 \text{ mm}$
	$\eta_D = -3.76 \text{ mm}$		$\eta_D = -4.816 \text{ mm}$		$\eta_D = -0.940 \text{ mm}$
	$\eta_E = -3.235 \text{ mm}$		$\eta_E = -4.438 \text{ mm}$		$\eta_E = -0.957 \text{ mm}$
	$\eta_H = -3.85 \text{ mm}$		$\eta_H = -4.689 \text{ mm}$		$\eta_H = -0.877 \text{ mm}$

BY SUPERPOSITION:

$$\eta_C = -9.049 \text{ mm}$$

$$\eta_D = -9.516 \text{ mm}$$

$$\eta_E = -8.624 \text{ mm}$$

$$\eta_H = -9.416 \text{ mm}$$

APPARENT MAXIMUM DEFLECTION AT MIDDLE OF BEAM AT D.



12-22

A23(a) AND A23-(b)

$$\textcircled{I} \quad \eta_{b1} = \frac{-P x^2}{6EI} (3L - x)$$

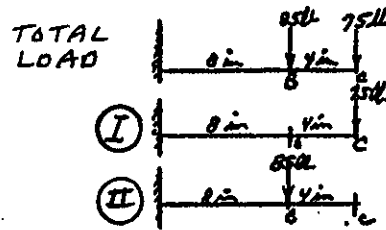
$$EI = (30 \times 10^6)(1.08734) = 2.62 \times 10^6$$

$$\eta_{b1} = \frac{-25(8)^2}{6(2.62 \times 10^6)} [3(12) - 8] = -0.0085 \text{ in}$$

$$\eta_{c1} = \frac{-P L^3}{3EI} = \frac{-25(12)^3}{3(2.62 \times 10^6)} = -0.0165 \text{ in}$$

$$\textcircled{II} \quad \eta_{b2} = \frac{-P a^2}{3EI} = \frac{-85(8)^2}{3(2.62 \times 10^6)} = -0.0055 \text{ in}$$

$$\eta_{c2} = \frac{-P a^2}{6EI} [3L - a] = \frac{-85(8)^2}{6(2.62 \times 10^6)} [3(12) - 8] = -0.0097 \text{ in}$$



TOTAL DEFLECTION

$$\eta_b = \eta_{b1} + \eta_{b2} = -0.0085 - 0.0055 = \underline{-0.0140 \text{ in}}$$

$$\eta_c = \eta_{c1} + \eta_{c2} = -0.0165 - 0.0097 = \underline{-0.0262 \text{ in}}$$

12-23

$$I = \frac{bh^3}{12} = \frac{20(80)^3}{12} = 8.533 \times 10^5 \text{ mm}^4$$

$$EI = [207 \times 10^3 \text{ N/mm}^2] [8.533 \times 10^5 \text{ mm}^4] = 1.766 \times 10^{11}$$

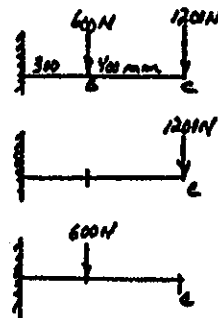
$$\textcircled{I} \quad \text{A23-(a)}$$

$$\eta_{c1} = \frac{-PL^3}{3EI} = \frac{-1200(700)^3}{3(1.766 \times 10^{11})} = -0.777 \text{ mm}$$

$$\textcircled{II} \quad \text{A23-(b)}$$

$$\eta_{c2} = \frac{-Pa^2}{6EI} [3L - a] = \frac{-600(300)^2}{6(1.766 \times 10^{11})} [3(700) - 300]$$

$$\eta_{c2} = -0.092 \text{ mm}$$



TOTAL DEFLECTION

$$\eta_c = \eta_{c1} + \eta_{c2} = -0.777 - 0.092 = \underline{-0.869 \text{ mm}}$$

12-24

DEFLECTIONS INVERSELY PROPORTIONAL TO E

$$\eta_c = -0.869 \text{ mm} \times \frac{E_s}{E_{Al}} = -0.869 \times \frac{207 \text{ GPa}}{73 \text{ GPa}} = \underline{-2.464 \text{ mm}}$$

12-25

$$\eta_c = -0.869 \text{ mm} \times \frac{E_s}{E_{Al}} = -0.869 \times \frac{207 \text{ GPa}}{45 \text{ GPa}} = \underline{-3.997 \text{ mm}}$$

12-26

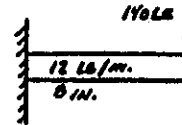
$$I = \pi d^4 / 64 = \pi (0.80)^4 / 64 = 0.0201 \text{ in}^4$$

$$W = \text{TOTAL LOAD} = wL = (12 \text{ LB/IN.})(8 \text{ IN.}) = 96 \text{ LB}$$

$$\frac{A23-(c)}{\gamma_1} = \frac{-wL^3}{8EI} = \frac{-96(8)^3}{8(30 \times 10^6)(0.0201)} = -0.0102 \text{ in}$$

$$\frac{A23-(a)}{\gamma_2} = \frac{-PL^3}{3EI} = \frac{-140(8)^3}{3(30 \times 10^6)(0.0201)} = -0.0396 \text{ in}$$

$$\text{TOTAL } \gamma = \gamma_1 + \gamma_2 = -0.0102 - 0.0396 = -0.0498 \text{ in}$$



12-27

FROM FIG P6-7,  $L = 9900 \text{ mm}$

$$L/360 = 27.5 \text{ mm}; \text{ LET } \gamma_{\text{MAX}} = -27.5 \text{ mm}$$

FROM PROBLEM 12-21,  $\gamma_{\text{MAX}} = -9.516 \text{ mm}$

FOR A W18-55 BEAM WITH  $I = 890 \text{ IN}^4$

DEFLECTION INVERSELY PROPORTIONAL TO  $I$ .

$$I_{\text{MIN}} = 890 \text{ IN}^4 \cdot \frac{9.516}{27.5} = 308 \text{ IN}^4$$

SPECIFY: W18x40-LIGHTEST

OR W14x43-LEAST DEPTH

RESULT COULD HAVE BEEN FOUND BY USING  
SUPERPOSITION APPROACH OUTLINED IN  
PROBLEM 12-21 WITH  $I$  TREATED AS AN

UNKNOWN. THEN SET  $\gamma_{\text{MAX}} = \frac{C}{I_{\text{MIN}}}$  AND

SOLVE FOR  $I_{\text{MIN}}$  FOR  $\gamma = 27.5 \text{ mm}$ .

12-28

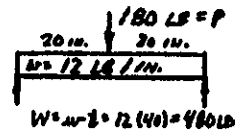
$$\gamma_{\text{MAX}} = -0.080 \text{ in} = \frac{A22-(a)}{48EI} - \frac{A22-(d)}{384EI}$$

$$48EI\gamma_{\text{MAX}} = -PL^3 - \frac{5wL^3}{8} = L^3[-P - 5w/8]$$

$$\text{REQ'D } I = \frac{L^3}{48E\gamma_{\text{MAX}}} [-P - 5w/8] = \frac{(40)^3}{48(10 \times 10^6)(-0.08)} [-180 - 5(410)/8] = 0.800 \text{ in}^4$$

USE A C4x2.33 ( $I_y = 1.02 \text{ in}^4$ )

OR C5x2.22 ( $I_y = 0.98 \text{ in}^4$ ) (LIGHTEST)



12-29

$$I = bh^3/12 = 1(2)^3/12 = 0.667 \text{ in}^4$$

$$M_{AB} = 750x$$

$$M_{BC} = -1250x + C$$

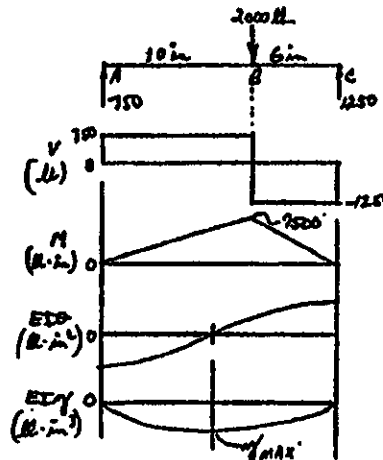
$$\text{at } x = 16, M = 0$$

$$0 = -1250(16) + C \therefore C = 20000$$

$$M_{BC} = -1250x + 20000$$

$$EI\theta_{AB} = \int 750x dx = 375x^2 + C_1$$

$$EI\theta_{BC} = \int (-1250x + 20000) dx = -625x^2 + 20000x + C_2$$



$$EI\theta_{AB} = (375x^2 + C_1) dx - 125x^3 + C_1x + C_3 = EI\theta_{AB}$$

$$EI\theta_{BC} = \int (-625x^2 + 20000x + C_2) dx = -208.3x^3 + 10000x^2 + C_3x + C_4 = EI\theta_{BC}$$

BOUNDARY CONDITIONS

$$\textcircled{1} \text{ at } x = 0, EI\theta_{AB} = 0$$

$$\textcircled{3} \text{ at } x = 10, EI\theta_{AB} = EI\theta_{BC}$$

$$\textcircled{2} \text{ at } x = 16, EI\theta_{BC} = 0$$

$$\textcircled{4} \text{ at } x = 10, EI\theta_{AB} = EI\theta_{BC}$$

SUBSTITUTING;

$$\textcircled{1} 0 = 0 + 0 + C_3 \therefore C_3 = 0$$

$$\textcircled{2} EI\theta_{BC} = 0 = -208.3(16)^3 + 10000(16)^2 + 16C_2 + C_4 = 1.707 \times 10^6 + 16C_2 + C_4 = 0$$

$$\textcircled{3} 375(10)^2 + C_1 = -625(10)^2 + 20000(10) + C_2$$

$$C_1 - C_2 = 1.0 \times 10^5$$

$$\textcircled{4} 125(10)^3 + 10C_1 + C_3 = -208.3(10)^3 + 10000(10)^2 + 10C_2 + C_4$$

$$10C_1 - 10C_2 - C_4 = 6.667 \times 10^5$$

IN  $\textcircled{2}$  AND  $\textcircled{3}$  SOLVE FOR  $C_1$  AND  $C_4$  IN TERMS OF  $C_2$

$$C_1 = C_2 + 1.0 \times 10^5$$

$$C_4 = -16C_2 - 1.707 \times 10^6$$

SUBSTITUTE INTO  $\textcircled{4}$

$$10[C_2 + 1.0 \times 10^5] - 10C_2 - (-16C_2 - 1.707 \times 10^6) = 6.667 \times 10^5$$

$$10C_2 + 1.0 \times 10^6 - 10C_2 + 16C_2 + 1.707 \times 10^6 = 6.667 \times 10^5$$

$$C_2 = -2.440 \times 10^6 / 16 = -1.275 \times 10^5 = -127500 = C_2$$

$$C_1 = C_2 + 1.0 \times 10^5 = -1.275 \times 10^5 + 1.0 \times 10^5 = -0.275 \times 10^5 = -27500 = C_1$$

$$C_4 = -16C_2 - 1.707 \times 10^6 = -16[-1.275 \times 10^5] - 1.707 \times 10^6 = 333000 = C_4$$

(CONTINUED NEXT PAGE)

12-29 (CONTINUED)

FINAL EQUATIONS

$$EI\theta_{AB} = 375x^2 - 27500$$

$$EI\theta_{BC} = -625x^2 + 20000x - 127500$$

$$EI\eta_{AB} = 125x^3 - 27500x$$

$$EI\eta_{BC} = -208.3x^3 + 10000x^2 - 127500x + 333000$$

MAX  $\eta$  OCCURS WHERE  $EI\theta = 0$

$$\text{SET } EI\theta_{AB} = 0 = 375x^2 - 27500$$

$$x = \sqrt{27500/375} = 8.56 \text{ in}$$

$$\text{MAX } \eta = (EI\eta_{AB})_{x=8.56} = 125(8.56)^3 - 27500(8.56) = -156997 \text{ lb}\cdot\text{in}^3$$

$$\text{MAX } \eta = \frac{(EI\eta)_{\text{MAX}}}{EI} = \frac{-156997 \text{ lb}\cdot\text{in}^3}{(30 \times 10^6)(0.667)(44 \text{ in}^4)} = \frac{-0.0078 \text{ in at } x=8.56 \text{ in}}{}$$

$$\text{CHECK: } \sigma = \frac{M}{I} = \frac{(2750 \text{ lb}\cdot\text{ft})(12 \text{ in})}{0.667 \text{ in}^4} = 11244 \text{ psi OK FOR STEEL}$$

12-30

$$I = 890 \text{ in}^4 \quad \underline{W18 \times 55}$$

$$M = 20x - 200$$

$$EI\theta = \int (20x - 200) dx = 10x^2 - 200x + C_1$$

$$\text{at } x=0, EI\theta = 0 \therefore C_1 = 0$$

$$EI\eta = \int (10x^2 - 200x) dx = 3.333x^3 - 100x^2 + C_2$$

$$\text{at } x=0, EI\eta = 0 \therefore C_2 = 0$$

$$\text{THEN } EI\eta = 3.333x^3 - 100x^2$$

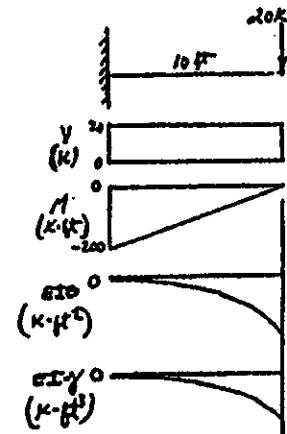
$$\text{at } x=10 \text{ ft}$$

$$EI\eta_{\text{MAX}} = 3.333(10^3) - 100(10)^2 = -6667 \text{ K}\cdot\text{ft}^3$$

$$\eta_{\text{MAX}} = \frac{-6667 \text{ K}\cdot\text{ft}^3}{(30 \times 10^6 \text{ lb/in}^2)(890 \text{ in}^4)} \times \frac{100 \text{ lb}}{\text{ft}^3} \times \frac{1728 \text{ in}^3}{\text{ft}^3} = -0.43 \text{ in}$$

$$\text{CHECK: } \sigma = \frac{M}{S} = \frac{(200 \text{ K}\cdot\text{ft})(12 \text{ in})}{98.3 \text{ in}^3} = 24415 \text{ psi}$$

$\sigma < S_y$   
BUT HIGH FOR A36  
STRUCTURAL STEEL.



12-31

$$V = -4x + 4 \quad I = 1.530 \text{ in}^4 \times 4.162 \times 10^5 \frac{\text{mm}^4}{\text{in}^4} = 6.368 \times 10^5 \text{ mm}^4$$

$$M = -2x^2 + 4x + C_1 = -2x^2 + 4x - 2$$

$$\text{at } x = 10, M = 0$$

$$0 = -2 + 4 + C_1 \therefore C_1 = -2$$

$$EI\theta = \int (-2x^2 + 4x - 2) dx = -\frac{2x^3}{3} + 2x^2 - 2x + C_2$$

$$\text{at } x = 0, EI\theta = 0 \therefore C_2 = 0$$

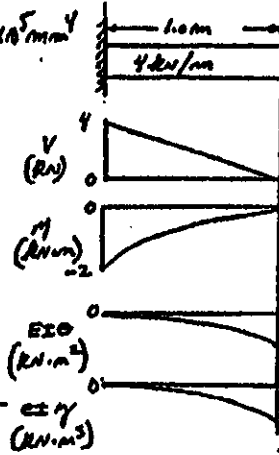
$$EI\eta = \int EI\theta dx = \frac{1}{6}x^4 + \frac{2}{3}x^3 - x^2 + C_3$$

$$\text{at } x = 10 \text{ m}$$

$$EI\eta = -1/6 + 2/3 - 1 = -0.50 \text{ kN}\cdot\text{m}^3$$

$$\eta = \frac{-0.50 \text{ kN}\cdot\text{m}^3 (10^3 \text{ N/kN}) (10^5 \text{ mm}^5/\text{m}^5)}{(207 \times 10^3 \text{ N/m}^2) (6.368 \times 10^5 \text{ mm}^4)} = -3.79 \text{ mm}$$

$$\text{CHECK: } \sigma = M/S = 115 \text{ MPa} - \text{OK.}$$



12-32

$$\text{SHEAR: } V = -4x + 60$$

$$\text{MOMENT: } M = \int V dx = -2x^2 + 60x + C$$

$$\text{at } x = 0, M = -400 \text{ K}\cdot\text{ft} \therefore C = -400$$

$$M = -2x^2 + 60x - 400$$

$$EI\theta = \int M dx = -\frac{2x^3}{3} + 30x^2 - 400x + C_2$$

$$\text{at } x = 0, EI\theta = 0 \therefore C_2 = 0$$

$$EI\eta = \int EI\theta dx = -\frac{x^4}{6} + 10x^3 - 200x^2 + C_3$$

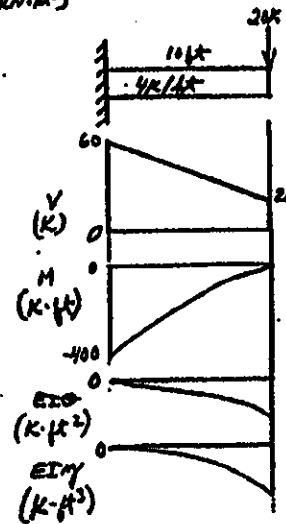
$$\text{at } x = 0, EI\eta = 0 \therefore C_3 = 0$$

$$\text{at } x = 10 \text{ ft,}$$

$$EI\eta_{\text{max}} = \frac{-10^4}{6} + 10(10^3) - 200(10^2) = -11667 \text{ K}\cdot\text{ft}^3$$

$$\eta_{\text{max}} = \frac{-11667 \text{ K}\cdot\text{ft}^3 (1000 \text{ lb/K}) (12 \text{ in})^3}{(30 \times 10^6 \text{ lb/in}^2) (2100 \text{ in}^4)} = -3.20 \text{ in}$$

$$\text{CHECK: } \sigma = \frac{M}{S} = 27273 \text{ psi} - \text{N.K.N}$$



12-33

$$V = -4x + 6$$

$$M = \int V dx = -2x^2 + 6x + C_1$$

$$\text{at } x = 0, M = -4 \therefore C_1 = -4$$

$$M = -2x^2 + 6x - 4$$

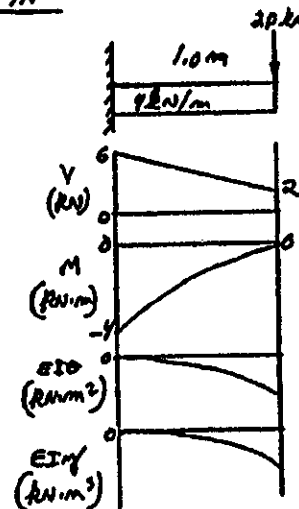
$$EI\theta = \int M dx = -\frac{2x^3}{3} + 3x^2 - 4x + C_2$$

$$\text{at } x = 0, EI\theta = 0 \therefore C_2 = 0$$

$$EI\eta = \int EI\theta dx = -\frac{x^4}{6} + x^3 - 2x^2 + C_3$$

$$\text{at } x = 0, EI\eta = 0 \therefore C_3 = 0$$

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12-33 (CONTINUED)

at  $x = 1.0 \text{ m}$ ,

$EI\gamma_{\text{MAX}} = -\frac{1}{6} + 1 - 2 = -1.167 \text{ kN}\cdot\text{m}^3$

For  $\gamma_{\text{MAX}} = 5.0 \text{ mm}$

REQ'D  $I = \frac{-1.167 \text{ kN}\cdot\text{m}^3}{EI\gamma_{\text{MAX}}} = \frac{-1.167 \times 10^3 \text{ N}\cdot\text{m}^3}{(207 \times 10^9 \text{ N/m}^2)(5.0 \text{ mm})} = +1.127 \times 10^6 \text{ mm}^4$

$I = \pi D^4/64 \therefore D = \sqrt[4]{64 I/\pi} = \sqrt[4]{64 (1.127 \times 10^6)/\pi} = 69.2 \text{ mm}$

CHECK:  $\sigma = \frac{Mc}{I} = \frac{(4.0 \text{ kN}\cdot\text{m})(34.6 \text{ mm})}{1.127 \times 10^6 \text{ mm}^4} = 122.8 \text{ MPa} < \sigma_K \text{ FOR STEEL}$

12-34

$V_{AB} = -50x + 55$

$V_{BC} = 60$

$M_{AB} = \int V_{AB} dx = -25x^2 + 55x + C_1$

$M_{BC} = \int V_{BC} dx = 60x + C_2$

at  $x = 4, M = 0$

$0 = 60(4) + C_2 = 240 + C_2 \therefore C_2 = -240$

$M_{BC} = 60x - 240$

$EI\theta_{AB} = \int M_{AB} dx = -\frac{25}{3}x^3 + 27.5x^2 + C_1$

$EI\theta_{BC} = \int M_{BC} dx = 30x^2 - 240x + C_2$

$EI\gamma_{AB} = \int EI\theta_{AB} dx = -\frac{25}{12}x^4 + \frac{55}{6}x^3 + C_1x + \frac{1}{2}x^2$

$EI\gamma_{BC} = \int EI\theta_{BC} dx = 10x^3 - 120x^2 + C_2x + C_4$

BOUNDARY CONDITIONS

① at  $x = 0, EI\gamma_{AB} = 0$  ③ at  $x = 3, EI\gamma_{BC} = 0$

② at  $x = 3, EI\gamma_{AB} = 0$  ④ at  $x = 3, EI\theta_{AB} = EI\theta_{BC}$

SUBSTITUTING:

①  $C_1 = 0$

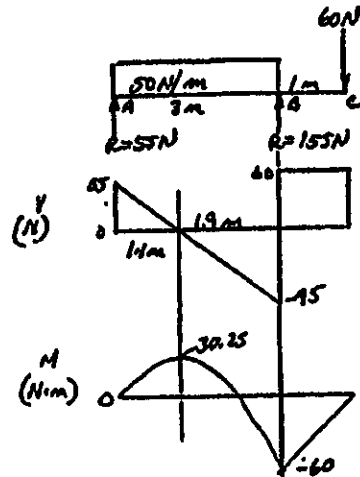
③  $0 = 10(3)^3 - 120(3)^2 + 3C_2 + C_4$   
 $3C_2 + C_4 = 810$

②  $0 = -\frac{25}{12}(3)^4 + \frac{55}{6}(3)^3 + 3C_1$   
 $C_1 = -78.75/3 = -26.25 = C_1$

④  $-\frac{25}{3}(3)^3 + 27.5(3)^2 - 26.25 = 30(3)^2 - 240(3) + C_2$

$C_2 = 446.25$

FROM ③:  $C_4 = 810 - 3C_2 = 810 - 3(446.25) = -528.75 = C_4$



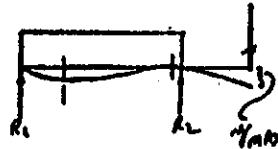
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# 12-34 (CONTINUED)

## FINAL EQUATIONS

$$\begin{aligned} E I \theta_{AB} &= -(25/3)x^3 + 27.5x^2 - 26.25 \\ E I \theta_{BC} &= 30x^2 - 240x + 446.25 \\ E I \psi_{AB} &= -(25/2)x^4 + 35/6 x^3 - 26.25x \\ E I \psi_{BL} &= 10x^3 - 120x^2 + 446.25 - 528.75 \end{aligned}$$

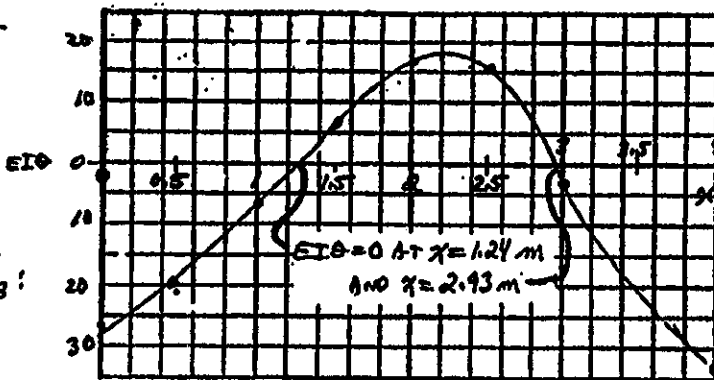
## SHAPE OF DEFLECTION CURVE



$y_{MAX}$  OCCURS WHERE  $E I \theta = 0$  OR AT RIGHT END OF OVERHANG.

EVALUATE  $E I \theta_{AB}$  AT SEVERAL POINTS + USE SOLVER.

$x =$	$E I \theta =$
0	-26.25
0.5	-20.417
1.0	-2.083
1.5	7.50
2.0	17.083
2.5	15.417
3.0	-3.75



ROOTS FOR  $E I \theta_{AB}$ :

$$x_1 = 1.2351 \text{ m}$$

$$x_2 = 2.9341 \text{ m}$$

EVALUATE  $E I \psi$  AT  $x = 1.24 \text{ m}$ ,  $x = 2.93 \text{ m}$ , AND  $x = 4.0 \text{ m}$ .

$$[E I \psi]_{x=1.24 \text{ m}} = -2.083(1.24)^4 + 9.167(1.24)^3 - 26.26(1.24) = -20.0 \text{ N}\cdot\text{m}^3$$

$$[E I \psi]_{x=2.93 \text{ m}} = -2.083(2.93)^4 + 9.167(2.93)^3 - 26.26(2.93) = +0.1121 \text{ N}\cdot\text{m}^3$$

$$[E I \psi]_{x=4} = 10(4)^3 - 120(4)^2 + 446.25 - 528.75 = -23.75 \text{ N}\cdot\text{m}^3 \text{ MAX}$$

$$\text{REQ'D } I = \frac{-23.75 \text{ N}\cdot\text{m}^3}{E \psi_{MAX}} = \frac{-23.75 \text{ N}\cdot\text{m}^3}{(207 \times 10^9 \text{ N/m}^2)(-1 \text{ mm})} = 1.15 \times 10^5 \text{ mm}^4$$

$$\text{CONVERT } I = 1.15 \times 10^5 \text{ mm}^4 \times 2.403 \times 10^{-6} \frac{\text{in}^4}{\text{mm}^4} = 0.276 \text{ in}^4$$

## POSSIBLE BEAM DESIGNS:

1.  $1\frac{1}{2}$  IN. SCH 40 PIPE:  $I = 0.3099 \text{ in}^4$

2. C3x6 CHANNEL:  $I_y = 0.305 \text{ in}^4$

3. 2x2x $\frac{1}{4}$  HOLLOW STEEL TUBE:  $I = 0.766 \text{ in}^4$

CHECK:  $\sigma = M/S$  - OK FOR ALL DESIGNS.

12-35

SELECT ALUMINUM I-BEAM;  $\sigma_d = 120 \text{ MPa}$

$$S = \frac{M}{\sigma} = \frac{17.6 \times 10^3 \text{ N}\cdot\text{m}}{120 \text{ N/mm}^2} \times \frac{10^3 \text{ mm}^3}{1 \text{ m}} = 1.467 \times 10^5 \text{ mm}^3$$

$$S = 1.467 \times 10^5 \text{ mm}^3 \times 6.102 \times 10^{-6} \frac{\text{m}^3}{\text{mm}^3} = 8.95 \text{ in}^3$$

USE I 7 x 5.800 ALUM I-BEAM

$$S = 12.25 \text{ in}^3; I = 42.89 \text{ in}^4$$

MOMENT -

$$M_{AB} = 28X$$

$$M_{BC} = 8X + C \text{ BUT AT } X = 0.4, M = 11.2$$

$$11.2 = 8(0.4) + C$$

$$C = 11.2 - 3.2 = 8.0$$

$$M_{CD} = 8X + 8$$

$$M_{CD} = -22X + C \text{ BUT AT } X = 2.0, M = 0$$

$$0 = -22(2.0) + C$$

$$C = 44$$

$$M_{CD} = -22X + 44$$

SLOPE -

$$EI\theta_{AB} = \int M_{AB} dx = 14x^2 + C_1$$

$$EI\theta_{BC} = \int M_{BC} dx = 4x^2 + 8x + C_2$$

$$EI\theta_{CD} = \int M_{CD} dx = -11x^2 + 44x + C_3$$

DEFLECTION -

$$EI\eta_{AB} = \int EI\theta_{AB} dx = \left(\frac{14}{3}\right)x^3 + C_1x + C_4$$

$$EI\eta_{BC} = \int EI\theta_{BC} dx = \left(\frac{4}{3}\right)x^3 + 4x^2 + C_2x + C_5$$

$$EI\eta_{CD} = \int EI\theta_{CD} dx = -\left(\frac{11}{3}\right)x^3 + 22x^2 + C_3x + C_6$$

BOUNDARY CONDITIONS

$$\textcircled{1} \text{ AT } X=0, EI\eta_{AB}=0$$

$$\textcircled{2} \text{ AT } X=0.4, EI\theta_{AB}=EI\theta_{BC}$$

$$\textcircled{3} \text{ AT } X=1.2, EI\theta_{BC}=EI\theta_{CD}$$

$$\textcircled{4} \text{ AT } X=2.0, EI\eta_{CD}=0$$

$$\textcircled{5} \text{ AT } X=0.4, EI\eta_{AB}=EI\eta_{BC}$$

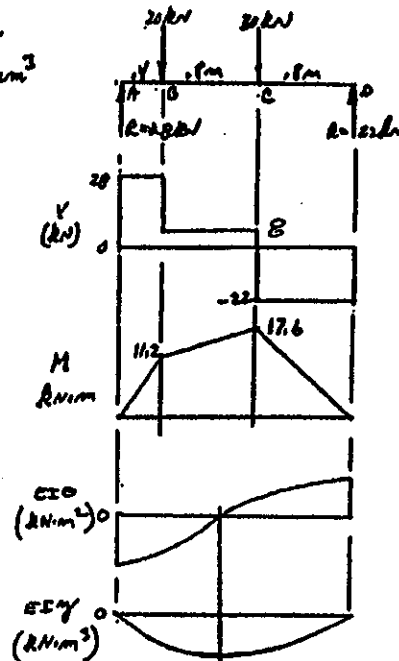
$$\textcircled{6} \text{ AT } X=1.2, EI\eta_{BC}=EI\eta_{CD}$$

SUBSTITUTING

$$\textcircled{1} 0 = C_4$$

$$\textcircled{2} EI\eta_{CD}=0 = -\left(\frac{11}{3}\right)(2)^3 + 22(2)^2 + 2C_3 + C_6$$

$$2C_3 + C_6 = -58.66$$



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12-35 CONTINUED

$$\textcircled{3} \quad 14(0.4)^2 + C_1 = 4(0.4)^2 + 8(0.4) + C_2$$

$$C_1 - C_2 = 1.60$$

$$\textcircled{4} \quad (14/3)(0.4)^3 + 0.4C_1 + R_4^0 = (4/3)(0.4)^3 + 4(0.4)^2 + 0.4C_2 + C_5$$

$$0.4C_1 - 0.4C_2 - C_5 = 0.4266$$

$$\textcircled{5} \quad 4(1.2)^2 + 1.2C_1 + C_2 = -11(1.2)^2 + 44(1.2) + C_3$$

$$C_2 - C_3 = 21.6$$

$$\textcircled{6} \quad (4/3)(1.2)^3 + 4(1.2)^2 + 1.2C_2 + C_5 = -(11/3)(1.2)^3 + 22(1.2)^2 + 1.2C_3 + C_6$$

$$1.2C_2 + C_5 - 1.2C_3 - C_6 = 17.28$$

SOLVE SIMULTANEOUSLY

$$C_1 = -11.56 \quad C_4 = 0$$

$$C_2 = -12.16 \quad C_5 = 0.2133$$

$$C_3 = -33.76 \quad C_6 = 8.8533$$

FINAL EQUATIONS

$EI\theta_{AB} = 14x^2 - 10.56$	$EI\eta_{AB} = 4.667x^3 - 10.56x$
$EI\theta_{BC} = 4x^2 + 8x - 12.16$	$EI\eta_{BC} = (4/3)x^3 + 4x^2 - 12.16x + 0.2133$
$EI\theta_{CD} = -11x^2 + 44x - 33.76$	$EI\eta_{CD} = -(11/3)x^3 + 22x^2 - 33.76x + 8.8533$

$$\text{SET } EI\theta_{BC} = 0 = 4x^2 + 8x - 12.16 = x^2 + 2x - 3.04$$

$$x = \frac{-2 \pm \frac{1}{2} \sqrt{4 - 4(-3.04)}}{2} = -1 \pm 2.01 = +1.01 \text{ m} - \text{VALID POINT}$$

THEN MAX  $\eta$  OCCURS AT  $x = 1.01 \text{ m}$

$$[EI\eta_{BC}]_{x=1.01} = (4/3)(1.01)^3 + 4(1.01)^2 - 12.16(1.01) + 0.2133 = -6.615 \text{ kN}\cdot\text{m}^3$$

$$\eta_{\text{MAX}} = \frac{-6.615 \text{ kN}\cdot\text{m}^3}{EI} = \frac{-6.615 \times 10^3 \text{ N}\cdot\text{m}^3}{(69 \times 10^9 \text{ N/m}^2)(1.785 \times 10^8 \text{ mm}^4)} \left( \frac{10 \text{ mm}}{1} \right)^5 = -5.37 \text{ mm}$$

$$I = 42.89 \text{ in}^4 \times 4.162 \times 10^5 \text{ mm}^4/\text{in}^4 = 1.785 \times 10^7 \text{ mm}^4$$

$$M_{AB} = -20X$$

$$M_{BC} = 20X - 160$$

$$M_{CD} = -10X + 80$$

$$M_{DE} = 20X - 280$$

$$EI\theta_{AB} = -10X^2 + C_1$$

$$EI\theta_{BC} = 10X^2 - 160X + C_2$$

$$EI\theta_{CD} = -5X^2 + 80X + C_3$$

$$EI\theta_{DE} = 10X^2 - 280X + C_4$$

$$EI\eta_{AB} = -\left(\frac{10}{3}\right)X^3 + C_1X + C_5$$

$$EI\eta_{BC} = \left(\frac{10}{3}\right)X^3 - 80X^2 + C_2X + C_6$$

$$EI\eta_{CD} = -\left(\frac{5}{3}\right)X^3 + 40X^2 + C_3X + C_7$$

$$EI\eta_{DE} = \left(\frac{10}{3}\right)X^3 - 140X^2 + C_4X + C_8$$

BOUNDARY AND CONTINUITY CONDITIONS

① at  $X=4$ ;  $EI\eta_{AB} = 0$

⑤ at  $X=4$ ;  $EI\theta_{AB} = EI\theta_{BC}$

② at  $X=4$ ;  $EI\eta_{BC} = 0$

⑥ at  $X=8$ ;  $EI\theta_{BC} = EI\theta_{CD}$

③ at  $X=12$ ;  $EI\eta_{CD} = 0$

⑦ at  $X=8$ ;  $EI\eta_{BC} = EI\eta_{CD}$

④ at  $X=12$ ;  $EI\eta_{DE} = 0$

⑧ at  $X=12$ ;  $EI\theta_{CD} = EI\theta_{DE}$

CONSTANTS OF INTEGRATION FROM SIMULTANEOUS SOLUTION OF ① THRU ⑧

$$C_1 = 306.66$$

$$C_5 = -1013.33$$

$$C_2 = 626.66$$

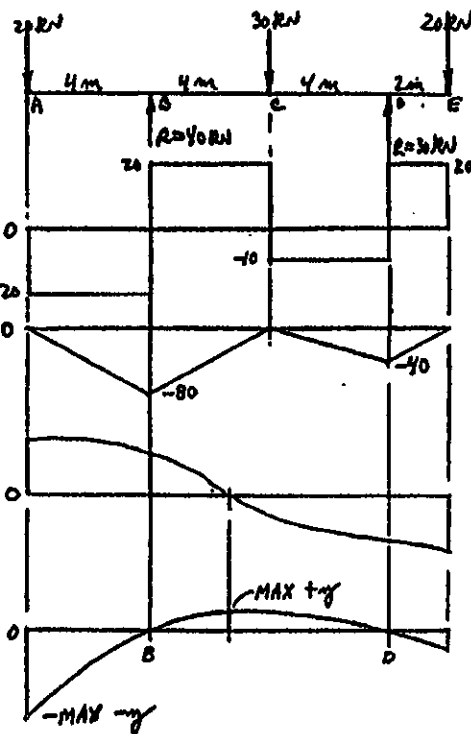
$$C_6 = -14.40$$

$$C_3 = -333.33$$

$$C_7 = 11.20$$

$$C_4 = 1826.66$$

$$C_8 = -752.0$$



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12-36 (CONTINUED)

FINAL EQUATIONS

$EI\theta_{AB} = -10x^2 + 306.66$	$EI\eta_{AB} = -(10/3)x^3 + 306x - 1013.33$
$EI\theta_{BC} = 10x^2 - 160x + 626.66$	$EI\eta_{BC} = (10/3)x^3 - 80x^2 + 626x - 1440$
$EI\theta_{CD} = -5x^2 + 80x - 333.33$	$EI\eta_{CD} = -(5/3)x^3 + 40x^2 - 333x + 1120$
$EI\theta_{DE} = 10x^2 - 280x + 1826.66$	$EI\eta_{DE} = (10/3)x^3 - 140x^2 + 1826x - 7520$

$$\text{Set } EI\theta_{BC} = 0 = 10x^2 - 160x + 626 = x^2 - 16x + 62.6$$

$$x = \frac{16 \pm \sqrt{16^2 - 4(62.6)}}{2} = 8 \pm \frac{1}{2}(27.37) = 9.16 \text{ OR } \boxed{6.845}$$

INVALID - NOT IN  
SEGMENT BC

MAX  $\eta$  AT  $x = 0$  OR  $x = 6.84 \text{ m}$  OR  $x = 14 \text{ m}$

$$\text{at } x = 0; EI\eta_{AB} = -1013.33 \text{ kN}\cdot\text{m}^3 \quad \text{MAX NEG. } \eta$$

$$\text{at } x = 6.84 \text{ m}; EI\eta_{BC} = +170 \text{ kN}\cdot\text{m}^3 \quad \text{MAX POS. } \eta$$

$$\text{at } x = 14; EI\eta_{DE} = -240 \text{ kN}\cdot\text{m}^3$$

$$E = 207 \times 10^9 \text{ N/m}^2$$

$$I = 245 \text{ m}^4 \times 4.162 \times 10^5 \text{ mm}^4/\text{m}^4 = 1.020 \times 10^8 \text{ mm}^4$$

$$\text{at } x = 0; \eta = \frac{-1013 \times 10^3 \text{ N}\cdot\text{m}^3 (10^6 \text{ mm}^5/\text{m}^5)}{(207 \times 10^9 \text{ N/m}^2)(1.020 \times 10^8 \text{ mm}^4)} = -48.0 \text{ mm}$$

$$\text{at } x = 6.82 \text{ m}; \eta = -48.0 \times \frac{170}{-1013} = +8.07 \text{ mm}$$

$$\text{at } x = 14 \text{ m}; \eta = -48.0 \text{ mm} \times \frac{-240}{-1013} = -11.37 \text{ mm}$$

$$\text{CHECK: } \sigma = \frac{M}{S}$$

$$M = 80 \text{ kN}\cdot\text{m} = \frac{10^3 \text{ N}}{\text{m}} \times \frac{8.85 \text{ LB}\cdot\text{ft}}{\text{N}\cdot\text{m}} = 7.08 \times 10^5 \text{ LB}\cdot\text{ft}$$

$$\sigma = \frac{7.08 \times 10^5 \text{ LB}\cdot\text{ft}}{35.3 \text{ in}^3} = 20,057 \text{ psi} \quad \text{OK}$$

12-37

MOMENT:  $M_{AB} = -X$

$M_{BC} = 2.2X - 0.6Y$

$M_{CD} = -1.8X + 0.96$

$M_{DE} = 3.0X - 4.8$

$EI\theta_{AB} = \int M_{AB} dX = -0.5X^2 + C_1$

$EI\theta_{BC} = \int M_{BC} dX = 1.1X^2 - 0.6YX + C_2$

$EI\theta_{CD} = \int M_{CD} dX = -0.9X^2 + 0.96X + C_3$

$EI\theta_{DE} = \int M_{DE} dX = 1.5X^2 - 4.8X + C_4$

$EI\gamma_{AB} = \int EI\theta_{AB} dX = -0.16\bar{6}X^3 + C_1X + C_5$

$EI\gamma_{BC} = \int EI\theta_{BC} dX = 0.36\bar{6}X^3 - 0.32YX^2 + C_2X + C_6$

$EI\gamma_{CD} = \int EI\theta_{CD} dX = -0.3X^3 + 0.48X^2 + C_3X + C_7$

$EI\gamma_{DE} = \int EI\theta_{DE} dX = 0.5X^3 - 2.4X^2 + C_4X + C_8$

# BOUNDARY CONDITIONS

① at  $X = 0.2$ ,  $EI\gamma_{AB} = 0$

⑤ at  $X = 0.2$ ,  $EI\theta_{AB} = EI\theta_{BC}$

② at  $X = 0.2$ ,  $EI\gamma_{BC} = 0$

⑥ at  $X = 0.4$ ,  $EI\theta_{BC} = EI\theta_{CD}$

③ at  $X = 1.2$ ,  $EI\gamma_{CD} = 0$

⑦ at  $X = 1.2$ ,  $EI\theta_{CD} = EI\theta_{DE}$

④ at  $X = 1.2$ ,  $EI\gamma_{DE} = 0$

⑧ at  $X = 0.4$ ,  $EI\gamma_{BC} = EI\gamma_{CD}$

CONSTANTS - FROM SIMULTANEOUS SOLUTION OF ① THROUGH ⑧

$C_1 = 0.490\bar{6}$

$C_5 = -0.0968$

$C_2 = 0.158\bar{6}$

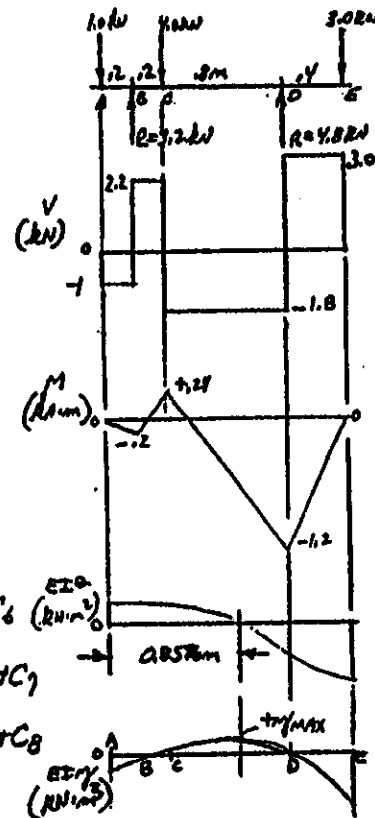
$C_6 = -0.0218\bar{6}$

$C_3 = -0.161\bar{3}$

$C_7 = 0.0208$

$C_4 = 3.294\bar{6}$

$C_8 = -1.316$



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# 12-31 CONTINUED

## FINAL EQUATIONS

$EI\theta_{AB} = -0.5x^2 + 1.4096$	$EI\eta_{AB} = -0.166x^3 + 0.4906x - 0.0968$
$EI\theta_{BC} = 1.1x^2 - 0.64x + 0.1586$	$EI\eta_{BC} = 0.366x^3 - 0.32x^2 + 0.1586x - 0.02$
$EI\theta_{CD} = -0.9x^2 + 0.96x - 0.1613$	$EI\eta_{CD} = -0.3x^3 + 0.48x^2 - 0.1613x + 0.0208$
$EI\theta_{DE} = 1.5x^2 - 4.8x + 3.2946$	$EI\eta_{DE} = 0.5x^3 - 2.4x^2 + 3.2946x - 1.3616$

$$\text{SET } EI\theta_{CD} = 0 = -0.9x^2 + 0.96x - 0.1613 \Rightarrow x^2 - 1.066x + 0.1792$$

$$x = \frac{1.06 \pm \sqrt{1.06^2 - 4(1)(0.1792)}}{2} = 0.5335 \pm 0.325 = \underline{0.8577 \text{ m}} \text{ or } \underline{0.2413 \text{ m}}$$

OUTSIDE CD

DEFLECTION AT  $x = 0.8577 \text{ m}$  IN SEGMENT CD

$$EI\eta_{CD} = -0.3(0.8577)^3 + 0.48(0.8577)^2 - 0.1613(0.8577) + 0.0208$$

$$= 0.0462 \text{ kN}\cdot\text{m}^3$$

AT  $x = 0$

$$EI\eta_{AB} = -0.166(0) + 0.4906(0) - 0.0968 = -0.0968 \text{ kN}\cdot\text{m}^3$$

AT  $x = 1.6 \text{ m}$

$$EI\eta_{DE} = 0.5(1.6)^3 - 2.4(1.6)^2 + 3.2946(1.6) - 1.362 = -0.1862 \text{ kN}\cdot\text{m}^3$$

MAXIMUM

$$\text{FOR } \eta_{\text{MAX}} = -0.13 \text{ mm} \Rightarrow E = 207 \times 10^3 \text{ N/mm}^2$$

$$I = \frac{EI\eta}{E\eta} = \frac{-0.186 \times 10^3 \text{ N}\cdot\text{m}^3}{(207 \times 10^3 \text{ N/mm}^2)(-0.13 \text{ mm})} = 6.91 \times 10^6 \text{ mm}^4$$

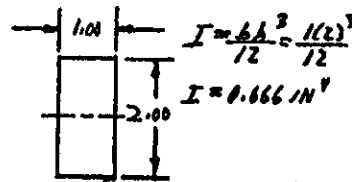
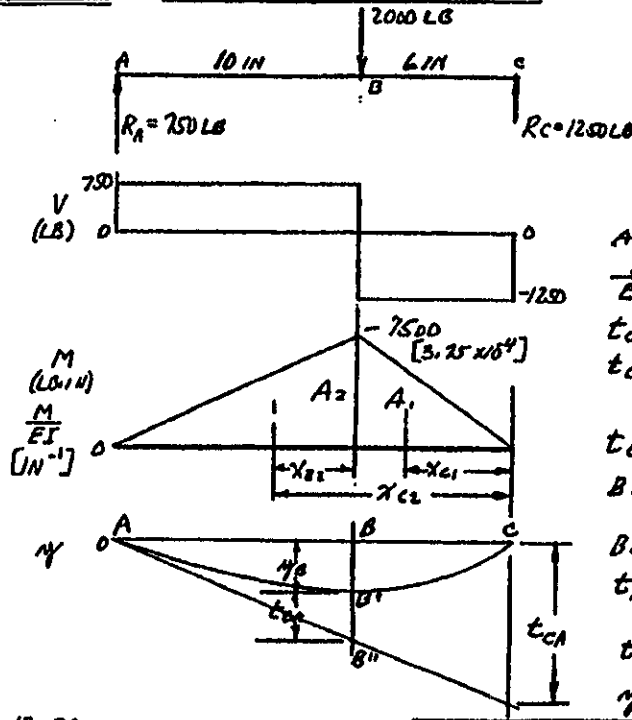
$$I = \pi D^4 / 64$$

$$D = \left[ \frac{64I}{\pi} \right]^{1/4} = \left[ \frac{64(6.91 \times 10^6)}{\pi} \right]^{1/4} = 109 \text{ mm}$$

$$\text{CHECK: } \sigma = \frac{Mc}{I} = \frac{(1.2 \text{ kN}\cdot\text{m})(109/2 \text{ mm})}{6.91 \times 10^6 \text{ mm}^4} \times \frac{(10^3 \text{ N})(10^3 \text{ mm})}{\text{N}\cdot\text{m}} = 9.96 \text{ MPa} \quad \underline{\text{OK}}$$

12-38

MOMENT-AREA METHOD



$$EI = 30 \times 10^6 \frac{\text{LB}}{\text{IN}^2} \times 0.666 \text{ IN}^4$$

$$EI = 2.0 \times 10^7 \text{ LB} \cdot \text{IN}^2$$

AT B:

$$\frac{M}{EI} = \frac{7500 \text{ LB} \cdot \text{IN}}{2.0 \times 10^7 \text{ LB} \cdot \text{IN}^2} = 3.75 \times 10^{-4} \text{ IN}^{-1}$$

$$t_{CA} = A_1 x_{C1} + A_2 x_{C2}$$

$$t_{CA} = \left(\frac{1}{2}\right)(3.75 \times 10^{-4})(6)(4) +$$

$$\left(\frac{1}{2}\right)(3.75 \times 10^{-4})(10)(3.33)$$

$$t_{CA} = 0.022 \text{ IN}$$

$$BB'' = t_{CA} \cdot \frac{AB}{AC} = 0.022 \cdot \frac{6}{16}$$

$$BB'' = 0.01375 \text{ IN}$$

$$t_{BA} = A_2 x_{B2}$$

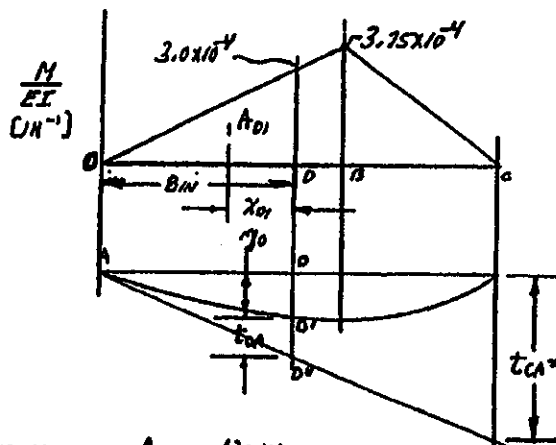
$$= (6.5)(3.75 \times 10^{-4})(10)(3.33)$$

$$t_{BA} = 0.00625 \text{ IN}$$

$$\gamma_B = BB'' - t_{BA} = 0.0075 \text{ IN}$$

12-39

SAME M/EI AS 12-38



$$DD'' = t_{CA} \cdot \frac{AD}{AC} = 0.022 \cdot \frac{6}{16} = 0.008125 \text{ IN}$$

$$t_{DA} = A_{D1} x_{D1}$$

$$= (0.5)(3.0 \times 10^4)(8)(2.667)$$

$$t_{DA} = 0.0032 \text{ IN}$$

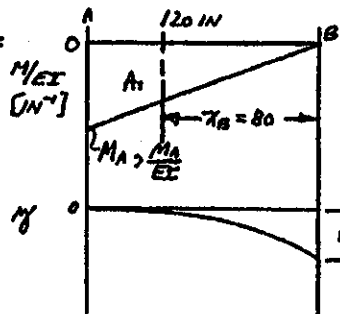
$$\gamma_D = DD'' - t_{DA} = 0.011 - 0.0032$$

$$\gamma_D = 0.0078 \text{ IN}$$

$$t_{CA} = 0.022 \text{ IN (FROM 12-38)}$$

12-40

M, M/EI  
(LB-IN) (IN<sup>-1</sup>)



$$M_A = 2.4 \times 10^6 \text{ LB} \cdot \text{IN}$$

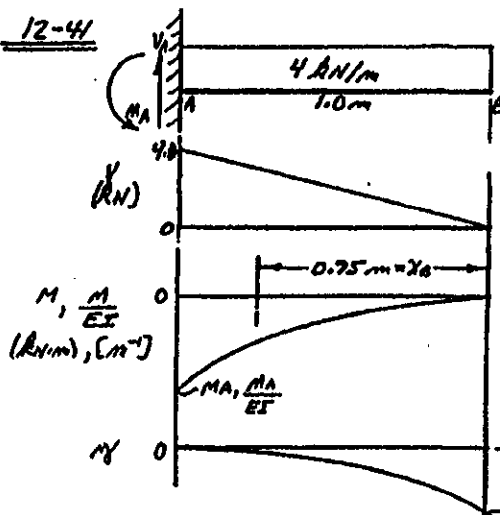
$$I = 890 \text{ IN}^4 \text{ (W18 x 55)}$$

$$\frac{M_A}{EI} = \frac{2.4 \times 10^6 \text{ LB} \cdot \text{IN}}{(30 \times 10^6 \text{ LB/IN}^2)(890 \text{ IN}^4)} = 89.9 \times 10^{-6} \text{ IN}^{-1}$$

$$t_{BA} = A_1 x_B = (0.5)(89.9 \times 10^{-6})(120)(80)$$

$$\gamma_B = 0.432 \text{ IN}$$

12-41



2 IN SCH 40 PIPE:

$$I = 1.530 \text{ IN}^4 \cdot \left( \frac{25.4 \text{ mm}}{(1 \text{ IN})} \right)^4 \cdot \frac{1 \text{ m}^4}{(10^3 \text{ mm})^4}$$

$$I = 6.37 \times 10^{-7} \text{ m}^4$$

$$M_A = 4.0 \text{ kN} \cdot 0.5 \text{ m} = 2.0 \text{ kN} \cdot \text{m}$$

$$\frac{M_A}{EI} = \frac{2.0 \times 10^3 \text{ N} \cdot \text{m}}{6.07 \times 10^9 \text{ N/m}^2 \cdot (6.37 \times 10^{-7} \text{ m}^4)}$$

$$\frac{M_A}{EI} = 0.0152 \text{ m}^{-1}$$

$$t_{BA} = \theta_B = A_1 \cdot x_B$$

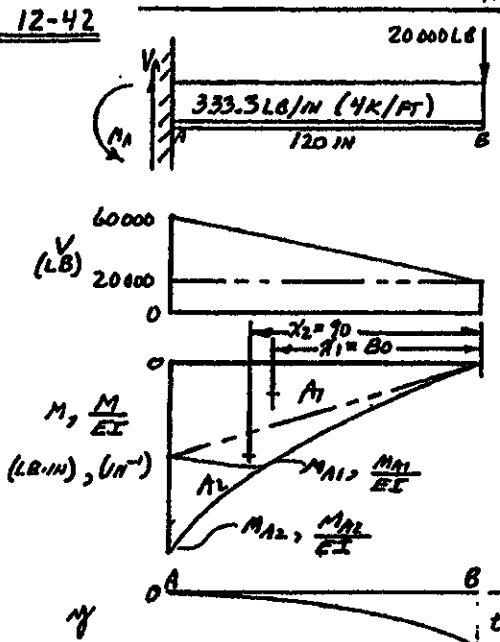
$$= \left( \frac{1}{3} \right) (0.0152) (1.0) (0.75) =$$

$$t_{BA} = 0.00379 \text{ m} = 3.79 \times 10^{-3} \text{ m}$$

$$\theta_B = 3.79 \text{ mm}$$

$$t_{BA} = \theta_B$$

12-42



W 24 x 76 :  $I = 2100 \text{ IN}^4$

$$M_{A1} = (20000 \text{ lb}) (120 \text{ in}) = 2.4 \times 10^6 \text{ lb} \cdot \text{in}$$

$$M_{A2} = M_{A1} + \left( \frac{1}{2} \right) (40000 \text{ lb}) (120 \text{ in})$$

$$M_{A2} = 2.4 \times 10^6 + 2.4 \times 10^6 = 4.8 \times 10^6 \text{ lb} \cdot \text{in}$$

$$\frac{M_{A1}}{EI} = \frac{2.4 \times 10^6 \text{ lb} \cdot \text{in}}{30 \times 10^6 \text{ lb/in}^2 \cdot (2100 \text{ in}^4)} = 3.810 \times 10^{-5} \text{ in}^{-1}$$

$$\frac{M_{A2}}{EI} = 2 \cdot \frac{M_{A1}}{EI} = 7.62 \times 10^{-5} \text{ in}^{-1}$$

$$t_{BA} = A_1 x_1 + A_2 x_2$$

$$= \frac{1}{2} (3.810 \times 10^{-5}) (120) (80) +$$

$$\frac{1}{2} (3.810 \times 10^{-5}) (120) (90)$$

$$t_{BA} = 0.1829 + 0.1371 = 0.320 \text{ in} = \theta_B$$

12-43

DIAGRAMS SAME FORM AS 12-42 : DIA. OF BAR = 100 mm = 0.10 m

$$I = \pi (0.1)^4 / 64 = 4.909 \times 10^{-6} \text{ m}^4 ; EI = (69 \times 10^9 \text{ N/m}^2) (4.909 \times 10^{-6} \text{ m}^4) = 3.39 \times 10^5 \text{ N} \cdot \text{m}^2$$

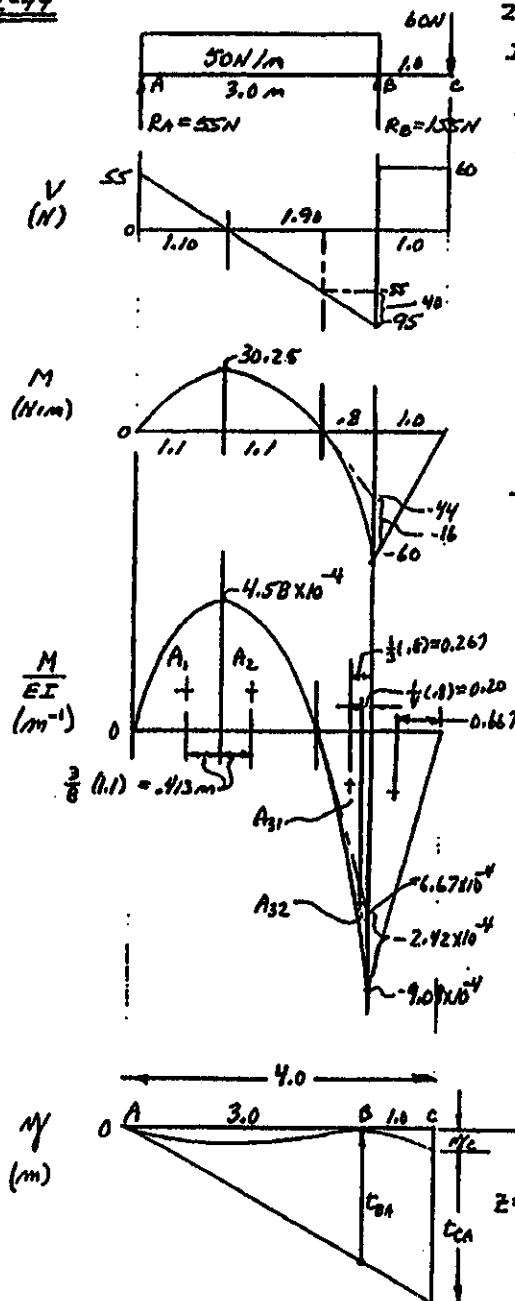
$$M_{A1} = (2000 \text{ N}) (10 \text{ m}) = 2000 \text{ N} \cdot \text{m} ; \frac{M_{A1}}{EI} = \frac{2000 \text{ N} \cdot \text{m}}{3.39 \times 10^5 \text{ N} \cdot \text{m}^2} = 5.91 \times 10^{-3} \text{ m}^{-1}$$

$$M_{A2} = \frac{1}{2} (4000 \text{ N}) (10 \text{ m}) + M_{A1} = 4000 \text{ N} \cdot \text{m} ; \frac{M_{A2}}{EI} = 1.18 \times 10^{-2} \text{ m}^{-1}$$

$$t_{BA} = A_1 x_1 + A_2 x_2 = \frac{1}{2} (5.91 \times 10^{-3}) (10) (0.667) + \frac{1}{2} (5.91 \times 10^{-3}) (10) (0.75)$$

$$t_{BA} = 1.968 \times 10^{-3} + 1.476 \times 10^{-3} = 3.445 \times 10^{-3} \text{ m} = 3.445 \text{ mm} = \theta_B$$

12-44



$$2 \times 2 \times \frac{1}{4} \text{ SQ TUBES: } I = 0.766 \text{ m}^4$$

$$I = 6.766 \text{ m}^4 \left( \frac{25.4 \text{ mm}}{1000} \right)^4 \frac{1 \text{ m}^4}{(10^3 \text{ mm})^4}$$

$$I = 3.19 \times 10^{-7} \text{ m}^4$$

$$EI = (207 \times 10^9 \text{ N/m}^2) (3.19 \times 10^{-7} \text{ m}^4) =$$

$$EI = 66.0 \times 10^3 \text{ N} \cdot \text{m}^2$$

$$A_1 = \frac{2}{3} (1.1) (4.58 \times 10^{-4}) = 3.36 \times 10^{-4}$$

$$A_2 = A_1 = 3.36 \times 10^{-4}$$

$$A_{31} = \frac{1}{2} (0.8) (6.67 \times 10^{-4}) = 2.67 \times 10^{-4}$$

$$A_{32} = \frac{1}{3} (0.8) (2.42 \times 10^{-4}) = 0.64 \times 10^{-4}$$

$$A_4 = \frac{1}{2} (1.0) (9.0 \times 10^{-4}) = 4.55 \times 10^{-4}$$

$$t_{BA} = A_1 x_{B1} + A_2 x_{B2} + A_{31} x_{B31} + A_{32} x_{B32}$$

$$x_{B1} = 1.9 + 0.413 = 2.313 \text{ m}$$

$$x_{B2} = 1.9 - 0.413 = 1.487 \text{ m}$$

$$x_{B31} = 0.267 \text{ m}$$

$$x_{B32} = 0.20 \text{ m}$$

$$\text{THEN } t_{BA} = 1.193 \times 10^{-3} \text{ m}$$

$$z = t_{BA} \cdot \frac{AC}{AB} = t_{BA} \cdot \frac{4.0}{2.0}$$

$$z = \frac{4}{2} (1.193 \times 10^{-3}) = 1.591 \times 10^{-3} \text{ m}$$

$$t_{CA} = A_1 x_{C1} + A_2 x_{C2} + A_{31} x_{C31} + A_{32} x_{C32} + A_4 x_{C4}$$

$$x_{C1} = 2.9 + 0.413 = 3.313 \text{ m}$$

$$x_{C2} = 2.9 - 0.413 = 2.487 \text{ m}$$

$$x_{C31} = 1.1 + 0.267 = 1.267 \text{ m}$$

$$x_{C32} = 1.0 + 0.20 = 1.200 \text{ m}$$

$$x_{C4} = 0.667 \text{ m}$$

$$\text{THEN } t_{CA} = 1.231 \times 10^{-3} \text{ m}$$

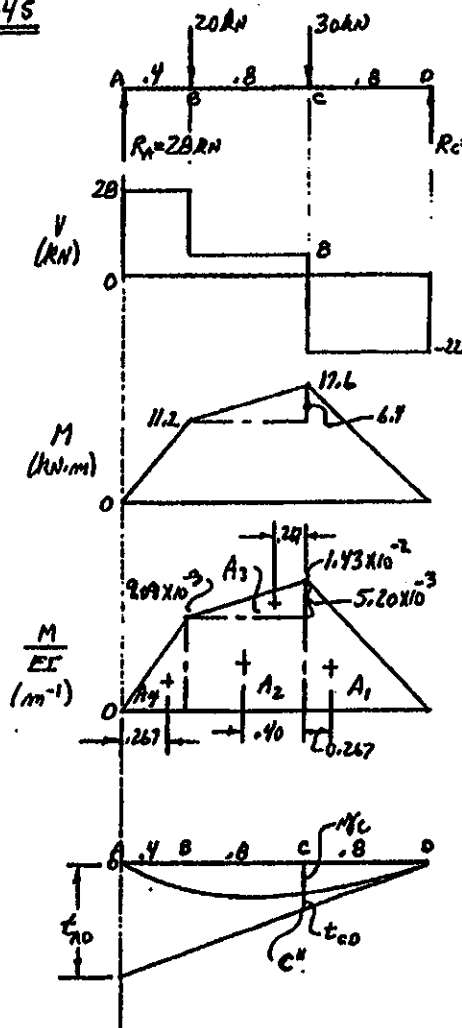
$$\eta_C = z - t_{CA}$$

$$= 1.591 \times 10^{-3} - 1.231 \times 10^{-3}$$

$$\eta_C = 0.360 \times 10^{-3} \text{ m} = 0.360 \text{ mm}$$



12-45



$$ALUM I 7 \times 5.800 : I = 42.89 \text{ IN}^4$$

$$I = (42.89 \text{ IN}^4) \left( \frac{25.4 \text{ mm}}{\text{IN}} \right)^4 \frac{1 \text{ m}^4}{(10^3 \text{ mm})^4}$$

$$I = 1.785 \times 10^{-5} \text{ m}^4$$

$$R_A = 28 \text{ kN} \quad R_D = 22 \text{ kN} \quad EI = (69 \times 10^9) (1.785 \times 10^{-5}) = 1.23 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$\frac{M_A}{EI} = \frac{11200 \text{ N} \cdot \text{m}}{1.23 \times 10^6 \text{ N} \cdot \text{m}^2} = 9.09 \times 10^{-3} \text{ m}^{-1}$$

SIMILARLY,

$$\frac{M_C}{EI} = 1.43 \times 10^{-2} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2} (0.8) (1.43 \times 10^{-2}) = 5.72 \times 10^{-3}$$

$$A_2 = (0.8) (9.09 \times 10^{-3}) = 7.27 \times 10^{-3}$$

$$A_3 = \frac{1}{2} (0.8) (5.20 \times 10^{-3}) = 2.08 \times 10^{-3}$$

$$A_4 = \frac{1}{2} (0.4) (9.09 \times 10^{-3}) = 1.82 \times 10^{-3}$$

$$t_{AD} = A_1 \chi_{A1} + A_2 \chi_{A2} + A_3 \chi_{A3} + A_4 \chi_{A4}$$

$$\chi_{A1} = 0.4 + 0.8 + 0.267 = 1.467 \text{ m}$$

$$\chi_{A2} = 0.4 + 0.4 = 0.80 \text{ m}$$

$$\chi_{A3} = 0.4 + 0.8 - 0.267 = 0.933 \text{ m}$$

$$\chi_{A4} = 0.267 \text{ m}$$

$$\text{THEN } t_{AD} = 16.63 \times 10^{-3} \text{ m}$$

$$CC'' = t_{AD} - \frac{CD}{AD} = t_{AD} - \frac{0.8}{2.0} = 6.65 \times 10^{-3} \text{ m}$$

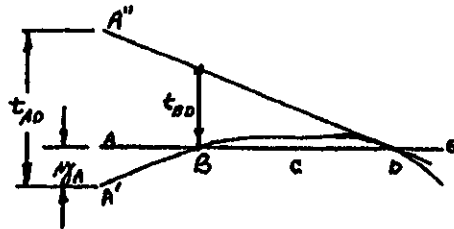
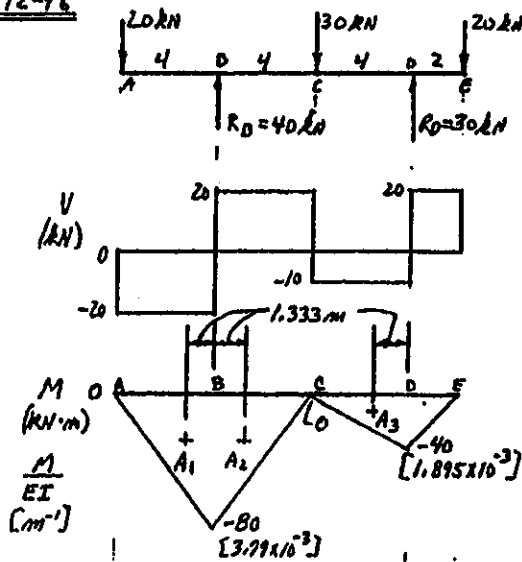
$$t_{CD} = A_1 \chi_{C1} = (5.72 \times 10^{-3}) (0.267 \text{ m})$$

$$t_{CD} = 1.52 \times 10^{-3} \text{ m}$$

$$\eta_C = CC'' - t_{CD} = (6.65 - 1.52) \times 10^{-3} \text{ m}$$

$$\eta_C = 5.13 \times 10^{-3} \text{ m} = 5.13 \text{ mm}$$

12-46



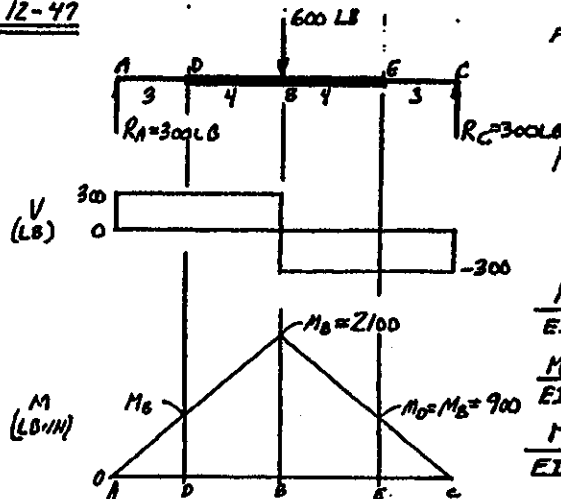
W14 X 26 :  $I = 245 \text{ in}^4$   
 $I = 245 \text{ in}^4 \left( \frac{25.4 \text{ mm}}{1 \text{ in}} \right)^4 \frac{1 \text{ m}^4}{(10^3 \text{ mm})^4}$   
 $I = 1.02 \times 10^{-4} \text{ m}^4$   
 $EI = (207 \times 10^9 \text{ N/m}^2)(1.02 \times 10^{-4} \text{ m}^4)$   
 $EI = 21.1 \times 10^6 \text{ N}\cdot\text{m}^2$   
 $\frac{M_A}{EI} = \frac{80000 \text{ N}\cdot\text{m}}{21.1 \times 10^6 \text{ N}\cdot\text{m}^2} = 3.79 \times 10^{-3} \text{ m}^{-1}$   
 $\frac{M_D}{EI} = \frac{1}{2} \frac{M_B}{EI} = 1.895 \times 10^{-3} \text{ m}^{-1}$   
 $t_{BD} = A_1 \chi_{01} + A_2 \chi_{03}$   
 $A_1 = \frac{1}{2} (3.79 \times 10^{-3})(4) = 7.58 \times 10^{-3}$   
 $A_2 = A_1$   
 $A_3 = \frac{1}{2} (1.895 \times 10^{-3})(4) = 3.79 \times 10^{-3}$   
 $\chi_{01} = 1.333 \text{ m}$   
 $\chi_{03} = 8.0 - 1.333 = 6.667 \text{ m}$   
 $\text{THEN } t_{BD} = 35.4 \times 10^{-3} \text{ m}$

$AA'' = t_{BD} \cdot \frac{AD}{BD} = t_{BD} \cdot \frac{12}{8}$   
 $AA'' = 53.1 \times 10^{-3} \text{ m}$

$t_{AD} = A_1 \chi_{A1} + A_2 \chi_{A2} + A_3 \chi_{A3}$   
 $\chi_{A1} = 4.00 - 1.333 = 2.667 \text{ m}$   
 $\chi_{A2} = 4.00 + 1.333 = 5.333 \text{ m}$   
 $\chi_{A3} = 12.00 - 1.333 = 10.667 \text{ m}$   
 $\text{THEN } t_{AD} = 101.1 \times 10^{-3} \text{ m}$

$\Delta_A = t_{AD} - AA'' = (101.1 - 53.1)(10^{-3}) \text{ m} = 48.0 \times 10^{-3} \text{ m} = 48.0 \text{ mm} = \Delta_A$

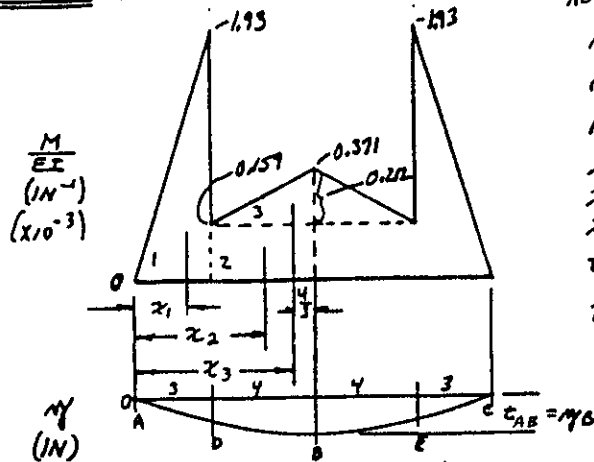
12-47



FOR AD AND EC:  
 $I_1 = \frac{\pi (0.75)^4}{64} = 0.0155 \text{ in}^4$   
 $EI_1 = (30 \times 10^6 \text{ psi})(0.0155 \text{ in}^4) = 4.66 \times 10^5 \text{ lb}\cdot\text{in}^2$   
 FOR DE:  
 $I_2 = \frac{\pi (1.40)^4}{64} = 0.1886 \text{ in}^4$   
 $EI_2 = (30 \times 10^6 \text{ psi})(0.1886 \text{ in}^4) = 5.66 \times 10^6 \text{ lb}\cdot\text{in}^2$   
 $\frac{M_D}{EI_1} = \frac{M_E}{EI_2} = \frac{900 \text{ lb}\cdot\text{ft}}{4.66 \times 10^5 \text{ lb}\cdot\text{in}^2} = 1.93 \times 10^{-3} \text{ in}^{-1}$   
 $\frac{M_D}{EI_2} = \frac{M_E}{EI_2} = \frac{900}{5.66 \times 10^6} = 0.159 \times 10^{-3} \text{ in}^{-1}$   
 $\frac{M_D}{EI_2} = \frac{2100}{5.66 \times 10^6} = 0.371 \times 10^{-3} \text{ in}^{-1}$

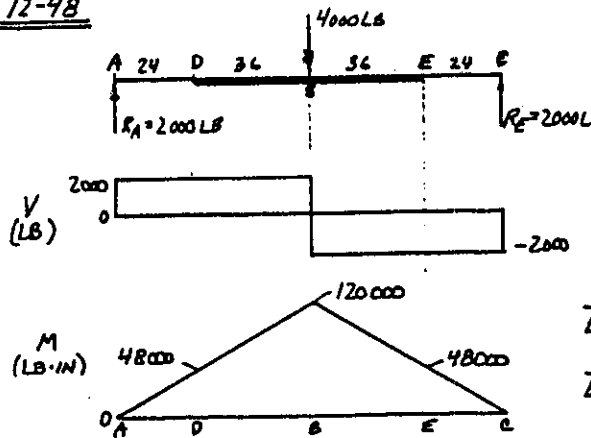
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12-47 (CONT.)

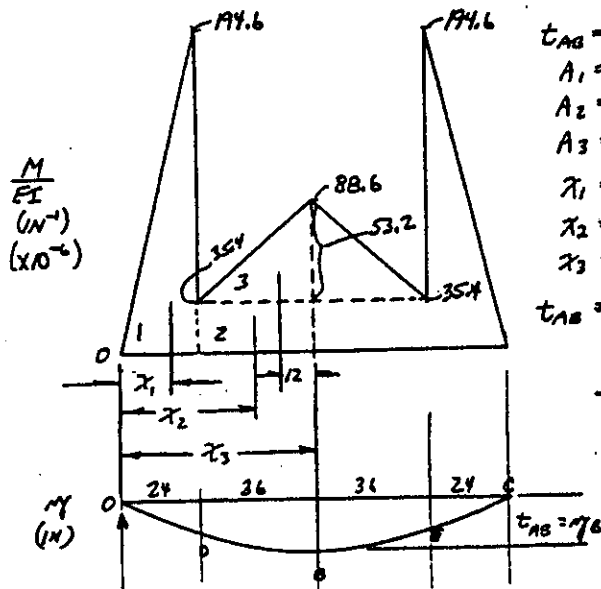


$$\begin{aligned}
 t_{AB} &= \eta_B = A_1 x_1 + A_2 x_2 + A_3 x_3 \\
 A_1 &= \frac{1}{2} (1.93 \times 10^{-3}) (3) = 2.895 \times 10^{-3} \\
 A_2 &= (0.159 \times 10^{-3}) (3) = 0.477 \times 10^{-3} \\
 A_3 &= \frac{1}{2} (0.212 \times 10^{-3}) (4) = 0.424 \times 10^{-3} \\
 x_1 &= (2/3) (3) = 2.000 \text{ in} \\
 x_2 &= 3 + 2 = 5.000 \text{ in} \\
 x_3 &= 7 - 4/3 = 5.667 \text{ in} \\
 t_{AB} &= 0.00580 + 0.00318 + 0.00240 \\
 t_{AB} &= \eta_B = 0.01138 \text{ in}
 \end{aligned}$$

12-48

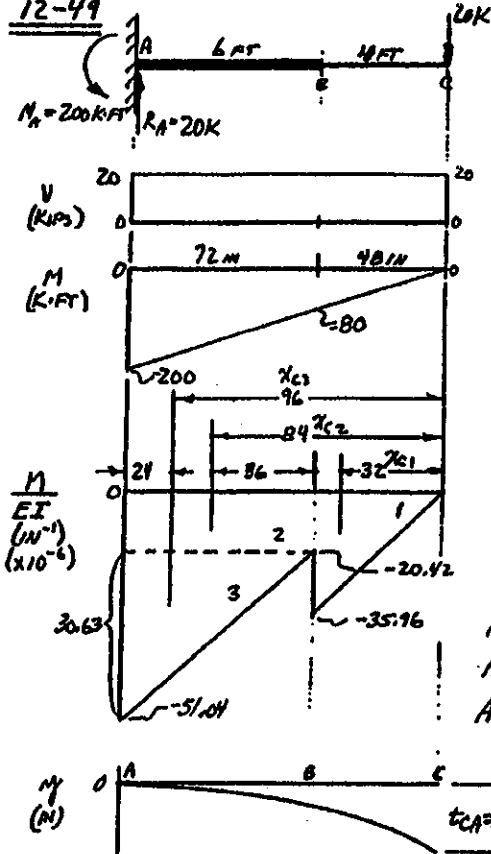


FOR AD AND EC:  $4 \times 4 \times 1/4$   
 $I_1 = 8.22 \text{ in}^4$   
 $EI_1 = (30 \times 10^6) (I_1) = 246.6 \times 10^6 \text{ LB} \cdot \text{in}^2$   
 $I_2 = 2 [I_1 + A d^2]$   
 $= 2 [8.22 + (3.59) (2)^2]$   
 $I_2 = 45.16 \text{ in}^4$   
 $EI_2 = (30 \times 10^6) (I_2) = 1355 \times 10^6 \text{ LB} \cdot \text{in}^2$   
 $\frac{M_0}{EI_1} = \frac{48000}{246.6 \times 10^6} = 194.6 \times 10^{-6} \text{ in}^{-1}$   
 $\frac{M_D}{EI_2} = \frac{48000}{1355 \times 10^6} = 35.4 \times 10^{-6} \text{ in}^{-1}$   
 $\frac{M_E}{EI_2} = \frac{120000}{1355 \times 10^6} = 88.6 \times 10^{-6} \text{ in}^{-1}$

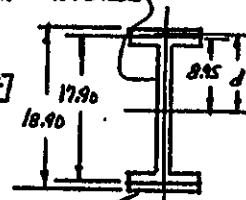


$$\begin{aligned}
 t_{AB} &= \eta_B = A_1 x_1 + A_2 x_2 + A_3 x_3 \\
 A_1 &= \frac{1}{2} (1.946 \times 10^{-3}) (3) = 2.919 \times 10^{-3} \\
 A_2 &= (0.154 \times 10^{-3}) (3) = 0.462 \times 10^{-3} \\
 A_3 &= \frac{1}{2} (0.212 \times 10^{-3}) (4) = 0.424 \times 10^{-3} \\
 x_1 &= (2/3) (3) = 2.000 \text{ in} \\
 x_2 &= 3 + 2 = 5.000 \text{ in} \\
 x_3 &= 7 - 4/3 = 5.667 \text{ in} \\
 t_{AB} &= \eta_B = 0.0374 + 0.0536 + 0.0459 \\
 \eta_B &= 0.1369 \text{ in}
 \end{aligned}$$

12-49



FOR BC:  $I_1 = 890 \text{ IN}^4$  W/18X55  
 $d = 8.95 + 0.25$   
 $d = 9.20 \text{ IN}$   
 $I_1 = I_2 + 2[I_0 + Ad^2]$   
 $I_{PL} = \frac{2(0.59)^3}{12}$   
 $I_{PL} = 0.0833 \text{ IN}^4$



$I_1 = 890 + 2[0.0833 + (4.0)(9.20)^2] \quad (\frac{1}{2} \times 8 \text{ PL.})$

$I_1 = 1567 \text{ IN}^4$

$EI_1 = (30 \times 10^6)(I_1) = 4.70 \times 10^{10} \text{ LB} \cdot \text{IN}^2$

$EI_2 = (30 \times 10^6)(I_2) = 2.67 \times 10^{10} \text{ LB} \cdot \text{IN}^2$

$\frac{M_A}{EI_1} = \frac{(20000)(12)}{4.70 \times 10^{10}} = 51.04 \times 10^{-6} \text{ IN}^{-1}$

$\frac{M_B}{EI_1} = \frac{(80000)(12)}{4.71 \times 10^{10}} = 20.42 \times 10^{-6} \text{ IN}^{-1}$

$\frac{M_B}{EI_2} = \frac{(80000)(12)}{2.67 \times 10^{10}} = 35.96 \times 10^{-6} \text{ IN}^{-1}$

$A_1 = \frac{1}{2}(35.96 \times 10^{-6})(48) = 0.8629 \times 10^{-3}$

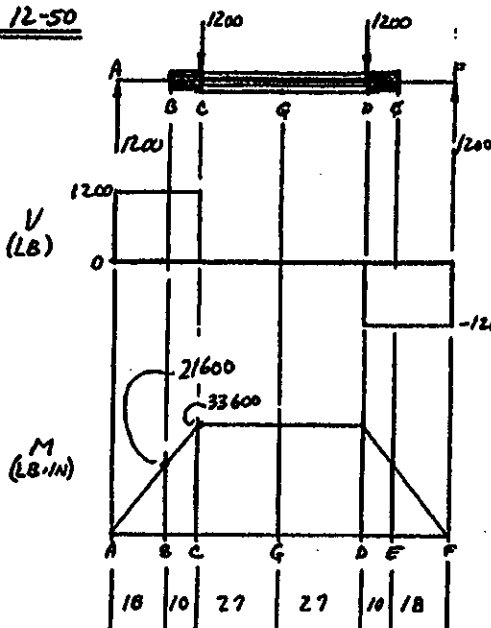
$A_2 = (20.42 \times 10^{-6})(72) = 1.470 \times 10^{-3}$

$A_3 = \frac{1}{2}(30.63 \times 10^{-6})(72) = 1.103 \times 10^{-3}$

$t_{CA} = \eta_C = A_1 x_1 + A_2 x_2 + A_3 x_3$

$t_{CA} = \eta_C \quad \eta_C = 0.0276 + 0.1235 + 0.1058 = 0.2569 \text{ IN}$

12-50



FOR AB AND EF:  $3 \times 3 \times \frac{1}{4}$ ;  $I_1 = 3.16 \text{ IN}^4$

FOR CD:  $4 \times 4 \times \frac{1}{2}$ ;  $I_2 = 12.3 \text{ IN}^4$

FOR BC AND DE:

$I_2 = I_1 + I_3 = 15.46 \text{ IN}^4$

$EI_1 = (30 \times 10^6)(I_1) = 98.8 \times 10^6 \text{ LB} \cdot \text{IN}^2$

$EI_2 = 463.8 \times 10^6 \text{ LB} \cdot \text{IN}^2$

$EI_3 = 369 \times 10^6 \text{ LB} \cdot \text{IN}^2$

$\frac{M_B}{EI_1} = \frac{21600 \text{ LB} \cdot \text{IN}}{98.8 \times 10^6 \text{ LB} \cdot \text{IN}^2} = 227.8 \times 10^{-6} \text{ IN}^{-1}$

$\frac{M_B}{EI_2} = \frac{21600}{463.8 \times 10^6} = 46.57 \times 10^{-6} \text{ IN}^{-1}$

$\frac{M_C}{EI_2} = \frac{33600}{463.8 \times 10^6} = 72.45 \times 10^{-6} \text{ IN}^{-1}$

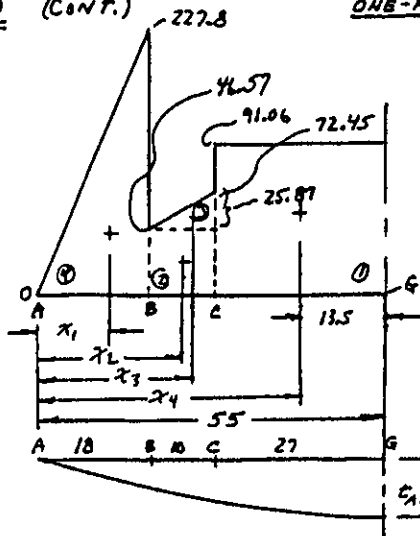
$\frac{M_C}{EI_3} = \frac{33600}{369 \times 10^6} = 91.06 \times 10^{-6} \text{ IN}^{-1}$

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12-50 (CONT.)

ONE-HALF OF DIAGRAMS DRAWN TWICE SIZE

$\frac{M}{EI}$   
( $1/N^4$ )  
( $\times 10^{-6}$ )



$$A_1 = (91.06 \times 10^{-6})(27) = 2.459 \times 10^{-3}$$

$$A_2 = (46.57 \times 10^{-6})(10) = .4657 \times 10^{-3}$$

$$A_3 = \frac{1}{2}(25.97 \times 10^{-6})(10) = 0.1294 \times 10^{-3}$$

$$A_4 = \frac{1}{2}(227.8 \times 10^{-6})(18) = 2.051 \times 10^{-3}$$

$$t_{A4} = t_{G4} = A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4$$

$$x_1 = 55 - 13.5 = 41.5 \text{ in.}$$

$$x_2 = 18 + 5 = 23.0$$

$$x_3 = 18 + (3/5)(10) = 24.667 \text{ in}$$

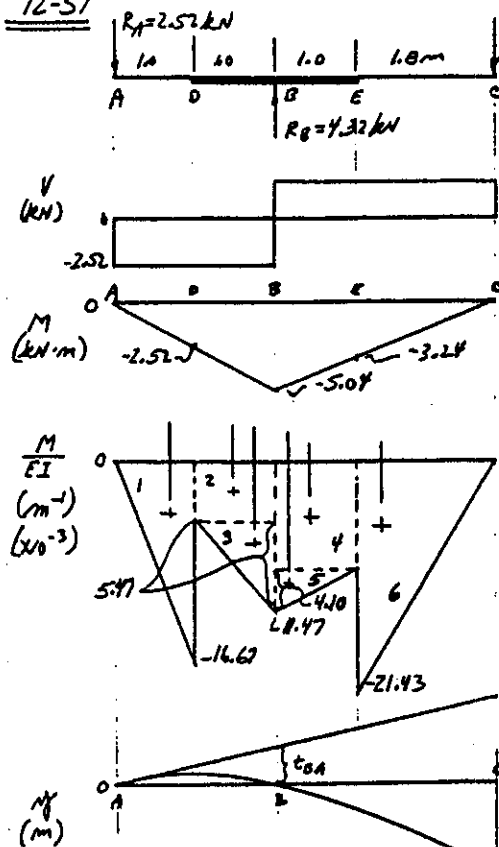
$$x_4 = (3/5)(18) = 12.0 \text{ in}$$

$$t_{A4} = t_{G4} = 0.1020 + 0.0107 + 0.0032 + 0.0246$$

$$\eta_{G4} = 0.1405 \text{ in}$$

$$t_{A4} = \eta_{G4}$$

12-51



FOR AD AND EC:

$$I_1 = (0.281)(1.0)^3 / 12 = 1.68 \times 10^{-6} \text{ m}^4$$

FOR DB AND BE:

$$I_2 = (0.281)(0.12)^3 / 12 = 48.82 \times 10^{-6} \text{ m}^4$$

$$EI_1 = (9 \times 10^9 \text{ N/m}^2)(1.68 \times 10^{-6} \text{ m}^4) = 1.51 \times 10^5 \text{ N}\cdot\text{m}^2$$

$$EI_2 = 4.39 \times 10^5 \text{ N}\cdot\text{m}^2$$

$$M_0/EI_1 = 16.67 \times 10^{-5} \text{ m}^{-1}$$

$$M_0/EI_2 = 5.74 \times 10^{-3} \text{ m}^{-1}$$

$$M_0/EI_2 = 16.47 \times 10^{-3} \text{ m}^{-1}$$

$$ME/EI_2 = 7.37 \times 10^{-3} \text{ m}^{-1}$$

$$ME/EI_1 = 21.43 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = 8.33 \times 10^{-3}; A_2 = 5.74 \times 10^{-3}$$

$$A_3 = 2.87 \times 10^{-3}; A_4 = 7.37 \times 10^{-3}$$

$$A_5 = 2.05 \times 10^{-3}; A_6 = 19.28 \times 10^{-3}$$

$$t_{BA} = A_1 x_{B1} + A_2 x_{B2} + A_3 x_{B3}$$

$$t_{BA} = 0.0111 + 0.00287 + 0.00096 = 0.01493 \text{ m}$$

$$z = t_{BA} \cdot \frac{AC}{AB} = 0.01493 \cdot \frac{4.0}{2.0} = 0.03584 \text{ m}$$

$$t_{CA} = A_1 x_{C1} + A_2 x_{C2} + A_3 x_{C3}$$

$$+ A_4 x_{C4} + A_5 x_{C5} + A_6 x_{C6}$$

$$t_{CA} = 0.0344 + 0.0189 + 0.0090$$

$$+ 0.0170 + 0.0057 + 0.0221$$

$$t_{CA} = 0.1075 \text{ m}$$

$$\eta_C = t_{CA} - z = 0.1075 \text{ m} - 0.0358 \text{ m} = 0.0717 \text{ m} = 71.7 \times 10^{-3} \text{ m}$$

$$\eta_C = 71.7 \text{ mm}$$

# CHAPTER 13 Statically Indeterminate Beams

13-1 CASE A-24 (a);  $P = 35 \text{ kN}$ ;  $L = 4.0 \text{ m}$

$$R_A = \left(\frac{11}{16}\right)P = \left(\frac{11}{16}\right)35000 \text{ N} = 24,063 \text{ N} = V_A$$

$$R_C = \left(\frac{5}{16}\right)P = \left(\frac{5}{16}\right)35000 \text{ N} = 10,938 \text{ N} = V_C$$

$$M_A = -\frac{3}{16} \cdot PL = -\frac{3}{16}(35000 \text{ N})(4.0 \text{ m}) = -26,250 \text{ N}\cdot\text{m}$$

$$M_B = \frac{5}{32} \cdot PL = \frac{5}{32}(35000)(4.0) = 21,875 \text{ N}\cdot\text{m}$$

$$M_B \text{ AT LOAD} = \frac{-7 PL^3}{768 EI} = \frac{-7(35000)(4)^3}{768 EI} = \frac{-20,417}{EI}$$

$$\text{AT D: } N = 0.447L = 0.447(4.0 \text{ m}) = 1.788 \text{ m}$$

$$M_{D0} = \frac{-PL^3}{107 EI} = \frac{-(35000)(4.0)^3}{107 EI} = \frac{-20,934}{EI}$$

$$\text{LET } \eta_{\text{MAX}} = \frac{L}{360} = \frac{4.0 \text{ m}}{360} = 0.0111 \text{ m}$$

SPECIFY STEEL BEAM  $E = 207 \times 10^9 \text{ Pa}$

$$\text{REQD. } I = \frac{20,934 \text{ N}\cdot\text{m}^3}{E \eta_{\text{MAX}}} = \frac{20,934 \text{ N}\cdot\text{m}^3}{207 \times 10^9 \text{ N/m}^2 (0.0111 \text{ m})} = 9.102 \times 10^{-6} \text{ m}^4$$

$$I = 9.102 \times 10^{-6} \text{ m}^4 \times \frac{(10^3 \text{ mm})^4}{1 \text{ m}^4} \times \frac{1 \text{ N}^4}{(25.4 \text{ mm})^4} = 21.87 \text{ IN}^4$$

$$\text{SPECIFY WB} \times 10: I = 30.8 \text{ IN}^4; S = 7.81 \text{ IN}^3 \times \frac{(25.4 \text{ mm})^3}{1 \text{ IN}^3} = 1.28 \times 10^5 \text{ mm}^3$$

CHECK STRESS:

$$\sigma = \frac{M}{S} = \frac{26,250 \text{ N}\cdot\text{m}}{1.28 \times 10^5 \text{ mm}^3} \times \frac{10^3 \text{ mm}}{1 \text{ m}} = 205 \text{ N/mm}^2 = 205 \text{ MPa}$$

$$\text{LET } \sigma = 0.66 S_y; \text{ REQD } S_y = \frac{\sigma}{0.66} = \frac{205 \text{ MPa}}{0.66} = 311 \text{ MPa}$$

SPECIFY ASTM A242,  $S_y = 345 \text{ MPa}$

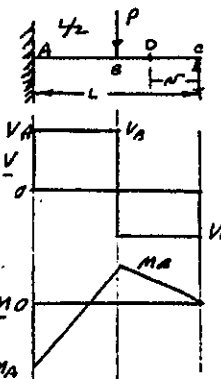
SUMMARY

REACTIONS:  $R_A = 24.06 \text{ kN}$ ,  $R_C = 10.94 \text{ kN}$

$V_{\text{MAX}} = V_A = 24.06 \text{ kN}$  AT A.  $M_{\text{MAX}} = 26.25 \text{ kN}\cdot\text{m}$  AT A.

REQD.  $I = 9.102 \times 10^{-6} \text{ m}^4$  (21.87 IN<sup>4</sup>) TO LIMIT DEFL. TO  $\frac{L}{360}$ .

SPECIFY LIGHTEST STEEL BEAM: WB  $\times 10$ ,  $I = 30.8 \text{ IN}^4$ ;  $S = 7.81 \text{ IN}^3$



13-2 CASE A-24 (b);  $P=35 \text{ kN}$ ,  $L=4.0 \text{ m}$ ,  $a=1.50 \text{ m}$

$$b = L - a = 4.0 - 1.5 = 2.5 \text{ m}$$

$$R_A = \frac{Pb}{2L^3} (L^2 - b^2) = \frac{35 \text{ kN} (2.5)}{2 (4.0)^3} (3 (4.0)^2 - 2.5^2) = 28.54 \text{ kN}$$

$$R_C = \frac{Pa^2}{2L^3} (b + 2L) = \frac{25 \text{ kN} (1.5)^2}{2 (4)^3} (2.5 + 2 (4)) = 6.46 \text{ kN}$$

$$M_A = \frac{-Pa^2b}{2L^2} (b + L) = \frac{-35 \text{ kN} (1.5)(2.5)}{2 (4.0)^2} (2.5 + 4.0) = -26.66 \text{ kN}\cdot\text{m}$$

$$M_B = \frac{Pa^2b}{2L^2} (b + 2L) = \frac{35 \text{ kN} (1.5)^2 (2.5)}{2 (4.0)^2} (2.5 + 2 (4.0)) = 16.15 \text{ kN}\cdot\text{m}$$

#### DEFLECTION

MAX. DEFLECTION OCCURS IN BC.

$$\eta_{BC} = \frac{-Pa^2N}{12EI} [3L^2b - N^2(3L - a)]$$

$N$  MEASURED FROM C - VARIABLE DISTANCE  
REDUCE TO EXPRESSION OF THE FORM:

$$\eta_{BC} = \frac{a_1 N^3 - a_2 N}{EI}$$

USE  $P=35000 \text{ N}$ , DISTANCES IN  $\text{m}$ .

$$\eta_{BC} = \frac{(-35000)(1.50)^2 N}{12 EI (4.0)^3} [3 (4.0)^2 (2.5) - N^2 (3 (4.0) - 1.5)]$$

$$= \frac{-102.54 N}{EI} [120 - 10.5 N^2] = \frac{1}{EI} [-12305 N + 1076.7 N^3]$$

$$\eta_{BC} = \frac{1}{EI} [1076.7 N^3 - 12305 N]$$

USING GRAPHING CALCULATOR, FUNCTION IS MINIMUM

AT  $N = 1.952 \text{ m}$  FROM C. VALUE OF FUNCTION THERE IS:

$$\eta_{\max} = \frac{-16010}{EI}$$

USE W8X10 - SAME AS IN PROB 13-1.  $E = 207 \times 10^9 \text{ N/m}^2$

$$I = 30.8 \text{ IN}^4 \times \frac{4.162 \times 10^5 \text{ mm}^4}{\text{IN}^4} = 1.28 \times 10^7 \text{ mm}^4$$

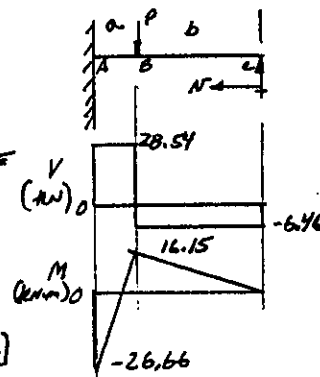
$$\eta_{\max} = \frac{-16010 \text{ N}\cdot\text{m}^3}{(207 \times 10^9 \text{ N/m}^2)(1.28 \times 10^7 \text{ mm}^4)} \cdot \frac{(10^3)^5 \text{ mm}^5}{\text{m}^5} = -6.03 \text{ mm}$$

#### STRESS

$$\sigma = \frac{M}{S} = \frac{26.66 \text{ kN}\cdot\text{m}}{1.28 \times 10^5 \text{ mm}^3} \times \frac{10^3 \text{ N}}{\text{kN}} \times \frac{10^3 \text{ mm}}{\text{m}} = 208 \text{ MPa}$$

FOR ASTM A242,  $S_y = 345 \text{ MPa}$ ;  $\sigma_b = 0.66 S_y = 227 \text{ MPa}$  OK

$$\text{AT LOAD, } \eta_b = -13939/EI = 5.25 \text{ mm}$$



13-3

CASE A-24(b);  $P=35 \text{ kN}$ ,  $L=4.0$ ,  $a=2.50 \text{ m}$

$$b = L - a = 4.0 - 2.5 = 1.5 \text{ m}$$

$$R_A = \frac{Pb}{2L^3} (3L^2 - b^2) = \frac{35 \text{ kN}(1.5)}{2(4)^3} [3(4)^2 - (1.5)^2] = 18.76 \text{ kN} = V_{AB}$$

$$R_C = \frac{Pa^2}{2L^3} (b + 2L) = \frac{35 \text{ kN}(2.5)^2}{2(4)^3} [1.5 + 2(4)] = 16.24 \text{ kN} = V_{BC}$$

$$M_A = \frac{-Pa^2b}{2L^3} (b + L) = \frac{-35(2.5)(1.5)}{2(4)^3} (1.5 + 4) = -22.56 \text{ kN}\cdot\text{m}$$

$$M_B = \frac{Pa^2b}{2L^3} (b + 2L) = \frac{35(2.5)^2(1.5)}{2(4)^3} (1.5 + 2(4)) = 24.35 \text{ kN}\cdot\text{m}$$

#### DEFLECTION

FROM A TO B:  $P=35000 \text{ N}$ ; DISTANCES IN M.

$$\eta_{AB} = \frac{-Px^2b}{12EI L^3} (3C_1 - C_2x)$$

$$C_1 = aL(L+b) = 2.5(4)[4+1.5] = 55$$

$$C_2 = (L+a)(L+b) + aL = (6.5)(5.5) + 10 = 45.75$$

$$\eta_{AB} = \frac{(-25000)(x^2)(1.5)[3(55) - 45.75x]}{12EI(4)^3}$$

$$= \frac{-68.36x^2[165 - 45.75x]}{EI}$$

$$\eta_{AB} = \frac{3127.4x^3 - 11279x^4}{EI}$$

USING A GRAPHING CALCULATOR FUNCTION IS A MINIMUM

AT  $x = 2.404 \text{ m}$  FROM A. THEN  $\eta_{\text{MAX}}$  IS:

$$\eta_{\text{MAX}} = \frac{-21734}{EI}$$

$$\text{AT } x = 2.5, \eta_B = \frac{-21628}{EI}$$

USING W8X10 STEEL BEAM - SAME AS PROB 13-1,

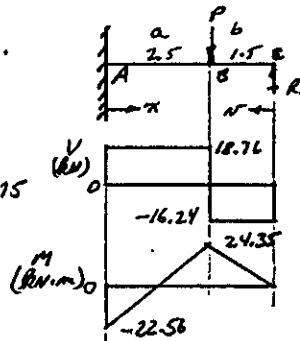
$$\eta_{\text{MAX}} = \frac{-21734 \text{ N}\cdot\text{m}^3}{(207 \times 10^3 \text{ N/m}^2)(1.28 \times 10^3 \text{ mm}^4)} \times \frac{10^{15} \text{ mm}^5}{\text{m}^5} = -8.19 \text{ mm}$$

$$\eta_B = \frac{-21628}{EI} = -8.15 \text{ mm}$$

#### STRESS:

$$\sigma = \frac{M}{S} = \frac{24.35 \text{ kN}\cdot\text{m}}{1.28 \times 10^3 \text{ mm}^3} \times \frac{10^3 \text{ N}}{\text{kN}} \times \frac{10^3 \text{ mm}}{\text{m}} = 190 \text{ MPa}$$

(OK - 50%)  
(13-2)





13-4

CASE A-24 (C):  $w = 400 \text{ lb/ft}$ ,  $L = 14.0 \text{ ft}$

$$W = wL = (400 \text{ lb/ft})(14.0 \text{ ft}) = 5600 \text{ lb}$$

$$R_A = 5/8 W = 3500 \text{ lb} = V_A; R_B = 3/8 W = 2100 \text{ lb} = V_B$$

$$M_A = -0.125 WL = -0.125(5600)(14.0) = -9800 \text{ lb}\cdot\text{ft}$$

$$M_E = 0.0703 WL = 0.0703(5600)(14.0) = 5512 \text{ lb}\cdot\text{ft}$$

POINT E IS  $5/8 L$  FROM A (FIXED END)

$$x_E = 5/8 (14.0 \text{ ft}) = 8.75 \text{ ft}$$

DEFLECTION:

$$\text{AT C AT } x = 0.579 L = 0.579(14.0) = 8.11 \text{ ft}$$

$$\Delta y_C = \Delta y_{\max} = \frac{-WL^3}{185EI} = \frac{-5600 \text{ lb} \times (14 \text{ ft})^3}{185EI}$$

$$\Delta y_{\max} = \frac{-83061 \text{ lb}\cdot\text{ft}^3}{EI}$$

$$\text{LET } \Delta y_{\max} \leq \frac{L}{360} = \frac{14.0 \text{ ft} (12 \text{ in/ft})}{360} = 0.467 \text{ in}$$

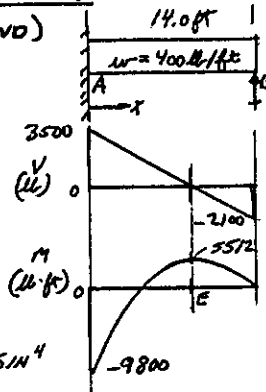
$$\text{REQD. } I = \frac{-83061 \text{ lb}\cdot\text{ft}^3}{(30 \times 10^6 \text{ lb/in}^2) \times (-0.467 \text{ in})} \times \frac{(12 \text{ in})^3}{1 \text{ ft}^3} = 10.25 \text{ in}^4$$

W 8 X 10 STEEL BEAM IS LIGHTEST,  $I = 30.8 \text{ in}^4$ ,  $S = 7.81 \text{ in}^3$

$$\text{ACTUAL } \Delta y_{\max} = \frac{-83061 \text{ lb}\cdot\text{ft}^3}{(30 \times 10^6 \text{ lb/in}^2)(30.8 \text{ in}^4)} \times \frac{(12 \text{ in})^3}{1 \text{ ft}^3} = 0.155 \text{ in.}$$

$$\text{STRESS: } \sigma = \frac{M}{S} = \frac{9800 \text{ lb}\cdot\text{ft}}{7.81 \text{ in}^3} \times \frac{12 \text{ in}}{1 \text{ ft}} = 15058 \text{ psi}$$

$$\text{CAN USE ASTM A36; } \sigma_u = 0.66 (36 \text{ ksi}) = 23760 \text{ psi} - \text{OK}$$



13-5

CASE A-24 (C):  $w = 50 \text{ lb/in}$ ,  $L = 16.0 \text{ in}$

$$W = wL = (50 \text{ lb/in})(16.0 \text{ in}) = 800 \text{ lb}$$

$$R_A = 5/8 W = 500 \text{ lb}; R_B = 3/8 W = 300 \text{ lb}$$

$$M_A = -0.125 WL = -0.125(800)(16) = -1600 \text{ lb}\cdot\text{in}$$

$$M_E = 0.0703 WL = 0.0703(800)(16) = 900 \text{ lb}\cdot\text{in}$$

$$x_E = 5/8 L = 5/8 (16) = 10.0 \text{ in}$$

DEFLECTION:

$$\text{AT C } \Delta y = 0.579 L = 0.579 (16) = 9.264 \text{ in}$$

$$\Delta y_C = \Delta y_{\max} = \frac{-WL^3}{185EI} = \frac{-800(16)^3}{185EI} = \frac{17712}{EI}$$

DESIGN COULD RESULT IN MULTIPLE SOLUTIONS.

SKETCH SIMILAR  
TO 13-4.

13-6 CASE A-24(d):  $P = 350 \text{ kN}$ ,  $L = 10.8 \text{ m}$ ,  $a = 2.50 \text{ m}$ .

$$R_A = \frac{-3Pa}{2L} = \frac{-3(350)(2.50)}{2(10.8)} = -721.5 \text{ kN DOWN}$$

$$R_B = P \left( 1 + \frac{3a}{2L} \right) = 350 \text{ kN} \left[ 1 + \frac{3(2.50)}{2(10.8)} \right] = 471.5 \text{ kN UP}$$

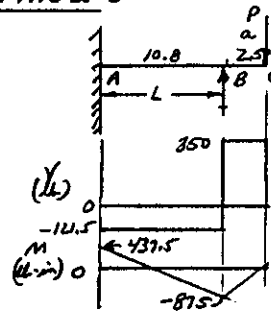
$$M_A = Pa/2 = (350)(2.50)/2 = 437.5 \text{ kN}\cdot\text{m}$$

$$M_B = -Pa = -350(2.5) = -875 \text{ kN}\cdot\text{m}$$

$$M_C = \frac{-PL^3}{EI} \left[ \frac{a^2}{4L^2} + \frac{a^3}{3L^3} \right]$$

$$= \frac{-350(10.8)^3}{EI} \left[ \frac{2.5^2}{4(10.8)^2} + \frac{2.5^3}{3(10.8)^3} \right]$$

$$M_C = \frac{7729 \text{ kN}\cdot\text{m}^3}{EI}$$



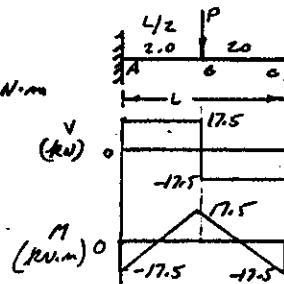
13-7 CASE A-24(e):  $P = 35 \text{ kN}$ ;  $L = 4.0 \text{ m}$

$$R_A = R_C = P/2 = 35/2 = 17.5 \text{ kN}$$

$$M_A = M_B = M_C = \frac{PL}{8} = \frac{(35000 \text{ N})(4.0 \text{ m})}{8} = 17500 \text{ N}\cdot\text{m}$$

$$M_B = M_{\text{max}} = \frac{-PL^3}{192EI} = \frac{-35000(4.0)^3}{192EI}$$

$$M_B = \frac{11667 \text{ N}\cdot\text{m}^3}{EI}$$



13-8 CASE A-24(f):  $P = 35 \text{ kN}$ ,  $L = 4.0 \text{ m}$ ,  $a = 1.50 \text{ m}$ ,  $b = 4.0 - a = 2.5 \text{ m}$ .  
THIS LOADING IS THE MIRROR IMAGE OF THAT IN 13-9.  
NOTATION OF CASE A-24(f) REQUIRES  $a > b$ . THEN  
CALCULATIONS ARE THE SAME.

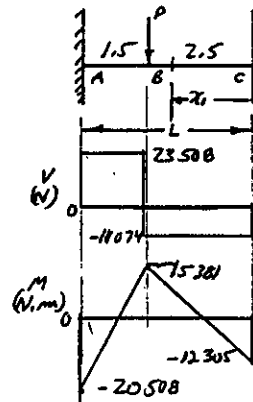
$$R_A = 23926 \text{ N}, R_C = 11074 \text{ N}$$

$$M_A = -20508 \text{ N}\cdot\text{m} \quad M_B = 15381 \text{ N}\cdot\text{m}$$

$$M_C = -12305 \text{ N}\cdot\text{m}$$

$$M_{\text{max}} = M_D = \frac{-10127 \text{ N}\cdot\text{m}^3}{EI}$$

$$x_1 = 2.222 \text{ m FROM C TO D}$$



13-9CASE A-24(f):  $P = 35000 \text{ N}$ ,  $L = 4.0 \text{ m}$ ,  $a = 2.5 \text{ m}$ ,  $b = L - a = 1.5 \text{ m}$ 

$$R_A = \frac{Pb^2}{L^3} (3a+b) = \frac{35000(1.5)^2}{(4.0)^3} (3(2.5)+1.5) = 11074 \text{ N}$$

$$R_C = \frac{Pa^2}{L^3} (3b+a) = \frac{35000(2.5)^2}{(4.0)^3} (3(1.5)+2.5) = 23926 \text{ N}$$

$$M_A = -\frac{Pab^2}{L^2} = -\frac{35000(2.5)(1.5)^2}{(4.0)^2} = -12305 \text{ N}\cdot\text{m}$$

$$M_B = \frac{2Pa^2b^2}{L^3} = \frac{2(35000)(2.5)^2(1.5)^2}{(4.0)^3} = 15381 \text{ N}\cdot\text{m}$$

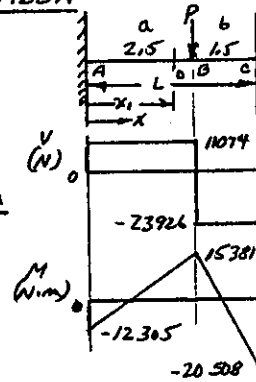
$$M_C = -\frac{Pa^3b}{L^2} = -\frac{35000(2.5)^2(1.5)}{(4.0)^2} = -20508 \text{ N}\cdot\text{m}$$

DEFLECTION:

$$\Delta y_{\text{MAX}} = \Delta y_D = \frac{-2Pa^3b^2}{3EI(3a+b)^2}$$

$$\Delta y_{\text{MAX}} = \frac{2(35000)(2.5)^3(1.5)^2}{3EI(3(2.5)+1.5)^2} = \frac{-10127 \text{ N}\cdot\text{m}^3}{EI}$$

$$x_1 = \frac{2aL}{3a+b} = \frac{2(2.5)(4)}{3(2.5)+1.5} = 2.222 \text{ m FROM A TO D.}$$

13-10CASE A-24(g):  $w = 400 \text{ lb/ft}$ ;  $L = 14.0 \text{ ft}$ 

$$W = wL = 400 \text{ lb/ft} (14.0 \text{ ft}) = 5600 \text{ lb}$$

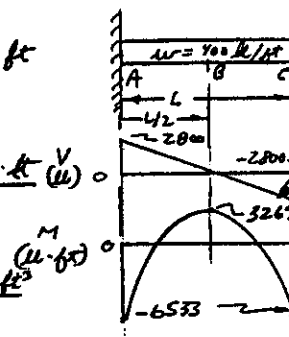
$$R_A = R_C = W/2 = 2800 \text{ lb}$$

$$M_A = M_C = -WL/12 = -5600(14)/12 = -6533 \text{ lb}\cdot\text{ft}$$

$$M_D = WL/24 = 3267 \text{ lb}\cdot\text{ft}$$

DEFLECTION:

$$\Delta y_D = \Delta y_{\text{MAX}} = \frac{-WL^3}{384EI} = \frac{-5600(14)^3}{384EI} = \frac{-40017 \text{ lb}\cdot\text{ft}^3}{EI}$$

13-11CASE A-24(h):  $w = 50 \text{ lb/m}$ ,  $L = 16.0 \text{ m}$ 

$$W = wL = 50 \text{ lb/m} (16 \text{ m}) = 800 \text{ lb}$$

$$R_A = R_C = W/2 = 400 \text{ lb} = V_A = V_C$$

$$M_A = M_C = -WL/12 = -800(16)/12 = -1067 \text{ lb}\cdot\text{m}$$

$$M_D = WL/24 = 523 \text{ lb}\cdot\text{m}$$

DEFLECTION:

$$\Delta y_D = \Delta y_{\text{MAX}} = \frac{-WL^3}{384EI} = \frac{-800(16)^3}{384EI} = \frac{-8533 \text{ lb}\cdot\text{m}^3}{EI}$$

(SKETCH SIMILAR TO B-10.)

13-12 CASE A-24(1):  $w = 400 \text{ lb/ft}$ ,  $L = 7.0 \text{ ft}$

$$W = wL = (400 \text{ lb/ft})(7 \text{ ft}) = 2800 \text{ lb ON 1 SPAN.}$$

$$R_A = R_C = 3W/8 = 3(2800)/8 = 1050 \text{ lb} = V_A = V_C$$

$$R_B = 1.25W = 1.25(2800) = 3500 \text{ lb}$$

$$V_B = 5W/8 = 5(2800)/8 = 1750 \text{ lb}$$

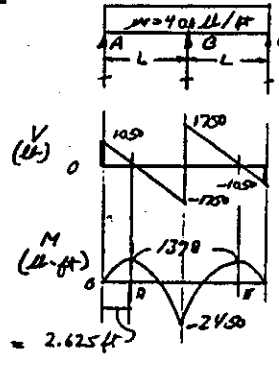
$$M_B = M_E = 0.0703 WL = 0.0703(2800)(7) = 1378 \text{ lb-ft}$$

$$M_D = -0.125 WL = -0.125(2800)(7) = -2450 \text{ lb-ft}$$

DEFLECTION: MAX AT  $X_1 = 0.4215L$  FROM A OR C.

$$X_1 = 0.4215(7.0 \text{ ft}) = 2.9505 \text{ ft}$$

$$\Delta_{\text{MAX}} = \frac{-wL^4}{185EI} = \frac{-(400)(7)^4}{185EI} = \frac{-5191 \text{ lb-ft}^3}{EI}$$



13-13 CASE A-24(1):  $w = 50 \text{ lb/in}$ ,  $L = 8.0 \text{ in}$

$$W = wL = (50 \text{ lb/in})(8 \text{ in}) = 400 \text{ lb ON EACH SPAN}$$

$$R_A = 3W/8 = 3(400)/8 = 150 \text{ lb} = R_C = V_A = V_C$$

$$R_B = 1.25W = 1.25(400) = 500 \text{ lb}$$

$$V_B = 5W/8 = 5(400)/8 = 250 \text{ lb}$$

$$M_B = M_E = 0.0703 WL = 0.0703(400)(8) = 225 \text{ lb-in}$$

$$M_D = -0.125 WL = -0.125(400)(8) = -400 \text{ lb-in}$$

DEFLECTION: MAX AT  $X_1 = 0.4215L$  FROM A OR C.

$$X_1 = 0.4215(8.0 \text{ in}) = 3.372 \text{ in}$$

$$\Delta_{\text{MAX}} = \frac{-wL^4}{185EI} = \frac{-50(8)^4}{185EI} = \frac{-1107}{EI}$$

(SKETCH SIMILAR TO 13-12.)

13-14 CASE A-24(2):  $w = 400 \text{ lb/ft}$ ,  $L = 56 \text{ ft}$ ,  $W = wL = 1867 \text{ lb}$

$$R_A = R_D = 0.4W = 0.4(1867 \text{ lb}) = 746.7 \text{ lb} = V_A = V_D$$

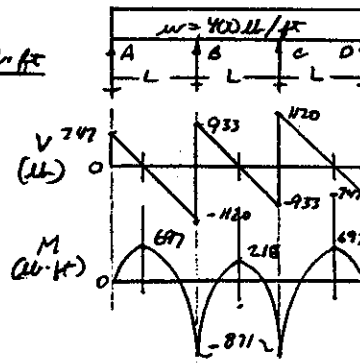
$$R_B = R_C = 1.10W = 1.10(1867) = 2053 \text{ lb}$$

$$M_E = M_F = 0.08 WL = 0.08(1867)(4.67 \text{ ft}) = 697 \text{ lb-ft}$$

$$M_B = M_C = -0.10 WL = -871 \text{ lb-ft}$$

$$M_G = 0.025 WL = 0.025(1867)(4.67 \text{ ft}) = 218 \text{ lb-ft}$$

DEFLECTION FORMULAS NOT AVAILABLE



13-15 CASE A24(L) :  $w = 50 \text{ lb/m}$ ,  $L = 5.333 \text{ m}$ ,  $W = wL = 266.7 \text{ lb}$

$R_A = R_D = 0.4W = 106.7 \text{ lb}$

$R_B = R_C = 1.10W = 293.3 \text{ lb}$

$M_F = M_E = 0.08WL = 0.08(266.7)(5.333) = 113.8 \text{ lb}\cdot\text{m}$

$M_B = M_C = -0.10WL = -0.10(266.7)(5.333) = -142.2 \text{ lb}\cdot\text{m}$

$M_G = 0.025WL = 0.025(266.7)(5.333) = 35.6 \text{ lb}\cdot\text{m}$

SKETCH  
SIMILAR TO  
13-14.

13-16 CASE A24(f) :  $w = 400 \text{ lb/ft}$ ,  $L = 3.5 \text{ ft}$ ,  $W = wL = 1400 \text{ lb}$  EACH SPAN

$R_A = R_E = 0.393W = 550 \text{ lb}$

$R_B = R_D = 1.143W = 1600 \text{ lb}$

$R_C = 0.928W = 1300 \text{ lb}$

$M_B = M_D = -0.1071WL = M_{\text{MAX}}$   
 $= -0.1071(1400)(3.5) = -525 \text{ lb}\cdot\text{ft}$

$M_F = M_I = 0.0772WL = 378 \text{ lb}\cdot\text{ft}$

$M_C = -0.0714WL = -350 \text{ lb}\cdot\text{ft}$

$M_G = M_H = 0.0364WL = 178 \text{ lb}\cdot\text{ft}$

13-17 CASE A24(f) :  $w = 50 \text{ lb/m}$ ,  $L = 4.0 \text{ m}$ ,  $W = wL = 200 \text{ lb}$

$R_A = R_E = 0.393W = 78.6 \text{ lb}$

$R_B = R_D = 1.143W = 228.6 \text{ lb}$

$R_C = 0.928W = 185.6 \text{ lb}$

$M_B = M_D = -0.1071WL = -85.68 \text{ lb}\cdot\text{m}$

$M_F = M_I = 0.0772WL = 61.76 \text{ lb}\cdot\text{m}$

$M_C = -0.0714WL = -57.12 \text{ lb}\cdot\text{m}$

$M_G = M_H = 0.0364WL = 29.12 \text{ lb}\cdot\text{m}$

SKETCH SIMILAR TO 13-16.

$M_{\text{MAX}} = 85.7 \text{ lb}\cdot\text{m} = M_B = M_D$

$V_{\text{MAX}} = 121.4 \text{ lb} = -V_B = +V_D$

$V_A = R_A = 78.6 \text{ lb}$

$-V_B = R_A - W = -121.4 \text{ lb}$

$+V_B = -121.4 + 228.6 = 107.2 \text{ lb}$

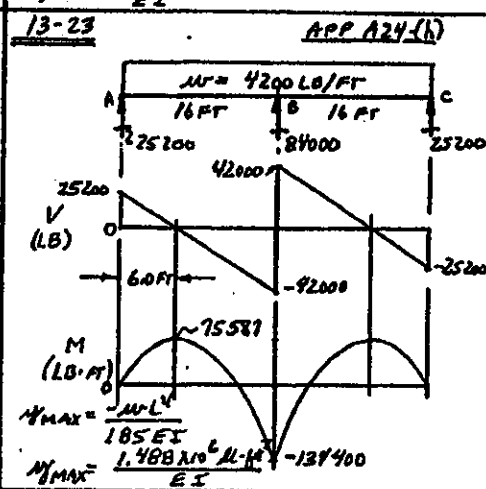
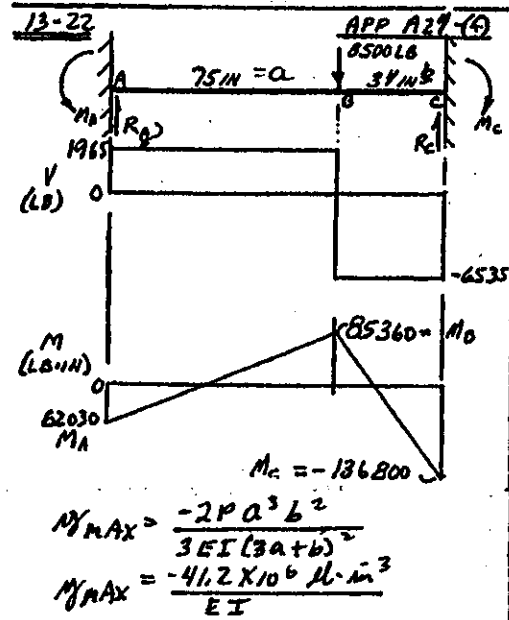
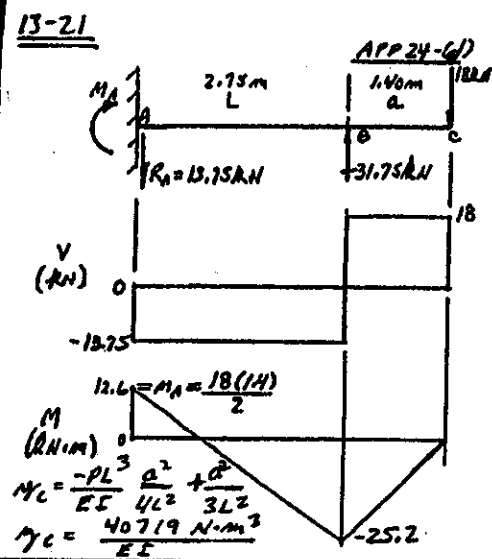
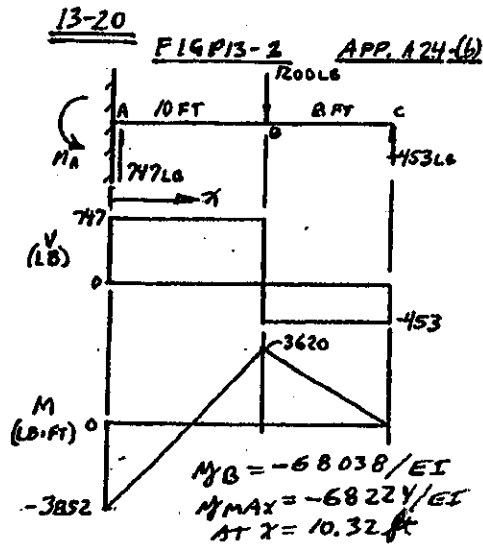
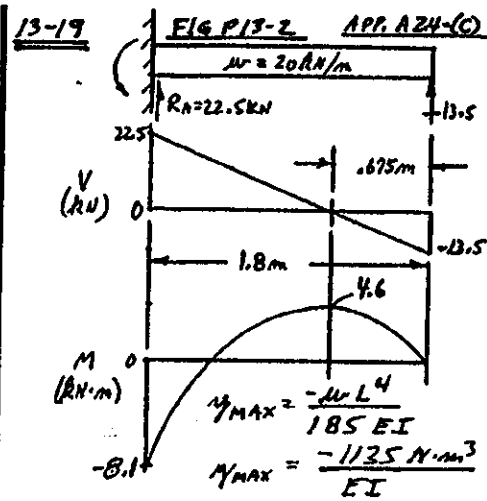
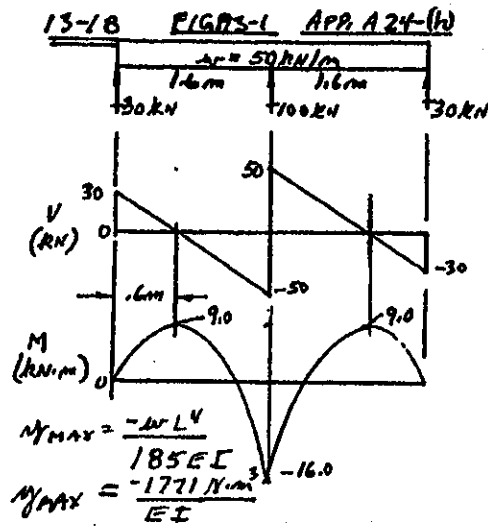
$-V_C = 107.2 - W = -92.8 \text{ lb}$

$+V_C = -92.8 + 185.6 = 92.8 \text{ lb}$

$-V_D = 92.8 - W = -107.2 \text{ lb}$

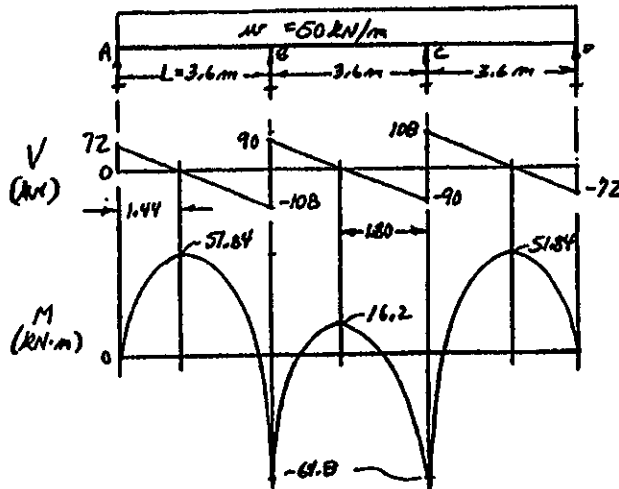
$+V_D = -107.2 + 228.6 = 121.4 \text{ lb}$

$V_E = 121.4 - W = -78.6 \text{ lb}$



13-24

APP. A24-(L)



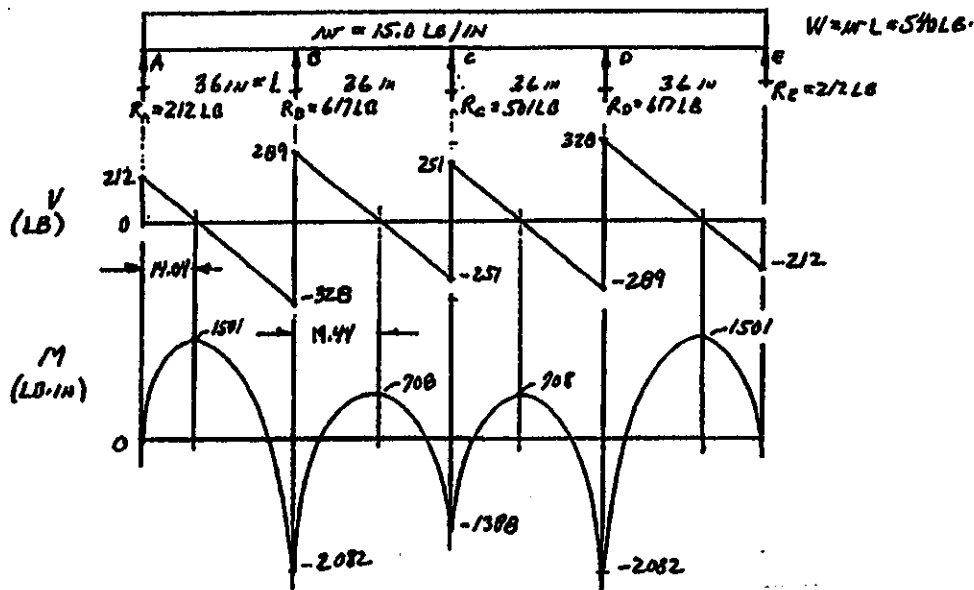
$$W = wL = (50)(3.6) = 180 \text{ kN}$$

$$R_A = R_D = 0.4W = 72 \text{ kN}$$

$$R_B = R_C = 1.10W = 198 \text{ kN}$$

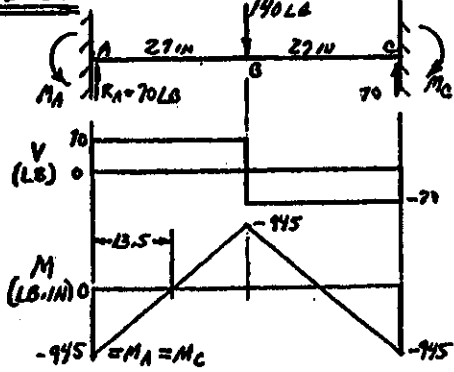
13-25

APP. A24-(L)



13-26

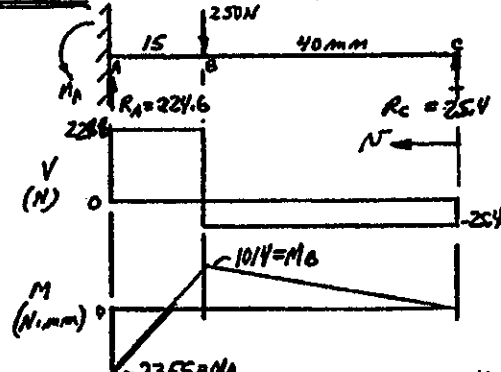
APP. A24-(E)



$$M_{\text{MAX}} = \frac{-PL^3}{192EI} = \frac{1.15 \times 10^5 \text{ lb-in}^3}{EI}$$

13-27

APP. A24-(L)



$$M_B = \frac{-Pa^3b^2}{12EIL^3}(3L+b) = \frac{-1.386 \times 10^{-4} \text{ N-m}^3}{EI}$$

$$M_{\text{MAX}} = -1.936 \times 10^{-4} \text{ N-m}^3/EI$$

AT  $N = 0.0284 \text{ m FROM C.}$

## 13-28

The objective is to compare the results of the beam loading and support conditions for five different beams in Figures 13-4, 13-10, 13-12, 13-14, and 13-16. Details were reported earlier in this solutions manual for each problem. Note that each beam design has a total length of 14.0 ft and carries a uniformly distributed load of 400 lb/ft resulting in a total load of 5600 lb. Changing the manner of support or adding additional supports affects the shearing force,  $V$ , the bending moment,  $M$ , and the maximum deflection,  $y$ , for a given  $EI$  value for the beam material and shape. Vertical shear stress and bending stress are directly proportional to the values of  $V$  and  $M$  respectively. Therefore, a reduction in either value will result in a reduction in stress or will allow the use of a smaller or lighter section for the beam. Comparisons are shown as ratios of  $V$ ,  $M$ , and  $y/EI$  to those values for the first design, a supported cantilever. The other designs are a fixed-end beam and continuous beams on 3, 4, and 5 supports.

Prob.	$V_{max}$	$V/V_1$	$M_{max}$	$M/M_1$	$y_{max}$	$y/y_1$
13-4	3500 lb	1.00	9800 lb in	1.00	-83061/EI	1.00
13-10	2800 lb	0.80	6533 lb in	0.667	-40017/EI	0.482
13-12	1750 lb	0.50	2450 lb in	0.250	-5191/EI	0.0625
13-14	1120 lb	0.32	871 lb in	0.089	N/A	-
13-16	850 lb	0.24	525 lb in	0.054	N/A	-

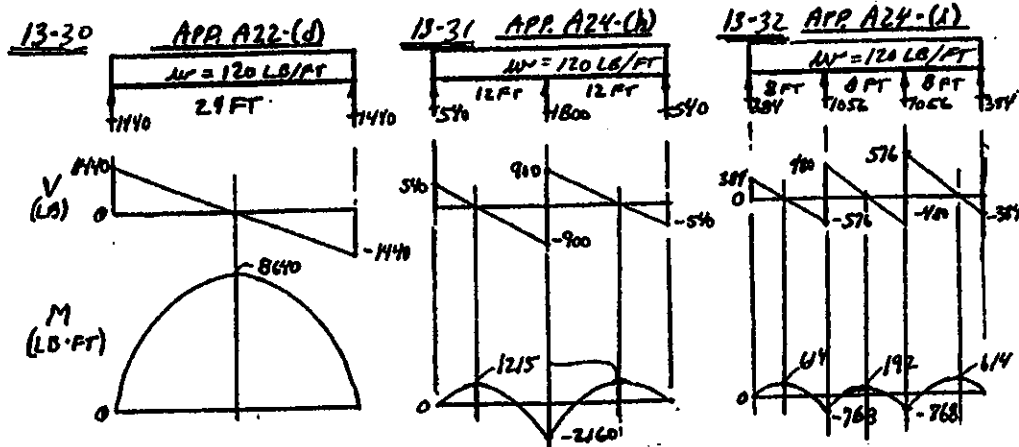
Note that maximum shearing force, bending moment, and deflection all decrease for successive designs. Deflection formulas are not available (N/A) in this book for the last two designs. But it stands to reason that deflection would be reduced by adding additional supports and reducing the span between adjacent supports. The comparison illustrates the advantages of using fixed ends for beams and of using more supports for a given load, thus reducing the effective span between adjacent supports. Fabrication problems and costs must also be considered when selecting a method of supporting a load on a beam.

## 13-29

This problem has the same objective as 13-28. Refer to that problem for a discussion of the objectives and the results. Data listed here are for different beam loadings ( $w = 50$  lb/in; total beam length = 16.0 in) but the support conditions are the same as in 13-28.

Problem	$V_{max}$	$V/V_1$	$M_{max}$	$M/M_1$	$y_{max}$	$y/y_1$
13-5	500 lb	1.00	1600 lb in	1.00	-17712/EI	1.00
13-11	400 lb	0.80	1067 lb in	0.667	-8533/EI	0.482
13-13	250 lb	0.50	400 lb in	0.250	-1107/EI	0.0625
13-15	160 lb	0.32	142 lb in	0.089	N/A	-
13-17	121 lb	0.24	86 lb in	0.054	N/A	-





13-33 COMPARISON OF 13-30 13-31 13-32

$$W = 120 \frac{\text{LB}}{\text{FT}} \times \frac{1 \text{ FT}}{12 \text{ IN}} = 10 \frac{\text{LB}}{\text{IN}}; L_a = 24 \text{ FT} \times \frac{12 \text{ IN}}{\text{FT}} = 288 \text{ IN}$$

DEFLECTIONS:

$$(13-30) \Delta_{\text{MAX}} = \frac{-5WL^4}{384EI} = \frac{-5(10)(288)^4}{384EI} = \frac{-896 \times 10^6}{EI} = \Delta_a$$

$$(13-31) \Delta_{\text{MAX}} = \frac{-WL^4}{185EI} = \frac{-(10)(288)^4}{185EI} = \frac{-372 \times 10^6}{EI} = \Delta_b = 0.415 \Delta_a \text{ IF EI IS EQUAL.}$$

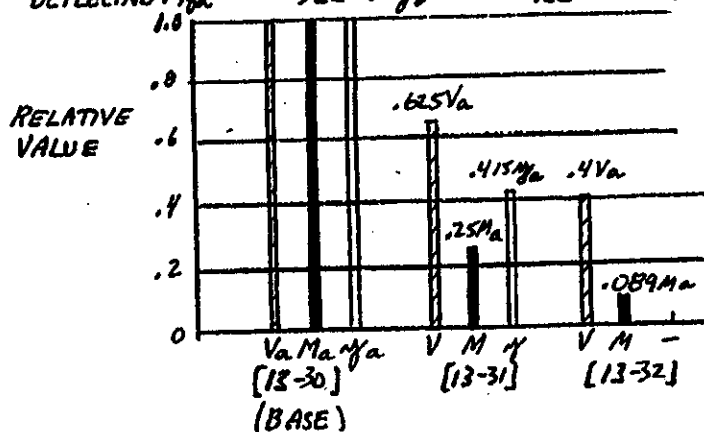
(13-32) DEFLECTION EQUATION NOT AVAILABLE; LESS THAN  $\Delta_b$

SUMMARY:

SHEARING FORCE:  $V_{a, \text{MAX}} = 1440 \text{ LB}$ ;  $V_{b, \text{MAX}} = 900 \text{ LB} = 0.625 V_a$ ;  $V_c = 576 \text{ LB} = 0.4 V_a$

MOMENT:  $M_{a, \text{MAX}} = 8640 \text{ LB-FT}$ ;  $M_b = 2160 \text{ LB-FT} = 0.25 M_a$ ;  $M_c = 768 \text{ LB-FT} = 0.089 M_a$

DEFLECTION:  $\Delta_a = 896 \times 10^6 / EI$ ;  $\Delta_b = 372 \times 10^6 / EI = 0.415 \Delta_a \text{ IF EI IS EQUAL.}$



(CONTINUED ON NEXT PAGE)

13-33 COMPARISON OF  
13-30, 13-31, 13-32 (CONTINUED)

BEAM SIZE: FOR NO. 2 SOUTHERN PINE:  $T_g = 70 \text{ psi}$ ;  $\sigma_g = 1000 \text{ psi}$

FOR A RECTANGLE:  $T_{MAX} = 3V/2A$ ;  $S = tA^2/6$

[13-30]  $V_{MAX} = 1140 \text{ LB}$ ;  $A_{MIN} = \frac{3V}{2T_g} = \frac{3(1140)}{2(70)} = 24.86 \text{ IN}^2$

$M_{MAX} = (8640 \text{ LB} \cdot \text{FT})(12 \text{ IN/FT}) = 103680 \text{ LB} \cdot \text{IN}$ ;  $\sigma = M/S$

$S_{MIN} = \frac{M}{\sigma_g} = \frac{103680 \text{ LB} \cdot \text{IN}}{1000 \text{ LB/IN}^2} = 103.7 \text{ IN}^3$  6x12 BEAM REQD  
 $I = 697 \text{ IN}^4$

[13-31]  $V_{MAX} = 900 \text{ LB}$ ;  $A_{MIN} = \frac{3(900)}{2(70)} = 19.3 \text{ IN}^2$

$M_{MAX} = 2160 \text{ LB} \cdot \text{FT} (12 \text{ IN/FT}) = 25920 \text{ LB} \cdot \text{IN}$

$S_{MIN} = \frac{M}{\sigma_g} = \frac{25920}{1000} = 25.9 \text{ IN}^3$  4x8 BEAM REQD  
 $I = 111 \text{ IN}^4$

[13-32]  $V_{MAX} = 576 \text{ LB}$ ;  $A_{MIN} = \frac{3(576)}{2(70)} = 12.34 \text{ IN}^2$

$M_{MAX} = 768 \text{ LB} \cdot \text{FT} (12 \text{ IN/FT}) = 9216 \text{ LB} \cdot \text{IN}$

$S_{MIN} = \frac{M}{\sigma_g} = \frac{9216}{1000} = 9.22 \text{ IN}^3$  2x10 BEAM REQD  
 $I = 98.9 \text{ IN}^4$

ACTUAL DEFLECTIONS:

[13-30]  $y_{MAX} = \frac{-896 \times 10^6}{EI} = \frac{-896 \times 10^6}{(1.3 \times 10^6)(697)} = -0.989 \text{ IN.}$

[13-31]  $y_{MAX} = \frac{-272 \times 10^6}{EI} = \frac{-272 \times 10^6}{(1.3 \times 10^6)(111)} = -2.58 \text{ IN. (LARGE) HIGHER I MAY BE REQUIRED FOR DEFLECTIONS.}$

13-34

APPENDIX A 24-(3) APP. A-1B (WOOD)

$V_{MAX}$  AND  $M_{MAX}$  OCCUR AT SECOND SUPPORT

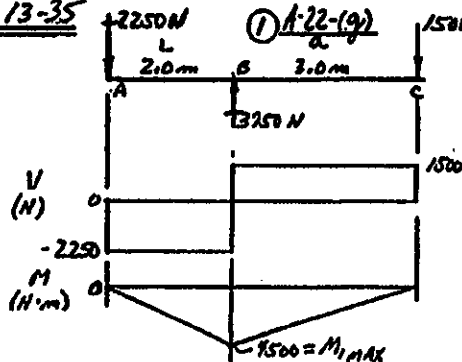
$V_{MAX} = 0.607 \text{ WL} = 0.607(100 \text{ LB/FT})(2 \text{ FT}) = 12.14 \text{ LB}$

$T_{MAX} = \frac{3V}{2A} = \frac{3(12.14 \text{ LB})}{2(1.50)(5.50) \text{ IN}^2} = 22.07 \text{ PSI}$ ;  $T_{ALLOW} = 70 \text{ PSI}$  - OK

$M_{MAX} = 0.107 \text{ WL}^2 = 0.107(100 \text{ LB/FT})(2.0 \text{ FT})^2 = 42.8 \text{ LB} \cdot \text{FT} \times 12 \text{ IN/FT} = 513.6 \text{ LB} \cdot \text{IN}$

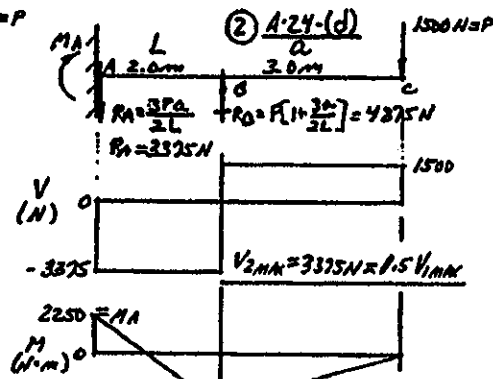
$\sigma = \frac{M}{S} = \frac{513.6 \text{ LB} \cdot \text{IN}}{(5.50)(1.50)^2/6} = 249 \text{ PSI}$ ;  $\sigma_{ALLOW} = 1000 \text{ PSI}$  - OK

13-35



$y_{C1} = \frac{-PA^2}{3EI} (L+a) = \frac{-1500(3)^2}{3EI} (2+3)$

$y_{C1} = \frac{22500}{EI}$



$y_{C2} = \frac{-PL^3}{EI} \left[ \frac{a^2}{4L^2} + \frac{a^3}{3L^3} \right]$

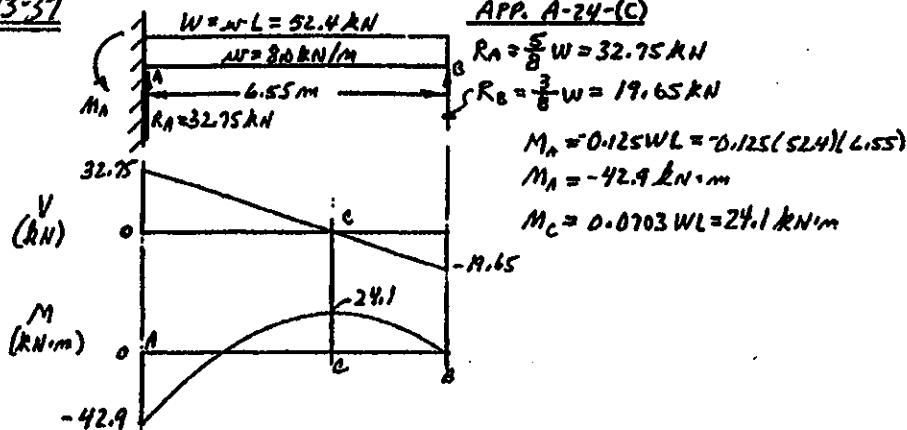
$y_{C2} = \frac{-1500(2)^3}{EI} \left[ \frac{3^2}{4(2)^2} + \frac{3^3}{3(2)^3} \right] = \frac{20250}{EI} = 0.9 y_{C1}$

NO MAJOR DIFFERENCE BETWEEN DESIGNS. (2) IS 10% STIFFER. (2) HAS 50% HIGHER SHEARING FORCE.

13-36

MULTIPLE DESIGNS POSSIBLE: USE  $V_{max}$  AND  $M_{max}$  FROM PROB. 13-35. SPECIFY MATERIAL, DESIGN STRESS, DESIGN FACTOR, SHAPE OF CROSS SECTION, DIMENSIONS. BECAUSE BENDING MOMENT IS EQUAL FOR BOTH DESIGNS, BOTH MAY BE THE SAME. BUT CHECK SHEAR.

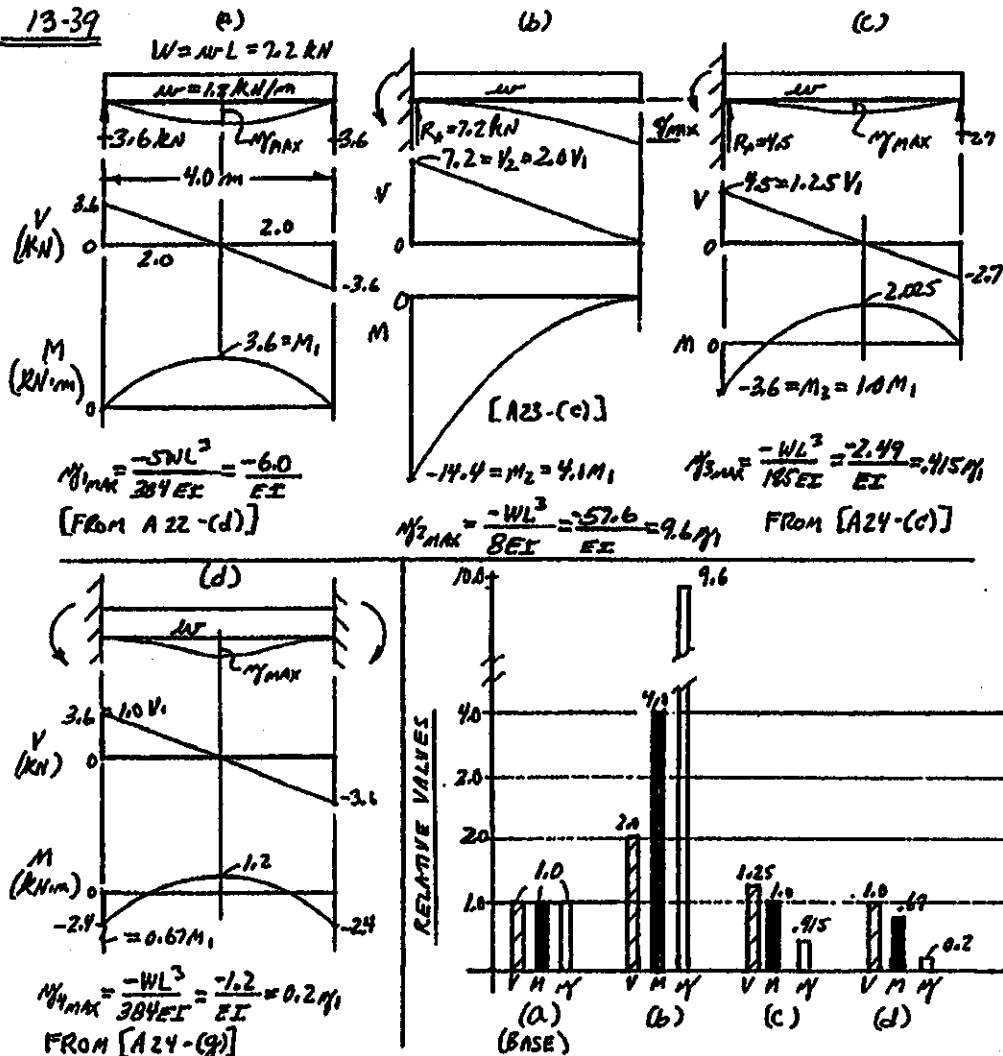
13-37



13-38

MULTIPLE DESIGNS POSSIBLE

13-39



# SUPERPOSITION METHOD

13-40

FIG. P13-1

$$\gamma_{c1} = \frac{-5WL^3}{384EI} \quad (\text{CASE (d)}) \quad \text{TABLE A-22}$$

$$\gamma_{c1} = \frac{-5(160 \times 10^3)(3200)^3}{384EI} = -6.827 \times 10^{13}$$

$$\gamma_{c2} = \frac{+P\delta^3}{48EI} = \frac{+R_B(3200)^3}{48EI} = 6.827 \times 10^8 (R_B) \quad (\text{A22 (a)})$$

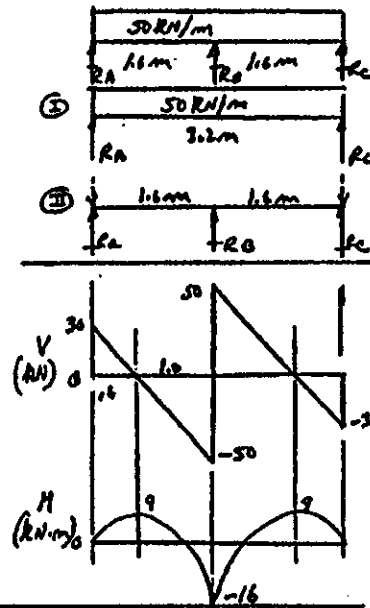
$$\gamma_{c1} + \gamma_{c2} = 0$$

$$-6.827 \times 10^{13} + 6.827 \times 10^8 (R_B) = 0$$

$$R_B = 6.827 \times 10^{13} / 6.827 \times 10^8 = 1.01 \times 10^5 \text{ N}$$

$$R_B = 100 \text{ kN}$$

$$\text{THEN } R_A = R_C = 30 \text{ kN}$$



13-41

FIG. P13-2

$$\gamma_{c1} = \frac{-WL^3}{8EI} = \frac{-(36 \times 10^3)(1800)^3}{8EI} = -2.624 \times 10^{13} \quad (\text{A23 (c)})$$

$$\gamma_{c2} = \frac{+P\delta^3}{384EI} = \frac{R_B(1800)^3}{384EI} = \frac{R_B(1.944 \times 10^9)}{EI} \quad (\text{A23 (a)})$$

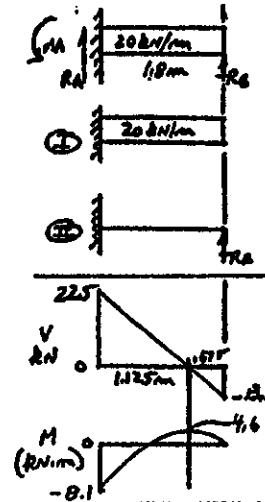
$$\gamma_{c1} + \gamma_{c2} = 0$$

$$-2.624 \times 10^{13} / EI + R_B(1.944 \times 10^9) / EI = 0$$

$$R_B = 2.624 \times 10^{13} / 1.944 \times 10^9 = 13.50 \text{ kN} = R_B$$

$$\text{THEN } R_A = 36 - 13.5 = 22.5 \text{ kN} = R_A$$

$$M_A = 36(0.9) - 13.5(1.8) = 8.1 \text{ kN-m (NEGATIVE)}$$



13-42 FIG. P13-3

$$\gamma_{c1} = \frac{-Pa^2}{6EI} (3L-a) = \frac{-(1200)(120)^2}{6EI} (3(216) - 120) = -1.52 \times 10^9 \quad (\text{A23 (b)})$$

$$\gamma_{c2} = \frac{R_c \delta^3}{384EI} = \frac{R_c(216)^3}{384EI} = \frac{3.359 \times 10^6 (R_c)}{EI} \quad (\text{A23 (a)})$$

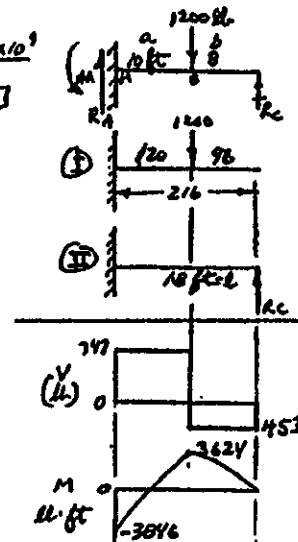
$$\gamma_{c1} + \gamma_{c2} = 0$$

$$-1.52 \times 10^9 / EI + 3.359 \times 10^6 (R_c) / EI = 0$$

$$R_c = 1.52 \times 10^9 / 3.359 \times 10^6 = 453 \text{ lb} = R_c$$

$$\text{THEN } R_A = 1200 - 453 = 747 \text{ lb} = R_A$$

$$M_A = 1200(10) - R_c(18) = 12000 - 453(18) = 3846 \text{ lb-ft (NEGATIVE)}$$



13-43

FIG. P13-7

SUPERPOSITION:  $\eta_{c1} + \eta_{c2} = 0$

$$\eta_{c1} = \frac{-Pa}{24EI} (3b^2 - 4a^2) \quad (\text{CASE (C)}) \quad \text{TABLE A-22}$$

$$\eta_{c1} = \frac{-800(36)}{24EI} \left[ (3)(144) - 4(36) \right] = \frac{-1.265 \times 10^8}{EI}$$

$$\eta_{c2} = + \frac{PL^3}{48EI} \quad (\text{CASE (A)})$$

$$\eta_{c2} = \frac{R_c (192)^3}{48EI} = \frac{1.475 \times 10^5 R_c}{EI}$$

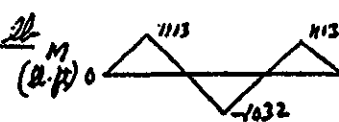
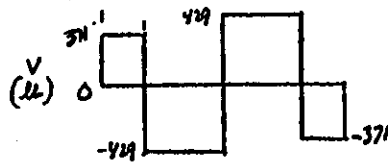
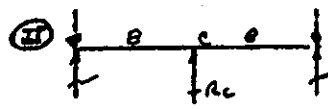
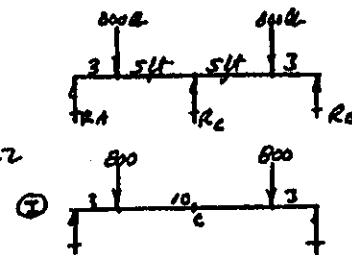
THEN

$$\eta_{c1} + \eta_{c2} = 0$$

$$\frac{-1.265 \times 10^8}{EI} + \frac{1.475 \times 10^5 R_c}{EI} = 0$$

$$R_c = 1.265 \times 10^8 / 1.475 \times 10^5 = 858 \text{ lb}$$

BECAUSE OF SYMMETRY:  $R_A = R_c = 371 \text{ lb}$



13-44

FIG. P13-8

$$\eta_{c1} = \frac{-Pa}{24EI} (3b^2 - 4a^2) = \frac{-1.265 \times 10^8}{EI}$$

$$\eta_{c2} = \frac{-5WL^3}{384EI} = \frac{-5(8000)(144)^3}{384EI} = \frac{-7.37 \times 10^8}{EI}$$

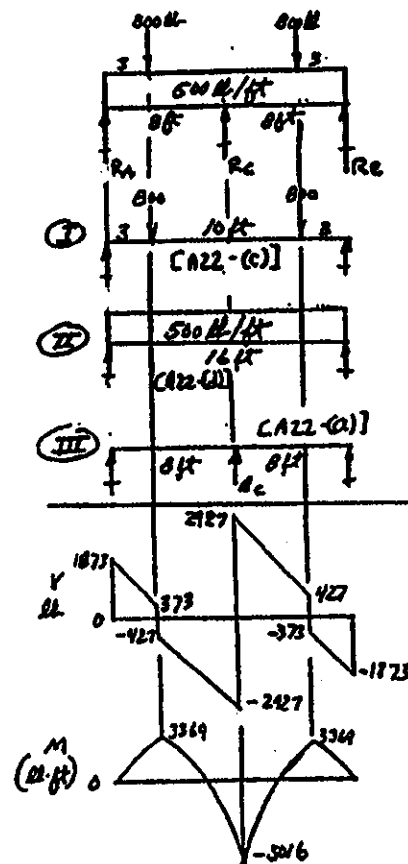
$$\eta_{c3} = + \frac{PL^3}{48EI} = \frac{R_c (192)^3}{48EI} = \frac{(1.475 \times 10^5) R_c}{EI}$$

THEN  $\eta_{c1} + \eta_{c2} + \eta_{c3} = 0$

$$\frac{-1.265 \times 10^8}{EI} - \frac{7.37 \times 10^8}{EI} + \frac{R_c (1.475 \times 10^5)}{EI} = 0$$

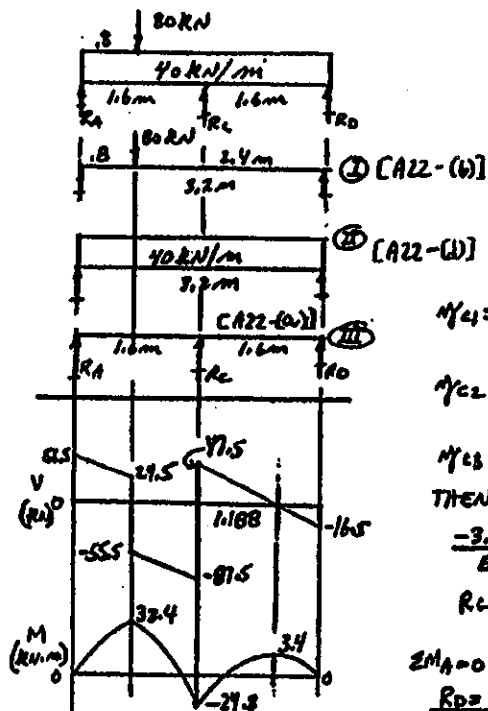
$$R_c = \frac{1.265 \times 10^8 + 7.37 \times 10^8}{1.475 \times 10^5} = 5854 \text{ lb} = R_c$$

THEN  $R_A = R_E = 1893 \text{ lb}$



13-45

FIG. P13-9



$$\eta_{c1} = \frac{-(80 \times 3.2)(3.2)(1.6)}{6EI(3.2)} \left[ \frac{3.2^3}{6} - 1.6^2 \cdot 3.2 \right] = \frac{-3.755 \times 10^{13}}{EI}$$

$$\eta_{c2} = \frac{-5WL^3}{384EI} = \frac{-5(120 \times 10^3)(3.2)^3}{384EI} = \frac{-5.46 \times 10^{13}}{EI}$$

$$\eta_{c3} = \frac{+PL^3}{48EI} = \frac{R_c(3.2)^3}{48EI} = \frac{R_c(6.827 \times 10^8)}{EI}$$

$$\text{THEN } \eta_{c1} + \eta_{c2} + \eta_{c3} = 0$$

$$\frac{-3.755 \times 10^{13}}{EI} - \frac{5.46 \times 10^{13}}{EI} + \frac{R_c(6.827 \times 10^8)}{EI} = 0$$

$$R_c = \frac{+3.755 \times 10^{13} + 5.46 \times 10^{13}}{6.827 \times 10^8} = 135 \text{ kN} = R_c$$

$$\Sigma M_A = 0 = 80(3.2) + 120(1.6) - 135(1.6) - R_D(3.2)$$

$$R_D = 16.5 \text{ kN}$$

$$\Sigma M_D = 0 = 80(2.4) + 120(1.6) - 135(1.6) - R_A(3.2)$$

$$R_A = 56.5 \text{ kN}$$

13-46

FIG. P13-10

$$W = w \cdot L = (40 \text{ kN/m})(3.6 \text{ m}) = 144 \text{ kN}$$

$$\eta_{c1} = \frac{-WL^3}{8EI} = \frac{-144(3.6)^3}{8EI} = \frac{-8.398 \times 10^{11}}{EI} \quad [A23-(c)]$$

$$\eta_{c2} = \frac{-Pa^2}{6EI}(3L-a) = \frac{-60(2.0)^2}{6EI}[3(3.6)-2.0] \quad [A23-(b)]$$

$$\eta_{c2} = \frac{-3.52 \times 10^{11}}{EI}$$

$$\eta_{c3} = \frac{+R_c(3.6)^3}{3EI} = \frac{R_c(6.555 \times 10^{11})}{EI} \quad [A23-(a)]$$

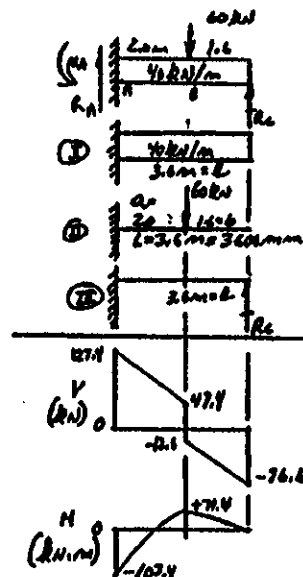
$$\eta_{c1} + \eta_{c2} + \eta_{c3} = 0$$

$$\frac{-8.398 \times 10^{11}}{EI} - \frac{3.52 \times 10^{11}}{EI} + \frac{R_c(6.555 \times 10^{11})}{EI} = 0$$

$$R_c = 76.6 \text{ kN}$$

$$R_A = 60 \text{ kN} + 144 \text{ kN} - 76.6 \text{ kN} = 127.4 \text{ kN} = R_A$$

$$M_A = 60(2.0) + 144(1.0) - R_c(3.6) = 103.44 \text{ kN}\cdot\text{m} \quad (\text{NEGATIVE})$$



13-47 FIG. P13-11

$$\gamma_{D1} = \frac{-wL^3}{8EI} = \frac{-4000(12)^3}{8EI} = -8.64 \times 10^8 / EI \quad [A23-(c)]$$

$$\gamma_{D2} = \frac{-Pa^2}{6EI} [3(1-a)] = \frac{-1200(36)^2}{6EI} [3(12)-36] \quad [A23-(b)]$$

$$\gamma_{D2} = 8.398 \times 10^8 / EI$$

$$\gamma_{D3} = \frac{-800(72)^2}{6EI} [3(120)-72] = 1.991 \times 10^9 / EI \quad [A23-(b)]$$

$$\gamma_{D4} = \frac{+R_D(120)^3}{3EI} = 5.76 \times 10^5 (R_D) / EI \quad [A23-(a)]$$

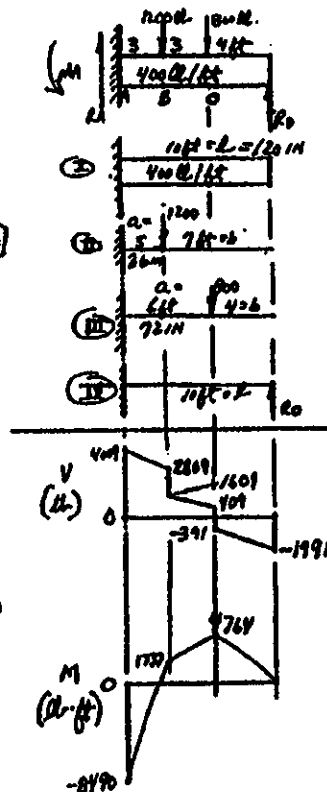
$$\gamma_{D1} + \gamma_{D2} + \gamma_{D3} + \gamma_{D4} = 0$$

$$\frac{-8.64 \times 10^8}{EI} - \frac{8.398 \times 10^8}{EI} - \frac{1.991 \times 10^9}{EI} + \frac{R_D(5.76 \times 10^5)}{EI} = 0$$

$$R_D = 1991 \text{ lb}$$

$$R_A = 1200 + 800 + 4000 - 1991 = 4009 \text{ lb} = R_A$$

$$M_A = 1200(3) + 800(6) + 4000(5) - 1991(10) = 8490 \text{ lb}\cdot\text{ft} \quad (\text{Negative})$$



THEOREM OF THREE MOMENTS

13-48

FIG. P13-1

USE EQ.(13-3) WITH  $M_A = M_C = 0$

SEE PROB 13-40 FOR SHEAR AND MOMENT DIAGRAM.

$$0 + 4M_B + 0 = -50(1.6)^3/2 = -64$$

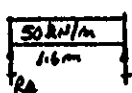
$$M_B = -64/4 = -16 \text{ kN}\cdot\text{m}$$

$$\sum M_B = 0 = 80(8) - R_A(1.6) - 16$$

$$R_A = 30 \text{ kN}$$

BECAUSE OF SYMMETRY,  $R_C = R_A = 30 \text{ kN}$

$$R_B = 50(3.2) - 2(30) = 100 \text{ kN}$$



13-49

FIG. P13-7

USE EQ.(13-4) WITH  $M_A = M_D = 0$

$$0 + 2M_C(8+8) + 0 = -\frac{800(3)}{8}(8^2-3^2) - \frac{800(3)}{8}(8^2-3^2)$$

$$32M_C = -33000 \quad ; \quad M_C = -33000/32 = -1031 \text{ lb}\cdot\text{ft}$$

NOTE:  $M_C$  IS THE MOMENT AT THE MIDDLE SUPPORT. SUBSCRIPTS IN EQ.(13-4) WERE ADJUSTED TO MATCH FIG. P13-7.

FOR REACTIONS:

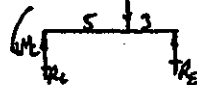
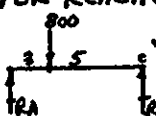
$$\sum M_C = 0 = 800(5) - R_A(8) - 1031$$

$$R_A = (4000 - 1031)/8 = 371 \text{ lb}$$

$$\sum M_C = 0 = 800(5) - R_D(8) - 1031$$

$$R_D = 371 \text{ lb}$$

$$R_C = 1600 - 2(371) = 858 \text{ lb} = R_C$$



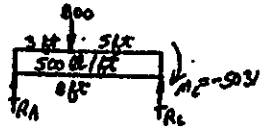
SHEAR AND MOMENT DIAGRAMS SAME AS PROB 13-43.

13-50      FIG P13-8

USE EQ. (13-6) WITH  $M_A = M_E = 0$

$$0 + 2M_C(8+8) + 0 = \frac{-800(2)}{8}(8^2-3^2) - \frac{800(2)}{8}(8^2-3^2) - \frac{500(8)^3}{4} - \frac{500(8)^3}{4}$$

$$32M_C = -161000 ; M_C = -161000/32 = -5031 \text{ lb}\cdot\text{ft}$$



$$\Sigma M_C = 0 = 800(5) + 4000(8) - R_A(8) - 5031$$

$$R_A = 1871 \text{ lb} = R_E \text{ BECAUSE OF SYMMETRY}$$

$$R_C = 800 + 8000 + 800 - 2(1871) = 5858 \text{ lb} = R_E$$

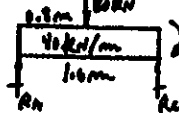
SEE PROB 13-44 FOR SHEAR AND MOMENT DIAGRAMS

13-51      FIG P13-9

USE EQ. (13-6) WITH  $M_A = M_D = 0$

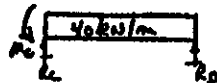
$$0 + 2M_C(1.6+1.6) = \frac{-80(0.8)}{1.6}(1.6^2-0.8^2) - \frac{40(1.6)^3}{4} - \frac{40(1.6)^3}{4}$$

$$M_C = -158.72/6.4 = -24.8 \text{ kN}\cdot\text{m}$$



$$\Sigma M_C = 0 = 80(0.8) + 64(0.8) - 24.8 - R_A(1.6)$$

$$R_A = 56.5 \text{ kN}$$



$$\Sigma M_C = 0 = 64(0.8) - 24.8 - R_D(1.6)$$

$$R_D = 16.5 \text{ kN}$$

$$R_C = 80 + 128 - 56.5 - 16.5 = 135 \text{ kN} = R_C$$

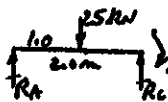
SEE PROB 13-45 FOR SHEAR AND MOMENT DIAGRAMS.

13-52      FIG. P13-12

EQ. (13-6) WITH  $M_A = M_D = 0$

$$0 + 2M_C(3.2) + 0 = \frac{-25(1.0)}{2.0}(2.0^2-1.0^2) - \frac{20(1.2)^3}{4} = -46.14$$

$$M_C = -46.14/6.4 = -7.21 \text{ kN}\cdot\text{m}$$



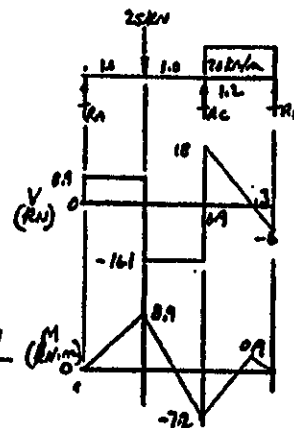
$$\Sigma M_C = 0 = 25(1.0) - 7.21 - R_A(3.2)$$

$$R_A = 8.90 \text{ kN}$$

$$\Sigma M_C = 0 = 24(0.6) - 7.21 - R_D(1.2)$$

$$R_D = 6.00 \text{ kN}$$

$$R_C = 25 + 24 - 8.90 - 6.00 = 34.1 \text{ kN}$$





13-53 FIG. P13-13

EQ(13-3) WITH  $M_A = 0$

$$M_C = (40 \text{ kN/m})(1.5 \text{ m})\left(\frac{1.5 \text{ m}}{2}\right) = 45 \text{ kN}\cdot\text{m}$$

$$0 + 4M_B - 45 = -40(3)^2/2 = -180$$

$$M_B = (-180 + 45)/4 = -33.75 \text{ kN}\cdot\text{m}$$

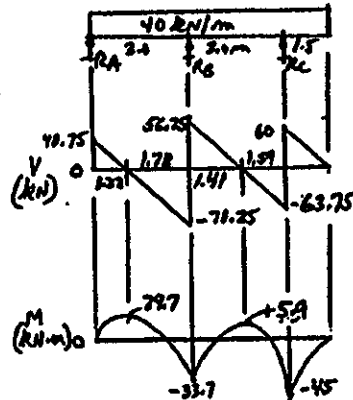
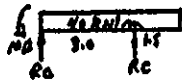
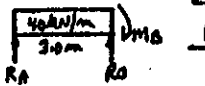
$$2M_B = 0 = 120(1.5) - 33.75 - R_A(3)$$

$$R_A = 48.75 \text{ kN}$$

$$\Sigma M_B = 0 = 180(2.25) - 33.75 - R_C(3)$$

$$R_C = 123.75 \text{ kN}$$

$$R_B = 300 - 48.75 - 123.75 = 127.5 \text{ kN}$$



13-54 FIG. P13-14

EQ.(13-6) WITH  $M_A = M_B = 0$

$$0 + 2M_C(3,2) + 0 = \frac{-20(1)(1.6^2/2)}{1.6} - \frac{20(1.6)^3}{6} - \frac{40(1.6)(1.6^2/2)}{1.6}$$

$$M_C = -78.98/6.4 = -12.34 \text{ kN}\cdot\text{m}$$

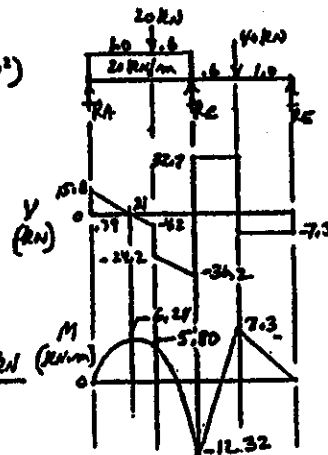
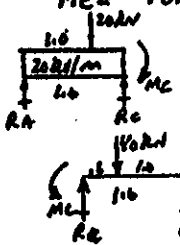
$$\Sigma M_C = 0 = 20(1.6) + 32(1.6) - 12.34 - R_A(1.6)$$

$$R_A = 15.8 \text{ kN}$$

$$\Sigma M_C = 0 = 40(1.6) - 12.34 - R_B(1.6)$$

$$R_B = 7.3 \text{ kN}$$

$$R_C = 20 + 32 + 40 - 15.8 - 7.3 = 68.9 \text{ kN}$$



13-55 FIG. P13-15

EQ(13-4) WITH  $M_A = 0$

$$M_B = -20(1.5) = -30 \text{ kN}\cdot\text{m}$$

$$0 + 2M_C(6) - 30(3) = \frac{-60(1.5)}{3}(3^2/1.5^2) - \frac{80(2.4)}{3.0}(3^2/2.4^2)$$

$$12M_C - 90 = -409.86$$

$$M_C = (-409.86 + 90)/12 = -26.7 \text{ kN}\cdot\text{m}$$

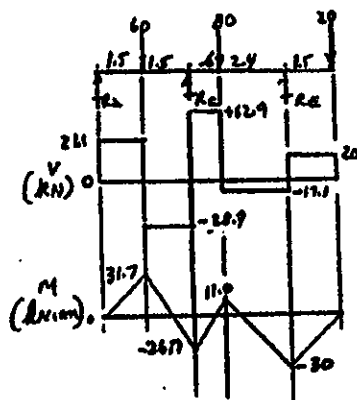
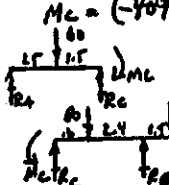
$$\Sigma M_C = 0 = 60(1.5) - 26.7 - R_A(3)$$

$$R_A = 21.1 \text{ kN}$$

$$\Sigma M_C = 0 = 80(1.5) + 20(4.5) - 26.7 - R_C(3)$$

$$R_C = 37.1 \text{ kN}$$

$$R_B = 60 + 80 + 20 - 21.1 - 37.1 = 101.8 \text{ kN}$$



14-1  $\frac{L_e}{r} = \frac{(1.0)(800)}{(20/4)} = 160$ ; FOR  $S_y = 331 \text{ MPa}$ ,  $C_c = 110$ ; LONG COLUMN (FIG. 14-3)  
 USE EULER EQ. (14-4):  $A = \pi D^2/4 = \pi (20)^2/4 = 314 \text{ mm}^2$   
 $P_{cr} = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2 (207 \times 10^3 \text{ N/mm}^2) (314 \text{ mm}^2)}{(160)^2} \times \frac{1 \text{ m}^2}{10^6 \text{ mm}^2} = 25.1 \text{ kN}$

14-2  $L_e/r = (0.0)(350)/(20/4) = 70$ ;  $C_c = 110$ ; SHORT COLUMN; EQ (14-6)  
 $P_{cr} = A S_y \left[ 1 - \frac{S_y (L_e/r)^2}{4 \pi^2 E} \right] = (314 \text{ mm}^2) (331 \text{ N/mm}^2) \left[ 1 - \frac{331 \times 10^3 (70)^2}{4 \pi^2 (207 \times 10^3)} \right]$   
 $P_{cr} = 83.3 \text{ kN}$

14-3  $L_e/r = 160$ ; FOR ALUM,  $S_y = 276 \text{ MPa}$  -  $C_c = 70$ ; FIG 14-4: LONG COLUMN  
 $P_{cr} = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2 (69 \times 10^3 \text{ N/mm}^2) (314 \text{ mm}^2)}{(160)^2} \times \frac{1 \text{ m}^2}{10^6 \text{ mm}^2} = 8.35 \text{ kN}$

14-4 FIXED ENDS:  $L_e/r = (0.65)(800)/(20/4) = 104$ ;  $C_c = 110$ ; SHORT COLUMN  
 $P_{cr} = (314)(331) \left[ 1 - \frac{(331 \times 10^3)(104)^2}{4 \pi^2 (207 \times 10^3)} \right] = 58.4 \text{ kN}$

14-5  $A = 314 \text{ mm}^2 = b^2$ ;  $b = \sqrt{314} = 17.7 \text{ mm}$ ;  $r = b/\sqrt{12} = 5.12 \text{ mm}$   
 $L_e/r = (1.0)(800)/5.12 = 156$ ;  $C_c = 138$ ; LONG COLUMN  
 $P_{cr} = \frac{\pi^2 (207 \times 10^3) (314)}{(156)^2 (10^6)} = 26.2 \text{ kN}$

14-6 1 IN SCH 40 PIPE:  $r = 0.421 \text{ in } (25.4 \text{ mm/in}) = 10.69 \text{ mm}$   
 $A = 0.494 \text{ in}^2 \times (25.4)^2 \text{ mm}^2/\text{in}^2 = 318.7 \text{ mm}^2$   
 $S_y = 331 \text{ MPa}$ ;  $C_c \approx 110$  - FIG 14-3.  
 (a) FIXED ENDS:  $L_e/r = (0.65)(2050 \text{ mm})/10.69 \text{ mm} = 124.6$  - LONG  
 $P_{cr} = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2 (207 \times 10^3 \text{ N/mm}^2) (318.7 \text{ mm}^2)}{(124.6)^2} = 41.5 \text{ kN}$   
 (b) FIXED-PINNED:  $L_e/r = (0.80)(2050)/10.69 = 153$  - LONG  
 $P_{cr} = \frac{\pi^2 (207 \times 10^3 \text{ N/mm}^2) (318.7 \text{ mm}^2)}{(153)^2} = 27.8 \text{ kN}$   
 (c) PINNED:  $L_e/r = 1.0(2050)/10.69 = 192$  - LONG  
 $P_{cr} = \frac{\pi^2 (207 \times 10^3) (318.7)}{(192)^2} = 17.7 \text{ kN}$   
 (d) FIXED-FREE:  $L_e/r = (2.0)(2050)/10.69 = 403$  LONG  
 $P_{cr} = \frac{\pi^2 (207 \times 10^3) (318.7)}{(403)^2} = 4.04 \text{ kN}$

14-7  $r_{min} = 12/\sqrt{12} = 3.46 \text{ mm}$ ;  $L_e/r = 1.0(210)/3.46 = 60.6$

FOR  $S_y = 469 \text{ MPa}$  - STEEL:  $C_c = 90$  - SHORT COLUMN

$A = (12)(25) = 300 \text{ mm}^2$

$P_{cr} = 800 \text{ mm}^2 (469 \text{ N/mm}^2) \left[ 1 - \frac{(469 \times 10^6 \text{ Pa})(60.6)^2}{4\pi^2 (207 \times 10^9 \text{ Pa})} \right] = 111 \text{ kN}$

14-8 FOR A36 STRUCTURAL STEEL:  $S_y = 248 \text{ MPa}$  -  $C_c = 122$ ;  $E = 200 \times 10^3 \text{ N/mm}^2$

FOR  $S6 \times 12.5$ :  $r_{min} = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1.82 \text{ in}^4}{3.67 \text{ in}^2}} = 0.704 \text{ in} \times 25.4 \text{ mm/in} = 17.9 \text{ mm}$

$L_e/r = (0.65)(5450)/17.9 = 198 > C_c$  - LONG:  $A = 3.67 \text{ in}^2 \times \frac{(25.4)^2 \text{ mm}^2/\text{in}^2}{1 \text{ in}^2} = 2368 \text{ mm}^2$

$P_a = \frac{\pi^2 EA}{1.92(L_e/r)^2} = \frac{\pi^2 (200 \times 10^3)(2368)}{1.92(198)^2} = 62.1 \text{ kN}$

14-9 3 in SCH 40 PIPE:  $r = 1.163 \text{ in}$ ;  $L_e/r = (20)(8 \text{ ft})(12 \text{ in/ft})/1.163 \text{ in} = 173$

$S_y = 30 \text{ ksi}$ ;  $C_c = 138$  - LONG COLUMN &  $E = 30 \times 10^6 \text{ psi}$  FOR STEEL.

$P_a = \frac{P_{cr}}{N} = \frac{\pi^2 EA}{N(L_e/r)^2} = \frac{\pi^2 (30 \times 10^6)(2.228)}{2(173)^2} = 7318 \text{ lb/colum}$

NO. OF COLUMNS =  $\frac{\text{TOTAL LOAD}}{7318 \text{ lb/colum}} = \frac{(75 \text{ lb/ft} \times 20 \times 40 \text{ ft})}{7318 \text{ lb/colum}} = 8.20$  - USE 9

14-10  $I_{10} \times 8.646$ :  $r_{min} = 1.42 \text{ in} \times 25.4 \text{ mm/in} = 36.1 \text{ mm}$

$A = 7.352 \text{ in}^2 \times (25.4)^2 \text{ mm}^2/\text{in}^2 = 4743 \text{ mm}^2$

$L_e/r = 1.0(2800)/36.1 = 77.6$ ; FOR  $S_y = 276 \text{ MPa}$  ALUM.,  $C_c = 70$  - LONG COLUMN

(EQ 14-18b)  $P_a = \frac{35200(A)}{(L_e/r)^2} = \frac{35200(4743)}{(77.6)^2} = 277 \text{ kN}$

14-11  $L_e/r = 1400/36.1 = 38.8$  - INTERMED. (EQ 14-17b)

$P_a = A[139 - 0.869(L_e/r)] = 4743 \text{ mm}^2 [139 - 0.869(38.8)] \text{ N/mm}^2 = 499 \text{ kN}$

14-12  $WB \times 15$ :  $r_{min} = \sqrt{I_y/A} = \sqrt{3.41 \text{ in}^4/4.44 \text{ in}^2} = 0.876 \text{ in}$

$C_c = \sqrt{\frac{2\pi^2 E}{S_y}} = \sqrt{\frac{2\pi^2 (29 \times 10^6 \text{ psi})}{36000 \text{ psi}}} = 126$

$L_e/r = (0.8 \times 12.5 \text{ ft})(12 \text{ in/ft})/0.876 \text{ in} = 137 > C_c$  - LONG

$P_a = \frac{\pi^2 EA}{1.92(L_e/r)^2} = \frac{\pi^2 (29 \times 10^6)(4.44)}{1.92(137)^2} = 35265 \text{ LB}$

14-13

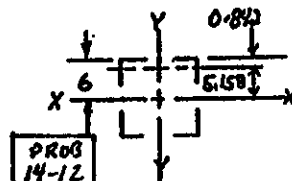
$I_{xx} = I_{yy} = 4[1.24 + 1.44(5.158)^2] = 158.2 \text{ in}^4$

$A = 4(1.44 \text{ in}^2) = 5.76 \text{ in}^2$

$r = \sqrt{I/A} = \sqrt{158.2/5.76} = 5.24 \text{ in}$

$L_e/r = (1.0)(18.4 \text{ ft})(12 \text{ in/ft})/5.24 \text{ in} = 42.1$  PROB 14-12  $C_c = 126$  JOHNSON EQ.

$P_a = \frac{P_{cr}}{N} = \frac{(5.76)(36000)}{3} \left[ 1 - \frac{36000(42.1)^2}{4\pi^2 (29 \times 10^6)} \right] = 65300 \text{ lb}$



14-14

$$I_{xx} = 2.44 \text{ in}^4$$

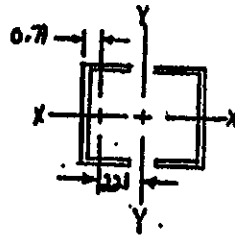
$$I_{yy} = 2[1.53 + 2.41(2.21)^2] = 26.6 \text{ in}^4$$

$$r_{yy} = \sqrt{I/A} = \sqrt{26.6/(2 \times 2.41)} = 2.35 \text{ in}$$

$$L_e = L = 10.5 \text{ Ft} (12 \text{ in/Ft}) = 126 \text{ in}$$

$$L_e/r = 126/2.35 = 53.6 \text{ (NT)}$$

$$EQ(14-7a) \quad P_a = 2(2.41)[20.2 - 0.126(53.6)] = 64.8 \text{ kips} = \underline{64,800 \text{ lb}}$$



14-15

$$F = mg = 1320 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 12950 \text{ N} = F_v$$

$$F_H = F/tan \theta = 12950/tan 22.6 = 31110 \text{ N}$$

$$\text{For } 56 \times 12.5, \quad r = 0.705 \text{ in} \times 25.4 \text{ mm/in} = 17.9 \text{ mm}$$

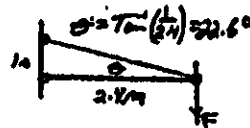
$$L_e/r = 2400/17.9 = 134 - \text{LONG} \quad [C_c = 126 \text{ FROM 14-12}]$$

$$P_a = \frac{\pi^2 EA}{(L_e/r)^2} = \frac{\pi^2 (200 \times 10^3 \text{ N/mm}^2)(2368 \text{ mm}^2)}{(134)^2} = 26016 \text{ N}$$

$$A = 2.67 \text{ in}^2 \times 645.16 \text{ mm}^2/\text{in}^2 = 2368 \text{ mm}^2$$

$$\text{IF } P_a = P_{cr}/N$$

$$N = \frac{P_{cr}}{P} = \frac{2.68 \times 10^5 \text{ N}}{31110 \text{ N}} = 8.57 \text{ OK}$$



14-16  $\lambda_{min} = \frac{0.125}{\sqrt{12}} = 0.036 \text{ IN} ; A = 0.25 \times 0.125 = 0.0313 \text{ IN}^2$   
 FOR 1040 CD:  $S_y = 82 \text{ ksi} ; C_c = 83 ; \frac{L_e}{\lambda} = \frac{8.40}{0.036} = 233 > C_c$  - LONG COLUMN  
 $P_{CR} = \frac{\pi^2 EA}{(L_e/\lambda)^2} = \frac{\pi^2 (30 \times 10^6) (0.0313)}{(233)^2} = 171 \text{ LB}$   
 $N = \frac{P_{CR}}{P} = \frac{171 \text{ LB}}{50 \text{ LB}} = 3.41 \text{ OK}$

14-17 FOR AISI 1141 OQT 1300:  $S_y = 469 \text{ MPa} ; C_c = 95 ; A = \frac{\pi D^2}{4} = \frac{\pi (12)^2}{4} = 113 \text{ mm}^2$   
 $\lambda = \frac{D}{4} = \frac{12}{4} = 3.0 \text{ mm} ; \frac{L_e}{\lambda} = \frac{(0.8 \times 90)}{3.0} = 50.7 < C_c$  - SHORT COLUMN  
 $P_{CR} = (113 \text{ mm}^2) (469 \text{ N/mm}^2) \left[ 1 - \frac{469 \text{ MPa} (50.7)^2}{4 \pi^2 (207 \times 10^3 \text{ MPa})} \right] = 45.2 \text{ kN}$   
 FOR  $N=3$ :  $P_a = \frac{P_{CR}}{3} = \frac{45.2 \text{ kN}}{3} = 15.1 \text{ kN}$

14-18 FOR AISI 1020 HR;  $S_y = 48 \text{ ksi} ; C_c = 105$   
 $\lambda = \frac{D}{4} = \frac{0.800 \text{ IN}}{4} = 0.200 \text{ IN} ; \frac{L_e}{\lambda} = \frac{28.5}{0.20} = 142.5$  - LONG ;  $A = \frac{\pi (0.8)^2}{4} = 0.508 \text{ IN}^2$   
 $P_{CR} = \frac{\pi^2 EA}{(L_e/\lambda)^2} = \frac{\pi^2 (30 \times 10^6) (0.508)}{(142.5)^2} = 7334 \text{ LB} ; N = \frac{P_{CR}}{P} = \frac{7334}{1375} = 5.33 \text{ OK}$

14-19 2 IN SCH 40 PIPE:  $\lambda = 0.787 \text{ IN} ; A = 1.075 \text{ IN}^2$  - FIXED/PINNED -  $k=0.8$   
 $\frac{L_e}{\lambda} = \frac{(0.8)(14 \text{ FT})(12 \text{ IN/FT})}{0.787} = 171$  ; FOR AISI 1040 HR,  $S_y = 82 \text{ ksi} ; C_c = 105$  - LONG  
 $P_{CR} = \frac{\pi^2 EA}{(L_e/\lambda)^2} = \frac{\pi^2 (30 \times 10^6) (1.075)}{(171)^2} = 10914 \text{ LB}$   
 EACH COLUMN CARRIES 5000 LB:  $N = \frac{P_{CR}}{P} = \frac{10914 \text{ LB}}{5000 \text{ LB}} = 2.18$  - MARGINAL

14-20 LACK OF RESTRAINT AT TOP OF COLUMNS MAKE THEM FREE.  
 FOR FIXED-FREE:  $\frac{L_e}{\lambda} = \frac{(240)(14 \text{ FT})(12 \text{ IN/FT})}{0.787 \text{ IN}} = 448$  - VERY LONG  
 $P_{CR} = \frac{\pi^2 EA}{(L_e/\lambda)^2} = \frac{\pi^2 (30 \times 10^6) (1.075)}{(448)^2} = 1584 \text{ LB} ; P_{ACTUAL} = 5000 \text{ LB}$  - FAILURE

14-21  
 $\lambda = 1.25 \text{ in} / \sqrt{12} = 0.361 ; A = (1.25 \text{ in})^2 = 1.563 \text{ in}^2$   
 COLUMNS ARE FIXED-FREE ;  $L_e = 2 \times 10 = 84 \text{ in}$   
 $L_e/\lambda = 84 / 0.361 = 233$  - LONG  
 ALUMINUM 6061-T6 ;  $S_y = 40000 \text{ psi} ; C_c = 70$   
 $P_{CR} = \frac{\pi^2 EA}{(L_e/\lambda)^2} = \frac{\pi^2 (10 \times 10^6) (1.563)}{(233)^2} = 2844 \text{ LB}$   
 EACH COLUMN CARRIES 1500 LB  
 $N = P_{ay}/P = 2844 / 1500 = 1.90$  - LOW

14-22 ASSUME FIXED-PINNED ENDS:  $L_{eff} = \frac{L}{\sqrt{2}} = \frac{2.00}{\sqrt{2}} = 0.577 \text{ IN}$

$\frac{L_c}{r} = \frac{(0.80)(12.75 \text{ FT})(12 \text{ IN/FT})}{0.577 \text{ IN}} = 212$ ; AISI 1040 Q&T 1100;  $S_y = 80 \text{ ksi}$ ;  $C_c = 85$  LONG

$P_{cr} = \frac{\pi^2 EA}{(L_c/r)^2} = \frac{\pi^2 (30 \times 10^6 \text{ lb/in}^2)(6.0 \text{ in}^2)}{(212)^2} = 39.525 \text{ LB}$

$N = \frac{P_{cr}}{P} = \frac{39.525}{12500} = 3.16 \text{ OK}$

THIS IS PROBABLY CONSERVATIVE BECAUSE PIN MAY PROVIDE SOME RESTRAINT AGAINST BUCKLING WITH RESPECT TO VERTICAL AXIS. THUS  $L_c/r$  WOULD BE SMALLER;  $P_{cr}$  AND  $N$  WOULD BE LARGER.

14-23 ASSUME A LONG COLUMN: EQ. 14-5:  $L_c = (6.8)(12.75 \text{ FT})(12 \text{ IN/FT}) = 172 \text{ IN}$

REQD  $I = \frac{N P L_c^2}{\pi^2 E} = \frac{(4.0)(12500 \text{ LB})(172 \text{ IN})^2}{\pi^2 (30 \times 10^6 \text{ lb/in}^2)} = 2.53 \text{ in}^4 = \frac{\pi D^4}{64}$

$D_{min} = \left[ \frac{64(2.53)}{\pi} \right]^{1/4} = 2.68 \text{ IN}$ ;  $r = \frac{D}{4} = \frac{2.68}{4} = 0.670$ ;  $\frac{L_c}{r} = \frac{172}{0.67} = 181$  - LONG

14-24 LET  $E = 29 \times 10^6 \text{ psi}$  FOR STRUCTURAL STEEL:  $S_y = 36 \text{ ksi}$ ;  $C_c = 730$

ASSUME COLUMN IS LONG:  $I_{min} = \frac{N P L_c^2}{\pi^2 E} = \frac{(4.0)(12500 \text{ LB})(172 \text{ IN})^2}{\pi^2 (29 \times 10^6 \text{ lb/in}^2)} = 2.60 \text{ in}^4$

USE 3 IN SCH. 40:  $I = 3.017 \text{ in}^4$ ;  $r = 1.163 \text{ IN}$ ;  $\frac{L_c}{r} = \frac{172}{1.163} = 148 < C_c$  - SHORT

$P_a = \frac{A S_y \left[ 1 - \frac{S_y (L_c/r)^2}{4 \pi^2 E} \right]}{N} = \frac{(2.228 \text{ in}^2)(36000 \text{ lb/in}^2) \left[ 1 - \frac{(36000)(148)^2}{4 \pi^2 (29 \times 10^6 \text{ lb/in}^2)} \right]}{4} = 13.114 \text{ LB}$  OK

14-25 FOR BUCKLING ABOUT Y-Y AXIS - FIXED-PINNED:  $L_c = (0.8)(12.75 \text{ FT})(12 \text{ IN/FT}) = 122 \text{ IN}$

FOR Y-Y AXIS - FIXED-FIXED:  $L_c = 0.65(12.75)(12) = 99.5 \text{ IN}$ ; ASSUME LONG COLUMN

$I_{Xmin} = \frac{N P L_c^2}{\pi^2 E} = \frac{4(12500)(122)^2}{\pi^2 (30 \times 10^6)} = 7.51 \text{ in}^4$  - I S X 3.700 BEAM SHAPE REQD

$I_{Ymin} = \frac{4(12500)(99.5)^2}{\pi^2 (30 \times 10^6)} = 5.02 \text{ in}^4$  - I X 5.800 REQD;  $r_y = 1.08 \text{ IN}$

$\frac{L_c}{r_y} = \frac{99.5 \text{ IN}}{1.08 \text{ IN}} = 92.1$ ; FOR 60K-T6:  $S_y = 40 \text{ ksi}$ ;  $C_c = 70$  - LONG COLUMN - OK

14-26  $L_c = 0.8L = 0.8(16.5 \text{ FT})(12 \text{ IN/FT}) = 158 \text{ IN}$ ;  $3 \times 3 \times \frac{1}{4}$  -  $r = 1.10 \text{ IN}$ ;  $A = 2.59 \text{ in}^2$

$L_c/r = 158/1.10 = 144$ ; FOR ASTM A500, GRADE B:  $S_y = 46 \text{ ksi}$ ;  $C_c = 110$  - LONG

$P_{cr} = \frac{\pi^2 EA}{(L_c/r)^2} = \frac{\pi^2 (29 \times 10^6 \text{ lb/in}^2)(2.59)}{(144)^2} = 35.750 \text{ LB}$ ;  $P_a = \frac{P_{cr}}{N} = \frac{35.750}{3} = 11.917 \text{ LB}$

14-27  $L_c = 0.8L = 0.8(16.5)(12) = 158 \text{ IN}$ ; FOR  $4 \times 2 \times \frac{1}{4}$ ,  $r_{min} = 0.770 \text{ IN}$ ;  $A = 2.59 \text{ in}^2$

$L_c/r = 158/0.77 = 205$ ;  $C_c = 110$  (PROB 14-28) - LONG

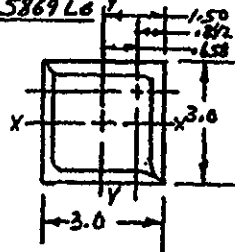
$P_{cr} = \frac{\pi^2 (29 \times 10^6 \text{ lb/in}^2)(2.59)}{(205)^2} = 17.606 \text{ LB}$ ;  $P_a = \frac{P_{cr}}{N} = \frac{17.606}{3} = 5.869 \text{ LB}$

14-28 FROM 14-26:  $L_c = 158 \text{ IN}$ ;  $S_y = 36 \text{ ksi}$ ;  $C_c = 130$

$I_x = I_y = 2[I + A d^2] = 2[1.24 + 1.44(0.65)^2] = 3.73 \text{ in}^4$

$r = \sqrt{I/A} = \sqrt{3.73/2.59} = 1.138 \text{ IN}$ ;  $\frac{L_c}{r} = \frac{158}{1.138} = 139$  - LONG

$P_a = \frac{P_{cr}}{N} = \frac{\pi^2 (29 \times 10^6 \text{ lb/in}^2)(2.59)}{(3)(139)^2} = 4.230 \text{ LB}$



14-29 AISI 1020 HR:  $S_y = 331 \text{ MPa}$ ;  $C_c = 105$ ;  $r = \frac{40}{\sqrt{12}} = 11.55 \text{ mm}$

FIXED-FIXED:  $L_e = 0.65(750) = 488 \text{ mm}$ ;  $\frac{L_e}{r} = \frac{488}{11.55} = 42.2$  - SHORT

$P_{cr} = A S_y \left[ 1 - \frac{S_y (L_e/r)^2}{4\pi^2 E} \right] = (2400 \text{ mm}^2) (331 \text{ N/mm}^2) \left[ 1 - \frac{331 \text{ MPa} (42.2)^2}{4\pi^2 (207 \times 10^3 \text{ MPa})} \right] = 737 \text{ kN}$

LET  $N=3$ :  $P_a = \frac{P_{cr}}{N} = \frac{737 \text{ kN}}{3} = 245 \text{ kN}$

14-30 6061-T4:  $S_y = 145 \text{ MPa}$ ;  $C_c = 98$ ;  $r = \frac{1}{\sqrt{12}} \sqrt{I_x} = \frac{1}{\sqrt{12}} \sqrt{1.478 \times 10^8 \text{ mm}^4} = 16.3 \text{ mm}$

$\frac{L_e}{r} = \frac{4258 \text{ mm}}{16.3 \text{ mm}} = 261$  - LONG;  $A = 1.478 \text{ in}^2 \frac{(25.4)^2 \text{ mm}^2}{\text{in}^2} = 954 \text{ mm}^2$

$P_a = \frac{P_{cr}}{N} = \frac{\pi^2 EA}{N(L_e/r)^2} = \frac{\pi^2 (69 \times 10^3 \text{ N/mm}^2) (954 \text{ mm}^2)}{4(261)^2} = 2376 \text{ N}$

14-31 NO IMPROVEMENT BECAUSE COLUMN IS STILL LONG AND BUCKLING LOAD FROM EULER FORMULA IS INDEPENDENT OF STRENGTH.  $E$  IS THE SAME.

14-32 FOR ASTM A36:  $S_y = 36 \text{ ksi}$ ;  $E = 29 \times 10^3 \text{ psi}$ ;  $C_c = \sqrt{\frac{2\pi^2 E}{S_y}} = 126$ ;  $r_{min} = r_y = 3.02 \text{ in}$

$\frac{L_e}{r} = \frac{0.8(22.5 \text{ ft})(12 \text{ in/ft})}{3.02 \text{ in}} = 71.5 < C_c$  - USE EOS. 14-8, 14-9

$FS = \frac{5}{3} + \frac{3(71.5)}{8(126)} - \frac{(71.5)^3}{8(126)^3} = 1.86$

$P_a = \left[ 1 - \frac{(L_e/r)^2}{2C_c^2} \right] \frac{S_y A}{FS} = \left[ 1 - \frac{71.5^2}{2(126)^2} \right] \frac{36000(14.1)}{1.86} = 310,721 \text{ LB}$

# COLUMN ANALYSIS - SUMMARY OF RESULTS OF PROBLEMS 14-33 TO 14-43

Prob. No.	L	K	L <sub>e</sub>	s <sub>y</sub>	E	C <sub>c</sub>	A	r	SR	N	P <sub>cr</sub>	Eqn.	P <sub>s</sub>
14-33	163.2 in	0.65	106.2 in	36 ksi	29E06 psi	126	2.09 in <sup>2</sup>	0.742	143	3	29267 lb	Euler	9756 lb
14-34	83.2 in	0.80	66.6 in	36 ksi	29E06 psi	126	2.09 in <sup>2</sup>	0.742	89.7	3	56202 lb	Johnson	18734 lb
14-35	163.2 in	0.80	130.6 in	36 ksi	29E06 psi	126	2.09 in <sup>2</sup>	0.742	176	3	19321 lb	Euler	6440 lb
14-36	163.2 in	0.65	106.1 in	36 ksi	29E06 psi	126	2.09 in <sup>2</sup>	1.1	96.4	3	65974 lb	Johnson	21991 lb
14-37	2650 mm	1.00	2650 mm	248 MPa	200 GPa	126	1671 mm <sup>2</sup>	19.6	135.5	3	179.7 kN	Euler	59.9 kN
14-38	2650 mm	1.00	2650 mm	248 MPa	200 GPa	126	1671 mm <sup>2</sup>	27.9	9408	3	297 kN	Johnson	99.1 kN

L<sub>e</sub> = 3/4 \* L for steel tube

## Problem 14-39 Truss Analysis + Design of Compression Members

Example data only. Many possible designs Listing of compression members only

Design factor on load = 3.0

Member	Load	L	Shape	Size	Material	P <sub>s</sub>
AC	1925 lb	40 in	Circular	15/16 in	ASTM A36	2253 lb
CD	750	25 in	Circular	9/16 in	ASTM A36	750 lb
CE	650	40 in	Circular	11/16 in	ASTM A36	654 lb

## Problem 14-40 Truss Analysis + Design of Compression Members

Example data only. Many possible designs Listing of compression members only

Design factor on load = 2.5

Member	Load	L	Shape	Size	Material	P <sub>s</sub>
AC	4609 lb	120 in	Steel tube	4x3x1/4	ASTM A501	5694 lb
CG	3625 lb	120 in	Steel tube	3x3x1/4	ASTM A501	4012 lb
EH	1101 lb	154 in	Steel tube	3x3x1/4	ASTM A501	2446 lb
EG	3625 lb	120 in	Steel tube	3x3x1/4	ASTM A501	4012 lb
BE	4141 lb	120 in	Steel tube	4x3x1/4	ASTM A501	5694 lb

## Problem 14-41 Truss Analysis + Design of Compression Members

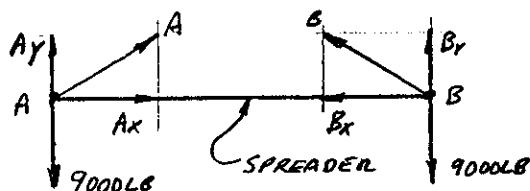
Example data only. Many possible designs Listing of compression members only

Design factor on load = 2.5

Member	Load	L	Shape	Size	Material	P <sub>s</sub>
DE	2300 N	.25 m	Square	10 mm	Alum. 6061-T6	3600 N
BD	2597 N	.297 m	Square	10 mm	Alum. 6061-T6	2600 N
EF	2300 N	.20 m	Square	8 mm	Alum. 6061-T6	2324 N
FG	550 N	.15 m	Square	5 mm	Alum. 6061-T6	630 N
CF	800 N	.16 m	Square	6 mm	Alum. 6061-T6	1149 N



- 14-42** The support cables for the sling act at  $30^\circ$  to the horizontal and exert a direct axial compressive force on the spreader as shown below. Assume central loading of a straight column. The horizontal (axial) component of the cable force is 15 588 lb.



$$A_y = B_y = 9000 \text{ LB}$$

$$A = B = 9000 \text{ LB} / \sin 30^\circ = 18000 \text{ LB}$$

$$A_x = B_x = 18000 \text{ LB} (\cos 30^\circ) = 15588 \text{ LB}$$

**Design decision:** Use a hollow steel tube made from ASTM A501 structural steel. The column buckling analysis spreadsheet (Figure 14-9) was used to determine that the lightest size with adequate capacity is a 3x3x1/4 hollow steel tube. Other results are summarized below.

$L = L_e = 96 \text{ in}$ ;  $r = 1.10 \text{ in}$ ;  $SR = 87.3$ ;  $A = 2.59 \text{ in}^2$ ;  $s_y = 36\,000 \text{ psi}$ ;  $E = 29 \times 10^6 \text{ psi}$ ;  $C_c = 126$ ; Column is long; Use  $N = 2.5$  (design decision); Column is short; Use Johnson formula;  $P_{cr} = 70\,900 \text{ lb}$ ;  $P_a = 28\,364 \text{ lb}$ .

- 14-43** The analysis is similar to Problem 14-42. With the angle of  $15^\circ$ , the axial force on the tube is 33 588 lb. The spreader now must be a 4x4x1/4 steel tube with:  $A = 3.59 \text{ in}^2$ ;  $r = 1.51 \text{ in}$ ;  $SR = 63.6$ ; Short column; From the Johnson formula,  $P_{cr} = 112\,814 \text{ lb}$ ;  $P_a = 45\,126 \text{ lb}$ .

### Crooked Columns

For Problems 14-44 to 14-49, loading data were taken from earlier problems as listed in the problem statements. The amount of initial crookedness is given. The Crooked Column Analysis spreadsheet (Figure 14-14) was used to determine the critical buckling load and the allowable load for a design factor of 3.0. The spreadsheet solves Equation 14-18. Results are summarized in the table on the following page.

### Eccentrically-Loaded Columns

For Problems 14-50 to 14-58, data from the problem statements were entered into the Eccentric Column Analysis spreadsheet (Figure 14-15). Where the problem asks for the maximum stress and deflection, the design factor  $N = 1.0$  was entered at the lower left column. For design problems, the requested design factor (typically  $N = 3.0$ ) was entered. Results are summarized in the table on the following page.

### Problem 14-59

Straight and crooked column analysis required for the 2-in schedule 40 steel pipe, 156 in long. The spreadsheets in Figures 14-9 (Straight columns) and 14-14 (crooked columns) were used to determine the following results.

- Straight pipe:  $SR = 198$ ;  $C_c = 126$ ; Long column;  $P_{cr} = 7831 \text{ lb}$ ;  $P_a = 2610 \text{ lb}$ .
- Crooked pipe:  $a = 1.25 \text{ in}$ ;  $C_1$  in Eqn. 14-18 =  $-21\,766$ ;  $C_2 = 3.36 \times 10^7$ ; Euler buckling load =  $7831 \text{ lb}$ ;  $P_a = 1676 \text{ lb}$ .

# CROOKED COLUMN ANALYSIS - SUMMARY OF RESULTS OF PROBLEMS 14-44 TO 14-49

Prob.	a	L	K	L <sub>e</sub>	s <sub>y</sub>	E	C <sub>c</sub>	A	r	SR	N	Equation 14-18			
												P <sub>cr</sub>	C <sub>1</sub>	C <sub>2</sub>	P <sub>a</sub>
14-44	4.00 mm	800 mm	1.00	800 mm	331 MPa	207 GPa	111	314 mm <sup>2</sup>	5.00 mm	160	3	25.06 kN	-56362	2.89E08	5.71 kN
14-45	1.60 mm	210 mm	1.00	210 mm	469 MPa	207 GPa	93.3	300 mm <sup>2</sup>	3.46 mm	60.6	3	111 kN	-146962	2.61E09	20.6 kN
14-46	14.0 mm	2800 mm	1.00	2800 mm	276 MPa	69 GPa	70.2	4743 mm <sup>2</sup>	36.1 mm	77.6	3	537 kN	-761829	7.81E10	122 kN
14-47	0.75 in	150 in	0.80	120 in	36 ksi	29E06 psi	126	4.44 in <sup>2</sup>	0.876 in	137	3	67778 lb	-120164	1.204E09	11030 lb
14-48	1.25 in	163.2 in	0.65	106.1 in	36 ksi	29E06 psi	126	2.09 in <sup>2</sup>	0.742 in	143	3	29267 lb	-56985	2.48E08	4678 lb
14-49	32.0 mm	2650 mm	1.00	2650 mm	248 MPa	200 GPa	126	1671 mm <sup>2</sup>	19.6 mm	135	1.1	180 kN	-875101	6.00E10	75.04 kN

----> iterated to find N for P<sub>a</sub> = 75 kN

## ECCENTRICALLY LOADED COLUMN ANALYSIS - SUMMARY OF RESULTS OF PROBLEMS 14-50 TO 14-58

Value of N in Eqn. 14-20 set equal to 1.0 to find maximum stress in column.

Prob.	e	L	K	L <sub>e</sub>	s <sub>y</sub>	E	C <sub>c</sub>	A	r	SR	N	Value of secant for:			
												Stress	Defl.	Stress	y <sub>max</sub>
14-50	0.60 in	42.0 in	1.00	42.0 in	21.0 ksi	10E06 psi	97	1.563 in <sup>2</sup>	0.361 in	116	1	1.1527	1.1527	3456 psi	0.092 in
14-51	150 mm	3200 mm	1.00	3200 mm	331 MPa	207 GPa	111	1437 mm <sup>2</sup>	29.5 mm	108	1	1.1719	1.1719	211 MPa	25.8 mm
14-52	0.30 in	14.75 in	1.00	14.75 in	40.0 ksi	28E06 psi	118	0.063 in <sup>2</sup>	0.072 in	204	1	1.1511	1.1511	6687 psi	0.045 in

## Problems 14-53 to 14-58 Eccentrically Loaded Column Analysis

Value of N = 3 used to evaluate safety

Prob.	e	L	K	L <sub>e</sub>	s <sub>y</sub>	E	C <sub>c</sub>	A	r	SR	N	Value of secant for:			
												Stress	Defl.	Reqd s <sub>y</sub>	y <sub>max</sub>
14-53	0.50 in	40.0 in	1.00	40.0 in	50.0 ksi	29E06 psi	107	3.59 in <sup>2</sup>	1.51 in	26.5	3	1.225	1.0667	96.3 ksi	0.033 in

Design is not safe because reqd s<sub>y</sub> is greater than given s<sub>y</sub>. Prop is redesigned to find lightest steel tube that is safe.

14-53 0.50 in 40.0 in 1.00 40.0 in 50.0 ksi 29E06 psi 107 7.59 in<sup>2</sup> 3.15 in 12.7 3 1.021 1.0069 35.7 ksi 0.003 in  
Square steel tube 8x8x1/4. Assumed pinned ends. Would be safer if ends are flat and parallel to the press ram.

14-54a 0.90 in 72 in 1.00 72.0 in 90.0 ksi 30E06 psi 81.1 1.28 in<sup>2</sup> 0.231 in 312 1 1.4289 1.4289 8319 psi 0.386 in  
This is the maximum stress in the column for an assumed design factor of N = 1.0.

14-54b 0.9 72 in 1.00 72.0 in 90.0 ksi 30E06 psi 81.1 1.28 in<sup>2</sup> 0.231 in 312 3 5.2212 1.4289 84.9 ksi 0.386 in  
This gives the required yield strength (84.9 ksi) of the material for N = 3. Specify AISI 1040 WQT 900 steel; s<sub>y</sub> = 90.0 ksi.

Problems 14-53 to 14-58 Eccentrically Loaded Column Analysis (Continued)

Value of  $N = 3$  used to evaluate safety

Prob.	e	L	K	L <sub>e</sub>	s <sub>y</sub>	E	C <sub>c</sub>	A	r	SR	N	Value of secant for:		
												Stress	Defl.	Reqd s <sub>y</sub> y <sub>max</sub>

14-55 0.4645 in 112 in 1.00 112 in 36.0 ksi 29E06 psi 126 2.84 in<sup>2</sup> 0.489 in 229 3 7.2758 1.4774 94.8 ksi 0.222 in  
 The design is not safe using a desired value of  $N = 3$ . Required yield strength is over 2.5 times higher than given yield strength.  
 The eccentricity is the distance from the centroidal axis and the middle of the flange width.

14-56 3.00 in 126 in 0.80 100.8 46.0 ksi 29E06 psi 112 6.36 in<sup>2</sup> 1.39 in 72.5 3 1.2579 1.0753 46.0 ksi 0.226 in  
 The load for these data is 19,870 lb, found by iterating Equation 14-20 using the spreadsheet until the required yield strength became less than the given 46,000 psi.

14-57 1.75 in 40.0 in 1.00 40.0 in 40.0 ksi 10E06 psi 70.2 0.600 in<sup>2</sup> 0.433 in 92.4 3 1.5127 1.1333 39.1 ksi 0.233 in  
 The load for these data is 675 lb, found by iterating Equation 14-20 using the spreadsheet until the required yield strength became less than the given 40,000 psi.

The analysis was done assuming that the loading in the plane of the drawing was critical. IT IS NOT! See the analysis below.

14-57b Buckling about the thickness of the bar is now checked assuming that the load is centrally applied.

Column analysis spreadsheet is used to determine allowable load for  $N = 3$ .

0	40.0 in	1.00	40.0 in	40.0 ksi	10E06 psi	70.2	0.60 in <sup>2</sup>	0.1154 in	347	3	-	-	-	-
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The allowable load for buckling about this axis is only 164 lb. This is the limiting load.

14-58 20 mm 750 mm 1.00 750 mm 931 MPa 200 GPa 65.1 314 mm<sup>2</sup> 7.29 mm 103 3 1.4512 1.1205 389 MPa 2.41 mm  
 The design is safe because the required yield strength is less than the actual yield strength of the given material.

Problem 14-59 has two-parts. The first analysis is for a straight pipe. The second analysis is for the crooked pipe.

Equation 14-18

Prob.	a	L	K	L <sub>e</sub>	s <sub>y</sub>	E	C <sub>c</sub>	A	r	SR	N	P <sub>cr</sub>	C <sub>1</sub>	C <sub>2</sub>	P <sub>a</sub>
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14-59a	0	156 in	1.00	156 in	36.0 ksi	29E06 psi	126	1.075 in <sup>2</sup>	0.787 in	198	3	7831	-	-	2610 lb
14-59b	1.25 in	156 in	1.00	156 in	36.0 ksi	29E06 psi	126	1.075 in <sup>2</sup>	0.787 in	198	3	7831	-21766	3.37E07	1676 lb

The allowable load decreases from 2610 lb to 1676 lb when the pipe is crooked.

# CHAPTER 15 Pressure Vessels

15-1

$$t = (200 - 184)/2 = 8.0 \text{ mm}$$

$$\text{LET } D = \text{MEAN DIA.} = (D_o + D_i)/2 = (200 + 184)/2 = 192 \text{ mm}$$

$$D/t = 192/8.0 = 24 \text{ - THIN-WALLED SPHERE}$$

$$\sigma = \frac{pD}{4t} = \frac{(19.2 \text{ N/mm}^2)(192 \text{ mm})}{4(8.0 \text{ mm})} = 115 \text{ MPa}$$

15-2

$$D_m = D_o - t = 10500 - 12 = 10488 \text{ mm}$$

$$\frac{D_m}{t} = \frac{10488 \text{ mm}}{12 \text{ mm}} = 874 \text{ - VERY THIN WALL ; AISI 1040 HR}$$

$$\sigma_y = \frac{S_y}{4} = \frac{414 \text{ MPa}}{4} = 103.5 \text{ MPa} ; \sigma_{\text{MAX}} = \frac{pD}{4t}$$

$$p = \frac{4t\sigma_y}{D_m} = \frac{4(12 \text{ mm})(103.5 \text{ MPa})}{10488 \text{ mm}} = 0.477 \text{ MPa} = 477 \text{ kPa}$$

15-3

$$D_o = 1200 \text{ mm - ASSUME THIN WALL ; FOR Ti-6AL-4V, } S_y = 1070 \text{ MPa}$$

$$\sigma_y = \frac{S_y}{4} = \frac{1070 \text{ MPa}}{4} = 267.5 \text{ MPa} ; \sigma = \frac{pD_m}{4t} = \frac{p(D_o - t)}{4t} = \frac{pD_o - pt}{4t}$$

$$t = \frac{pD_o}{4\sigma_y + p} = \frac{(4.20 \text{ MPa})(1200 \text{ mm})}{4(267.5 \text{ MPa}) + 4.2 \text{ MPa}} = 4.70 \text{ mm} \text{ USE } t = 5.01 \text{ mm CONVENIENT SIZE}$$

$$p_{\text{TE}} \approx \frac{1195}{5} = 239 \text{ - VERY THIN WALL}$$

15-4

$$\sigma_y = \frac{S_y}{4} = \frac{414 \text{ MPa}}{4} = 103.5 \text{ MPa}$$

$$t = \frac{pD}{4\sigma_y} = \frac{(4.20 \text{ MPa})(1200 \text{ mm})}{4(103.5 \text{ MPa})} = 12.2 \text{ mm} ; D/t = 1200/12.2 = 98 \text{ - THIN}$$

MASS = DENSITY x VOLUME OF SPHERE

$$\text{VOLUME} = 0.5236(D^3 - d^3) ; D = 1200 \text{ mm} ; d = D - 2t$$

$$\text{Ti : } d = 1200 - 2(4.71 \text{ mm}) = 1190 \text{ mm}$$

$$V = 0.5236(1200^3 - 1190^3) = 2.114 \times 10^7 \text{ mm}^3$$

$$\text{ALUM : } d = 1200 - (12.2 \times 2) = 1176 \text{ mm}$$

$$V = 0.5236(1200^3 - 1176^3) = 5.39 \times 10^7 \text{ mm}^3$$

$$\text{MASS OF Ti} = \frac{4430 \text{ kg}}{\text{m}^3} \times 2.114 \times 10^7 \text{ mm}^3 \times \frac{1 \text{ m}^3}{(10^3 \text{ mm})^3} = 92.6 \text{ kg}$$

LIGHTER

$$\text{MASS OF ALUM} = \frac{2770 \text{ kg}}{\text{m}^3} \times 5.39 \times 10^7 \text{ mm}^3 \times \frac{1 \text{ m}^3}{(10^3 \text{ mm})^3} = 149 \text{ kg}$$

15-5

$$D_o = 10.750 \text{ in} ; D_i = 10.020 \text{ in} ; D_m = (D_o + D_i)/2 = 10.385 \text{ in}$$

$$t = 0.365 \text{ in} ; D_m/t = 10.385/0.365 = 28.5 \text{ (THIN)}$$

$$\sigma = \frac{pD}{4t} = \frac{(50 \text{ psi})(10.385 \text{ in})}{4(0.365 \text{ in})} = 2134 \text{ psi}$$

15-6  $D_m = D_i + t = 80 + 3.5 = 83.5 \text{ mm}$ ;  $D_m/t = 83.5/3.5 = 23.9$  THIN  
 $\sigma = \frac{P D_m}{2t} = \frac{(6.85 \text{ MPa})(83.5 \text{ mm})}{2(3.5 \text{ mm})} = 84.0 \text{ MPa}$

15-7 ASSUME THIN WALL:  $\sigma_0 = \frac{S_y}{4} = \frac{565 \text{ MPa}}{4} = 141.3 \text{ MPa} = \frac{P D}{2t}$   
 $t = \frac{P D}{2 \sigma_0} = \frac{(1.7 \text{ MPa})(300 \text{ mm})}{2(141.3 \text{ MPa})} = 1.80 \text{ mm}$   
 $D_m = D_o - t = 300 - 1.81 = 298.2$ ;  $D_m/t = 298.2/1.80 = 164$  - VERY THIN

15-8  $t = \frac{P D}{2 \sigma_0} = \frac{(15.2 \text{ MPa})(250 \text{ mm})}{2(141.3 \text{ MPa})} = 13.45 \text{ mm}$   
 $D_m = D_i - t = 250 - 13.45 = 236.5 \text{ mm}$ ;  $D_m/t = 236.5/13.45 = 17.6$  - THICK WALL  
 USE EQ. FOR  $\sigma_2$  FROM TABLE 15-1. TRY  $t = 14.0 \text{ mm}$   
 $D_i = D_o - 2t = 250 - 2(14) = 222 \text{ mm}$ ;  $a = 111 \text{ mm}$ ,  $b = 125 \text{ mm}$ .  
 $\sigma_2 = \frac{P(b^2 + a^2)}{b^2 - a^2} = \frac{(15.2 \text{ MPa})(125^2 + 111^2)}{(125^2 - 111^2)} = 128.6 \text{ MPa}$   
 OK FOR  $\sigma_1 = 141.3 \text{ MPa}$  BUT SOMEWHAT LOW.  
 FOR  $t = 13.0 \text{ mm}$ ;  $D_i = 224 \text{ mm}$ ;  $a = 112 \text{ mm}$ ,  $b = 125 \text{ mm}$   
 $\sigma = 139.0 \text{ MPa}$  OK USE  $t = 13.0 \text{ mm}$

15-9  $D_m = D_o - t = 450 - 2.20 = 447.8 \text{ mm}$ ;  $D_m/t = 447.8/2.20 = 204$  - THIN  
 $\sigma_{\text{max}} = \frac{P D_m}{2t} = \frac{(250 \times 10^3 \text{ Pa})(447.8 \text{ mm})}{2(2.20 \text{ mm})} = 76.3 \text{ MPa}$   
 $N = \frac{S_y}{\sigma} = \frac{290 \text{ MPa}}{76.3 \text{ MPa}} = 3.80$

15-10 ASSUME THIN WALL:  $\sigma_0 = S_y/4 = 444/4 = 103 \text{ MPa} = \frac{P D}{2t}$   
 $t = \frac{P D}{2 \sigma_0} = \frac{(750 \times 10^3 \text{ Pa})(1800 \text{ mm})}{2(103 \times 10^6 \text{ Pa})} = 6.55 \text{ mm}$  LET  $t = 7.0 \text{ mm}$   
 $D_m = D_o - t = 1800 - 7.0 = 1793 \text{ mm}$ ;  $D_m/t = 1793/7.0 = 256$  - VERY THIN

15-11  $D_m = D_o - t = 250 - 18 = 232 \text{ mm}$ ;  $D_m/t = 232/18 = 12.9 < 20$  - THICK WALL  
 $b = 250/2 = 125 \text{ mm}$ ;  $a = b - t = 125 - 18 = 107 \text{ mm}$   
 $\sigma_{2, \text{max}} = \frac{P(b^2 + a^2)}{2(b^2 - a^2)} = \frac{(71.0 \text{ MPa})(125^2 + 107^2)}{2(125^2 - 107^2)} = 212 \text{ MPa}$  TANGENTIAL  
 $\sigma_{3, \text{max}} = -P = -70.0 \text{ MPa}$  RADIAL

15-12  $D_o = 0.840 \text{ in}$ ;  $D_i = 0.622$ ;  $D_m = (D_o + D_i)/2 = 0.731$ ;  $D_m/t = \frac{0.731}{0.1109} = 6.71$  (THICK)  
 $b = D_o/2 = 0.420 \text{ in}$ ;  $a = D_i/2 = 0.311 \text{ in}$   
 $\sigma_1 = \frac{P a^2}{b^2 - a^2} = \frac{(250 \text{ psi})(0.311)^2}{(0.420)^2 - (0.311)^2} = 303 \text{ psi}$  LONGITUDINAL  
 $\sigma_2 = \frac{P(b^2 + a^2)}{b^2 - a^2} = \frac{(250 \text{ psi})(0.420^2 + 0.311^2)}{0.420^2 - 0.311^2} = 857 \text{ psi}$  HOOP  
 $\sigma_3 = -P = -250 \text{ psi}$  RADIAL

15-13

$$\frac{D_m}{t} = \frac{(300+220)/2}{(300-220)/2} = 6.5 \quad (\text{THICK}) \quad a = 110 \text{ mm}; b = 150 \text{ mm}$$

$$\sigma_z = \frac{p a^2 (b^2 + r^2)}{r^2 (b^2 - a^2)} = \frac{b^2 + r^2}{r^2} \times \frac{p a^2}{b^2 - a^2} = \frac{b^2 + r^2}{r^2} \times \frac{(50 \text{ MPa}) (110)^2}{150^2 - 110^2} = 58.17 \left[ \frac{b^2 + r^2}{r^2} \right]$$

r	$\sigma_z$ HOOP STRESS
110 mm	166 MPa - INNER SURFACE - MAXIMUM
120	149 MPa
130	136 MPa
140	125 MPa
150	116 MPa - OUTER SURFACE

15-14

$$D_m = (D_o + D_i)/2 = (1.900 + 1.610)/2 = 1.755 \quad D_m/t = 1.755/0.145 = 12.1 \quad (\text{THICK})$$

USING THICK WALLED EQN,  $b = 1.90/2 = 0.95$ ;  $a = 1.61/2 = 0.805$

$$\sigma_z = \frac{p (b^2 + a^2)}{b^2 - a^2} = \frac{(0.0 \text{ MPa}) (0.95^2 + 0.805^2)}{(0.95^2 - 0.805^2)} = 60.9 \text{ MPa}$$

USING THIN-WALLED EQN.

$$\sigma_z = \frac{p D}{2t} = \frac{(0.0 \text{ MPa}) (1.755 \text{ mm})}{2 (0.145 \text{ mm})} = 60.5 \text{ MPa}$$

15-15

$$D_m = (D_o + D_i)/2 = (50 + 30)/2 = 40 \text{ mm}; t = (D_o - D_i)/2 = (50 - 30)/2 = 10 \text{ mm}$$

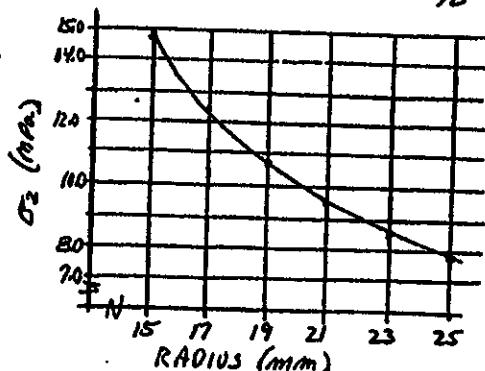
$$D_m/t = 40/10 = 4.0 \quad \text{THICK}; b = D_o/2 = 25 \text{ mm}; a = D_i/2 = 15 \text{ mm}$$

$$\sigma_z = \frac{p (b^2 + a^2)}{b^2 - a^2} = \frac{(0.0 \text{ MPa}) (25^2 + 15^2)}{(25^2 - 15^2)} = 14.88 \text{ MPa TANGENTIAL}$$

15-16

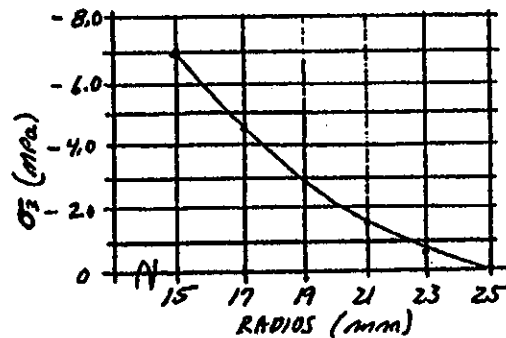
$$\sigma_z = \frac{p a^2 (b^2 + r^2)}{r^2 (b^2 - a^2)} = \frac{p a^2}{b^2 - a^2} \times \frac{(b^2 + r^2)}{r^2} = \frac{2.0 (15)^2}{(25^2 - 15^2)} \times \frac{(b^2 + r^2)}{r^2} = 3.94 \times \frac{(b^2 + r^2)}{r^2}$$

r	$\sigma_z$ (MPa)
15	14.88 INNER SURFACE
17	12.45
19	10.75
21	9.52
23	8.59
25	7.88 OUTER SURFACE



$$15-17 \quad \sigma_3 = \frac{-p a^2 (b^2 - r^2)}{r^2 (b^2 - a^2)} = \frac{-p a^2}{b^2 - a^2} \times \frac{b^2 - r^2}{r^2} = \frac{-(7.0)(15^2)}{(25^2 - 15^2)} \times \frac{b^2 - r^2}{r^2} = 2.94 \frac{b^2 - r^2}{r^2}$$

$r$	$\sigma_3$ (MPa) RADIAL
15	-7.00 INNER SURFACE
17	-4.58
19	-2.88
21	-1.64
23	-0.71
25	0 OUTER SURFACE



15-18 ASSUMING THIN WALL THEORY

$$\sigma_2 = \frac{p D_m}{2t} = \frac{(7.0 \text{ MPa})(40 \text{ mm})}{2(10 \text{ mm})} = 14.0 \text{ MPa} \quad (5.9\% \text{ LOW})$$

FROM PROB. 15-15: ACTUAL  $\sigma_{2 \text{ MAX}} = 14.88 \text{ MPa}$

15-19  $D_m = D_o - t = 500 - 40 = 460 \text{ mm}$ ;  $D_m/t = 460/40 = 11.5 < 20$  - THICK

$$\sigma_2 = \frac{S_y}{4} = \frac{931 \text{ MPa}}{4} = 232 \text{ MPa AISI 501 OQT 1000 \quad \left. \begin{array}{l} b = D_o/2 = 250 \text{ mm} \\ a = D_i/2 = 210 \text{ mm} \end{array} \right\}$$

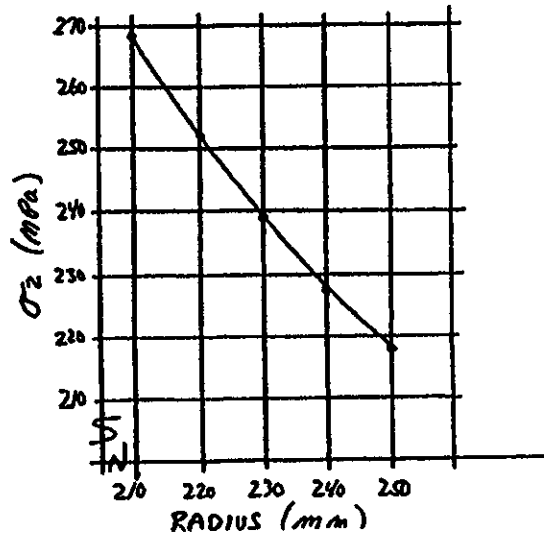
$$\sigma_{\text{MAX}} = \sigma_{2 \text{ MAX}} = \frac{p(b^3 + 2a^3)}{2(b^3 - a^3)} = p \frac{b^3 + 2a^3}{b^3 - a^3} = \frac{2(232 \text{ MPa})(250^3 - 210^3)}{[250^3 + 2(210^3)]}$$

$$\sigma_{\text{MAX}} = 86.8 \text{ MPa}$$

15-20 FROM PROB. 15-19: THICK WALL:  $b = 250 \text{ mm}$ ,  $a = 210 \text{ mm}$

$$\sigma_1 = \sigma_2 = \frac{p a^2 (b^2 + 2r^2)}{2r^2 (b^2 - a^2)} = \frac{p a^2}{b^2 - a^2} \times \frac{(b^2 + 2r^2)}{r^2} = \frac{100 \text{ MPa}(210^2)}{2(250^2 - 210^2)} \times \frac{(b^2 + 2r^2)}{r^2}$$

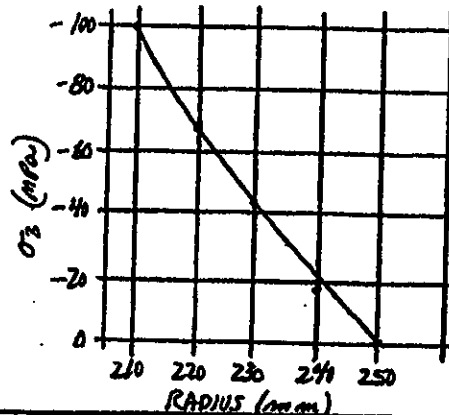
$r$	$\sigma_2$ (MPa) TANGENTIAL
210	268 INNER SURFACE
215	260
220	252
225	245
230	239
235	233
240	228
245	223
250	218 OUTER SURFACE



15-21 FROM PROB. 15-17: THICK WALL SPHERE:  $b=250 \text{ mm}$ ,  $a=210 \text{ mm}$

$$\sigma_r = \frac{-Pa^3(b^3 - r^3)}{r^3(b^3 - a^3)} = \frac{-Pa^3}{b^3 - a^3} \times \frac{(b^3 - r^3)}{r^3} = \frac{-(100 \text{ MPa})(210^3)}{(250^3 - 210^3)} \times \frac{(250^3 - r^3)}{r^3}$$

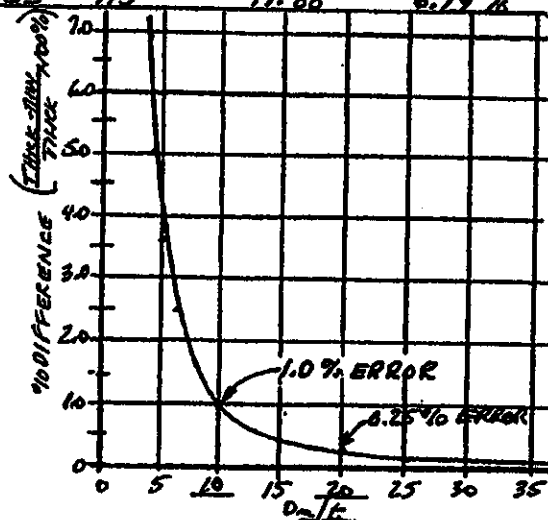
$r$	$\sigma_r$ (MPa) RADIAL
210	-100 INNER SURFACE
215	-83.3
220	-68.0
225	-54.1
230	-41.4
235	-29.7
240	-19.0
245	-9.1
250	0.0 OUTER SURFACE



15-22	$D_o = 400 \text{ mm}$		THIN		THICK	
$t$ (mm)	$D_m = D_o - t$ mm	$D_m/t$	$\sigma_r = \frac{PD_m}{2t}$	$a = \frac{D_o - 2t}{2}$	$\sigma_r = \frac{P(b^3 + a^3)}{b^3 - a^3}$	% = $\frac{\text{THICK} - \text{THIN}}{\text{THICK}} \times 100\%$
5	395	79	395 MPa	195	395.06 MPa	0.015%
15	385	25.67	128.3	185	128.53	0.179%
19.05	380.95	20.0	100.0	180.95	100.25	0.25%
25	375	15.0	75.0	175	75.33	0.44%
35	365	10.43	52.14	165	52.62	0.91%
36.36	363.63	10.0	50.0	163.63	50.50	0.99%
45	355	7.89	39.44	155	40.02	1.60%
55	345	6.27	31.36	145	32.16	2.49%
65	335	5.15	25.77	135	26.74	3.63%
75	325	4.33	21.67	125	22.82	5.04%
85	315	3.71	18.53	115	19.88	6.79%

#### NOTES:

- $D_m/t = 20.0$  ADDED TO SHOW THAT ARBITRARY DIVISION BETWEEN THICK AND THIN WALL CYLINDERS RESULTS IN LESS THAN 0.25% ERROR FOR THIN-WALLED THEORY.
- FOR  $D_m/t > 10$ , ERROR IS LESS THAN 1.0%
- ERROR INCREASES RAPIDLY FOR  $D_m/t < 10$ .





15-23  $D_o = 400 \text{ mm}$ ;  $D_i = 325 \text{ mm}$ ;  $D_m = 362.5 \text{ mm}$ ;  $D_m/t = 9.67$  - THICK  
 $t = 37.5 \text{ mm}$ ;  $b = D_o/2 = 200 \text{ mm}$ ;  $a = D_i/2 = 162.5 \text{ mm}$   
 $\sigma_z = \frac{p a^3 (b^3 + 2a^3)}{2b^3 (b^3 - a^3)}$ ;  $p = 10.0 \text{ MPa}$

$r \text{ (mm)}$	$\sigma_z \text{ (MPa)}$	$\sigma_3 \text{ (MPa)}$	PROB. 15-24
$162.5 = a$	22.85	10.0	
170.0	20.99	7.27	
177.5	19.84	4.98	
185.0	18.88	3.05	
192.5	18.16	1.41	
$200.0 = b$	17.35	0.00	

15-24 SAME DATA AS 15-23:  $\sigma_3 = \frac{-p a^3 (b^3 - r^3)}{r^3 (b^3 - a^3)}$

15-25 SCHEDULE 40 PIPE: APP. A-12:  $D_m = (D_o + D_i)/2$

NOM. SIZE	$D_m$ (IN)	$t$ (IN)	$D_m/t$		NOM. SIZE	$D_m$ (IN)	$t$ (IN)	$D_m/t$
1/8	.337	.068	4.96	THICK	3	3.284	.216	15.20 THICK
1/4	.452	.088	5.14		3 1/2	3.714	.226	16.70
3/8	.584	.091	6.42		4	4.263	.237	17.99
1/2	.731	.109	6.71		5	5.305	.258	20.56 THIN
3/4	.937	.113	8.29		6	6.345	.280	22.66
1	1.182	.133	8.89		8	8.303	.322	25.77
1 1/4	1.520	.140	10.86		10	10.385	.365	28.45
1 1/2	1.755	.145	12.10		12	12.344	.406	30.40
2	2.221	.154	14.42		16	15.50	.500	31.00
2 1/2	2.672	.203	13.16		18	17.438	.562	31.03

16-1 (a) FIG. 16-7 (a) ASD, GR1 RIVETS, A36 PLATE.

$$F_s = T_u \cdot A_s = (17500 \text{ LB/IN}^2)(2)(\pi(0.5 \text{ IN})^2/4) = 6872 \text{ LB} = \text{ALLOWABLE}$$

$$F_b = \sigma_{ba} \cdot A_b = 1.2 (58000)(2)(0.5)(0.375) = 26100 \text{ LB}$$

$$F_t = \sigma_{ta} \cdot A_t = 0.6(36000)[3 - 2(0.5 + 0.063)](0.375) = 15179 \text{ LB}$$

16-1 (b) FIG. 16-7 (b)

$$F_s = (17500)(3)(\pi(0.375)^2/4) = 5798 \text{ LB} = \text{FAIL}$$

$$F_b = (1.20)(58000)(3)(0.375)(0.375) = 29363 \text{ LB}$$

$$F_t = (0.6)(36000)[3 - 3(0.375 + 0.063)](0.375) = 13657 \text{ LB}$$

16-1 (c) FIG. 16-7 (c)

$$F_s = (17500)(4)(\pi(0.375)^2/4) = 7732 \text{ LB} = \text{FAIL}$$

$$F_b = (1.20)(58000)(2)(0.375)(0.375) = 19525 \text{ LB}$$

$$F_t = (0.6)(36000)[3 - 2(0.375 + 0.063)](0.375) = 17204 \text{ LB}$$

16-1 (d) FIG. 16-7 (d)

$$\text{SAME AS 16-1 (c)}; F_s = 7732 \text{ LB} = \text{FAIL}$$

16-2 (a) FIG. 16-10 (a) ASD, GR2 RIVETS, A242 PLATE

$$F_s = (22000)(4)(\pi(0.5)^2/4) = 17279 \text{ LB} = \text{FAIL}$$

$$F_b = (1.20)(70000)(4)(0.5)(0.5) = 84000 \text{ LB}$$

$$F_t = (0.6)(50000)[4 - 2(0.5 + 0.063)](0.5) = 43110 \text{ LB}$$

16-2 (b) FIG. 16-10 (b)

$$F_s = (22000)(6)(2)(\pi(0.375)^2/4) = 29158 \text{ LB} = \text{FAIL}$$

$$F_b = (1.20)(70000)(6)(0.375)(0.50) = 94500 \text{ LB}$$

$$F_t = (0.6)(50000)[4 - 3(0.375 + 0.063)](0.50) = 40290 \text{ LB}$$

16-2 (c) FIG. 16-10 (c)

$$F_s = (22000)(4)(2)(\pi(0.5)^2/4) = 34558 \text{ LB} = \text{FAIL}$$

$$F_b = (1.20)(70000)(4)(0.5)(0.5) = 84000 \text{ LB}$$

$$F_t = (0.6)(50000)[4 - 2(0.5 + 0.063)](0.50) = 43110 \text{ LB}$$

16-2 (d) FIG. 16-10 (d)

$$F_s = (22000)(2)(2)(\pi(0.75)^2/4) = 38877 \text{ LB}$$

$$F_b = (0.20)(70000)(2)(0.75)(0.5) = 63000 \text{ LB}$$

$$F_t = (0.6)(50000)[4 - 2(0.75 + 0.063)](0.5) = 35610 \text{ LB} = \text{FAIL}$$

16-3 (a) Fig. 16-9 (a) A325 BOLTS; A441 STEEL; BEARING TYPE

$$F_s = (30000)(2)(\pi(0.5)^2/4) = 11780 \text{ LB} = F_{\text{ALLOW.}}$$

$$F_b = (1.20)(70000)(2)(0.50)(0.375) = 31500 \text{ LB}$$

$$F_t = 0.6(50000)(3.0 - 2(0.5 + 0.063))(0.375) = 21083 \text{ LB}$$

16-3 (b) Fig. 16-9 (b)

$$F_s = (30000)(3)(\pi(0.375)^2/4) = 9940 = F_{\text{ALLOW.}}$$

$$F_b = (1.20)(70000)(3)(0.375)(0.375) = 35450 \text{ LB}$$

$$F_t = 0.6(50000)(3.0 - 3(0.375 + 0.063))(0.375) = 18968 \text{ LB}$$

16-3 (c) Fig. 16-9 (c)

$$F_s = (30000)(2)(2)(\pi(0.375)^2/4) = 13253 \text{ LB} = F_{\text{ALLOW.}}$$

$$F_b = (1.20)(70000)(2)(0.375)(0.375) = 23600 \text{ LB}$$

$$F_t = 0.6(50000)(3.0 - 2(0.375 + 0.063))(0.375) = 23895 \text{ LB}$$

16-3 (d) Fig. 16-9 (d) SAME AS 16-3 (c):  $F_s = 13253 \text{ LB} = F_{\text{ALLOW.}}$

16-4 (a) Fig. 16-10 (a) A490 BOLTS; A514 STEEL; FRICTION TYPE

$$F_s = (22000)(4)(\pi(0.5)^2/4) = 17279 \text{ LB} = F_{\text{ALLOW.}}$$

$$F_t = 0.6(100000)[4.0 - 2(0.50 + 0.063)](0.50) = 86220 \text{ LB}$$

BEARING STRESS NOT CONSIDERED IN FRICTION-TYPE JOINT.

16-4 (b) Fig. 16-10 (b)

$$F_s = (22000)(6)(2)(\pi(0.375)^2/4) = 29158 \text{ LB} = F_{\text{ALLOW.}}$$

$$F_t = 0.6(100000)[4.0 - 3(0.375 + 0.063)](0.5) = 80580 \text{ LB}$$

16-4 (c) Fig. 16-10 (c)

$$F_s = (22000)(4)(2)(\pi(0.5)^2/4) = 34538 \text{ LB} = F_{\text{ALLOW.}}$$

$$F_t = 0.6(100000)[4.0 - 2(0.5 + 0.063)](0.5) = 86220 \text{ LB}$$

16-4 (d) Fig. 16-10 (d)

$$F_s = (22000)(2)(2)(\pi(0.75)^2/4) = 38877 \text{ LB} = F_{\text{ALLOW.}}$$

$$F_t = 0.6(100000)[4.0 - 2(0.75 + 0.063)](0.5) = 71220 \text{ LB}$$

16-5

$$P = 6500 \text{ lb} ; M = 6500 \text{ lb} (48 \text{ in}) = 312,000 \text{ lb} \cdot \text{in}$$

$$R_p = P/6 = 1083 \text{ lb}$$

$$\sum (x^2 + y^2) = 6(2.0^2) + 4(2.5^2) = 49 \text{ in}^2$$

FOR UPPER RIGHT BOLT

$$R_{ix} = \frac{M y_i}{\sum (x^2 + y^2)} = \frac{(312,000 \text{ lb} \cdot \text{in})(2.5 \text{ in})}{49 \text{ in}^2} = 15,918 \text{ lb}$$

$$R_{iy} = \frac{M x_i}{\sum (x^2 + y^2)} = \frac{(312,000 \text{ lb} \cdot \text{in})(2.0 \text{ in})}{49 \text{ in}^2} = 12,735 \text{ lb}$$

$$R_{R1} = \sqrt{15,918^2 + (1083 + 12,735)^2} = 21,079 \text{ lb}$$

$$\text{REQ'D } A = R_{R1} / \tau_A = 21,079 \text{ lb} / 17,500 \text{ psi} = 1.205 \text{ in}^2$$

$$\text{REQ'D DIA} = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(1.205)}{\pi}} = 1.24 \text{ in} - \text{USE } 1\frac{1}{4} \text{ in.}$$

16-6

POSSIBLE SOLUTION -

$$P_x = P \sin 30^\circ = 13.0 \text{ kN}$$

$$P_y = P \cos 30^\circ = 22.5 \text{ kN}$$

$$M = (P_x)(800) = 19125 \text{ kN} \cdot \text{mm}$$

12-ASTM A325 BOLTS -

FRICTION TYPE CONNECTION

ON BOLT ①

$$\frac{P_x}{12} = \frac{13.0 \text{ kN}}{12} = 1.083 \text{ kN} \leftarrow$$

$$\frac{P_y}{12} = \frac{22.5 \text{ kN}}{12} = 1.875 \text{ kN} \uparrow$$

FORCES DUE TO MOMENT:

$$\sum (x^2 + y^2) = 6(50)^2 + 6(100)^2 + 6(100)^2 = 155,000 \text{ mm}^2$$

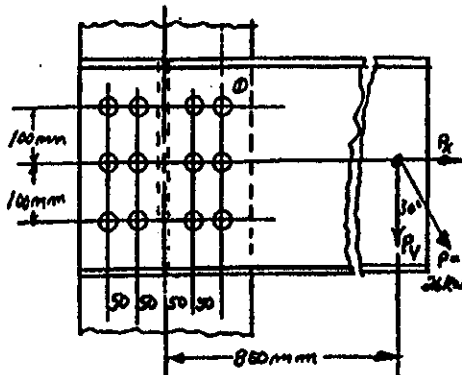
$$R_{ix} = \frac{M y_i}{\sum (x^2 + y^2)} = \frac{19,125 \text{ kN} \cdot \text{mm}(100 \text{ mm})}{155,000 \text{ mm}^2} = 12.34 \text{ kN} \leftarrow$$

$$R_{iy} = \frac{M x_i}{\sum (x^2 + y^2)} = \frac{19,125 \text{ kN} \cdot \text{mm}(50 \text{ mm})}{155,000 \text{ mm}^2} = 12.34 \text{ kN} \uparrow$$

$$R_1 = \left[ (1.083 + 12.34)^2 + (1.875 + 12.34)^2 \right]^{1/2} = 19.55 \text{ kN}$$

$$\text{REQ'D } A = \frac{R_1}{\tau_u} = \frac{19,550 \text{ N}}{121 \text{ N/mm}^2} = 162 \text{ mm}^2 = \pi D^2/4$$

$$D = \sqrt{4A/\pi} = \sqrt{4(162)/\pi} = 14.3 \text{ mm} - \text{USE } 16 \text{ mm}$$



16-7 FIG. 16-9(a)  
 $P = T_u L t = (8000 \text{ lb/in}^2)(6 \text{ in})(0.707)(0.3125 \text{ in}) = \underline{23860 \text{ lb - ON WELDS}}$

ON STRAP:  $P = \sigma_{tu}(2)(t) = (0.6)(36000 \text{ lb/in}^2)(3 \text{ in})(0.3125 \text{ in}) = 24300 \text{ lb}$   
 WELDS GOVERN JOINT STRENGTH

16-8 FIG. 16-10(c)  
 $P = T_u L t = (21000 \text{ lb/in}^2)(8 \text{ in})(0.707)(0.250 \text{ in}) = \underline{29700 \text{ lb - ON WELDS}}$   
 ON STRAP:  $P = (0.6)(50000 \text{ lb/in}^2)(0.5 \text{ in})(4 \text{ in}) = 60000 \text{ lb}$

THE SOLUTIONS SHOWN BELOW FOR PROBLEMS 16-9, 16-10, AND 16-11 ARE JUST SAMPLES OF MANY POSSIBLE SOLUTIONS. THE GENERAL CONFIGURATION AND NUMBER OF FASTENERS SHOWN IN FIG. 16-1 ARE USED, BUT OTHERS COULD BE USED.

16-9  
 FORCE ON JOINT  $= \left(\frac{1}{4}\right)(54.4 \times 10^3 \text{ kg})(9.81 \text{ m/s}^2) = 133400 \text{ N}$   
 USE 6 RIVETS - DESIGN FOR SHEAR STRENGTH  
 $F_s = T_u A_s$  - USE ASTM A502, GRADE 1 RIVETS  
 REQ'D  $A_s = F_s / T_u = 133400 \text{ N} / (21 \text{ N/mm}^2) = 1102 \text{ mm}^2$   
 DOUBLE SHEAR:  $A_s = (6)(2)(\pi D^2 / 4) = 3\pi D^2$   
 REQ'D DIA.  $= \sqrt{A_s / (3\pi)} = \sqrt{1102 \text{ mm}^2 / 3\pi} = 10.8 \text{ mm}$  - USE 12 mm

CHECK BEARING ON WEB OF TEE

$F_b = \sigma_{tu} A_b = 150(400 \text{ N/mm}^2)(6)(12 \text{ mm})(10.6 \text{ mm}) = 458 \text{ kN OK}$

FIND REQ'D THICKNESS OF STRAP,  $t$

$F_t = \sigma_{tu} A_t = \sigma_{tu}(n r - n D_r) PC$

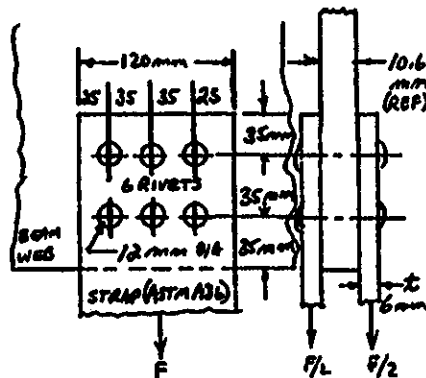
$t = \frac{F_t}{\sigma_{tu}(n r - n D_r)(2)}$

USE  $D_r = 12 + 2 = 14 \text{ mm}$

$t = \frac{133400 \text{ N}}{0.4248 \text{ N/mm}^2(120 - 3(14))(2)} = 5.75 \text{ mm}$

USE TWO STRAPS, 6 mm THICK EACH

USE ASTM A36 STEEL



16-10

FORCE ON JOINT = 133.4 kN (FROM PROB. 16-9)

USE 4 BOLTS - ASTM A490 - BEARING TYPE CONNECTION

$$\text{REQ'D. } A_s = F_s / T_u = 133400 \text{ N} / (152 \text{ N/mm}^2) = 878 \text{ mm}^2$$

$$A_s = (2)(4) \pi D^2 / 4 = 2 \pi D^2$$

$$\text{REQ'D. DIA} = \sqrt{A_s / 2\pi} = \sqrt{878 \text{ mm}^2 / 2\pi} = 11.8 \text{ mm} \quad \text{USE 12 mm}$$

CHECK BEARING ON STRAPS - TWO, 8 mm THICK, A36 STEEL

$$F_b = \sigma_{bL} A_b = 150 (400 \text{ N/mm}^2) (12 \text{ mm}) (8 \text{ mm}) (2)(4) = 461 \text{ kN} - \text{OK}$$

$$\text{USE } D_H = 12 + 2 = 14 \text{ mm}$$

$$F_t = \sigma_{tL} A_t = 0.6 (248 \text{ N/mm}^2) (40 - 2(14)) (2)(8 \text{ mm}) = 219 \text{ kN} - \text{OK}$$

TAB DESIGN - MAKE TAB 10 mm THICK TO MATCH BEAM WEB

REQ'D WIDTH,  $w$ , USING A36 STEEL

$$A_t = F_t / \sigma_{tL} = \frac{133400 \text{ N}}{0.6 (248 \text{ N/mm}^2)} = 896.5 \text{ mm}^2$$

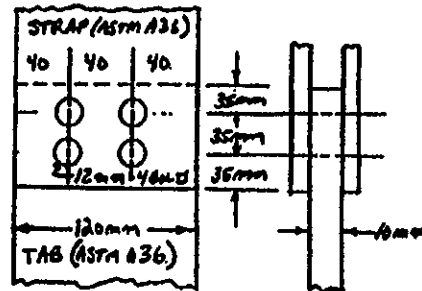
$$A_t = (w - 2D_H) t \quad \text{THEN } w = \frac{A_t}{t} + 2D_H$$

$$w = \frac{896.5 \text{ mm}^2}{10 \text{ mm}} + 2(14 \text{ mm}) = 118 \text{ mm} \quad \text{OK} - w = 120 \text{ mm}$$

CHECK BEARING ON TAB

$$F_b = \sigma_{bL} A_b = 155 S_u A_b$$

$$F_b = 15 (400) (4) (12) (10) = 288 \text{ kN} \quad \text{OK}$$



16-11

FORCE ON ONE TAB = 133.4 kN (FROM PROB 16-9)

TAB - A36 STEEL, 120 mm WIDE BY 10 mm THICK (PROB. 16-10)

WELD WITH E60 ELECTRODE

$$T_u = 124 \text{ MPa}$$

$$F = T_u L t$$

$$\text{REQ'D. } L = \frac{F}{T_u t}$$

USE 6 mm FILLET WELD

$$t = 0.707 (6 \text{ mm}) = 4.24 \text{ mm}$$

$$L = \frac{133400 \text{ N}}{(124 \text{ N/mm}^2) (4.24 \text{ mm})} = 254 \text{ mm}$$

$$\text{BUT } L = 2a + 120 \text{ mm}$$

$$\text{REQ'D. } a = \frac{L - 120 \text{ mm}}{2} = \frac{254 - 120}{2} = 67 \text{ mm} - \text{USE } 70 \text{ mm} = a$$

